THE STRUCTURAL DESIGN OF
Tall and Special Buildings

Optimum design of Tuned Mass Dampers for different earthquake ground motion parameters and models

Journal:	*The Structural Design of Tall and Special Buildings*
Manuscript ID	TAL-18-0093.R2
Wiley - Manuscript type:	Research Article
Date Submitted by the Author:	25-Jul-2019

Complete List of Authors:	Greco, Rita; Polytechnic of Bari Morga, Mariantonietta; Anglia Ruskin University Department of Engineering and the Built Environment

Keywords:	Tuned Mass Damper, non-stationary earthquake ground motion model, optimum design, covariance analysis, soil type, soil damping ratio

Abstract:	Tuned Mass Dampers are used for passive control of vibrations in civil structures subject to seismic and wind actions. Their efficiency depends on selection of their mechanical properties in relation to system and excitation characteristics. This paper proposes an optimum design strategy of single Tuned Mass Dampers to control vibrations of structures excited by earthquakes. The main purpose of the paper is to investigate the influence of the time modulation of earthquake excitation upon the optimal Tuned Mass Dampers design parameters: frequency and damping ratio. The study is based on numerical analyses carried out with stochastic models for earthquakes: a simple filtered White Noise model and two time modulated filtered White Noise models. The numerical analyses are carried out to solve an optimization problem with a performance index defined by the reduction of the standard deviation of either the structure displacement or its inertial acceleration as Objective Function. To complete the work the influence of the bandwidth excitation over the values of the optimal Tuned Mass Damper parameters is investigated, as well the optimum mass ratio and the structure frequency.

Optimum design of Tuned Mass Dampers for different earthquake ground motion parameters and models

Rita Greco[1], **Mariantonietta Morga**[2]

[1] **Rita Greco,** Ph.D
Associate Professor
Dipartimento di Ingegneria Civile, Ambientale, del Territorio, Edile e di Chimica
Technical University of Bari
via Orabona 4
70126 Bari, Italy
rita.greco@poliba.it;
phone +39 080 5963876; fax +39 080 5963414

[2] **Mariantonietta Morga**, Ph.D.
Senior lecturer,
School of Engineering and the Built Environment
Faculty of Science and Engineering
Anglia Ruskin University
Bishop Hall Lane
CM1 1SQ, Chelmsford, United Kingdom
mariantonietta.morga@anglia.ac.uk; mariantonietta.morga@gmail.com
phone +44 1245 683598;

Corresponding author: Rita Greco

“No funding Information available”

“No Acknowledgement”

Highligths

- The influence of ground motion models on TMD optimum design parameters is examined.
- TMD damping ratio and tuning ratio are defined as TMD optimum design parameters.
- Both stationary and non-stationary ground motions models are applied.
- The role of the ground motion models bandwidth in TMD optimum design is analysed.
- The role of TMD-structure mass ratio is studied for different earthquake models.

Abstract

Tuned Mass Dampers are frequently used for passive control of vibrations in civil structures subject to seismic and wind actions. Their efficiency depends on selection of their mechanical properties in relation to main system and excitation characteristics. This paper proposes an optimum design strategy of single Tuned Mass Dampers to control vibrations of principal mode of structures excited by earthquake ground motion. The main purpose of the paper is to investigate the influence of the time modulation of earthquake excitation upon the optimal

Tuned Mass Dampers design parameters: frequency and damping ratio. The study is based on numerical analyses carried out with different stochastic models for earthquakes: a simple filtered White Noise model and two time modulated filtered White Noise models. The numerical analyses are carried out to solve an optimization problem with a performance index defined by the reduction of the standard deviation of either the structure displacement or its inertial acceleration as Objective Function. To complete the work the influence of the bandwidth excitation over the values of the optimal Tuned Mass Damper parameters is investigated, as well the optimum mass ratio and the structure frequency. The results of the numeral analyses carried out infer the earthquake excitation characteristics, including its modulation in time domain, highly affect the optimum TMD design parameters values.

Keywords: Tuned Mass Damper, non-stationary earthquake ground motion model, optimum design, covariance analysis, passive protection, supplemental damping.

1 Introduction

The protection of new and existing structures had been a challenge tackled by scientists worldwide in the last few decades. Several technological solutions had been proposed to respond to it. They could be distinguished in energy dissipation systems and vibration mitigation devices and also in passive, semi-active and active systems. For sake of brevity, only a short introduction about the passive system is presented here.

The energy dissipation systems includes among the others hysteric devices and base isolation systems. Hysteretic devices increase strength and stiffness to the structure and hence they reduce the total plastic deformation of the structure measured in terms of inter-storey drift. The reduction of the plastic deformation of structure can be combined with reduction of the total floor acceleration by adding viscous, viscoelastic, friction or elastomeric devices in the frame structure [1, 2, 3, 4, 5]. The optimal effect of the combined action of additional damping devices together with energy dissipation ones is obtained through an integrated design of them and the frame structure. In [1] an example of this kind of design is proposed to guarantee the minimum damage. The work proposed by Apostolakis and Dargush offers an optimal design of a bracing system integrating yielding metallic and/or friction dampers to dissipate the energy transmitted to the structure by the ground shaking [2]. Other studies proposed the use of hysteretic metal shear panels to dissipate the earthquake energy in structures [6, 7]. In other studies, optimal design of either simply shaped steel bracing systems is proposed [8, 9, 10] or heretical metal connections linking frame elements [11]. Other scientists preferred introducing a fully stressed design algorithm to define the additional viscous damping needed for the structure to dissipate the seismic energy in the structure [12].

The aim of base isolation systems is the reduction of energy transmitted from the foundation soil to the structure. This effect is obtained by filtering the frequencies of the seismic signal, i.e. by damping those in the range of structure eigenfrequencies. The optimal design of those systems are intended to define the value of their design parameters that allow minimizing the earthquake energy transmitted by the foundation soil to the structure. An example of this optimal design is presented in [13]. Instead of using an energetic approach, other scientists proposed to minimize the structural acceleration as objective of the optimal design of a parameter of the base isolation system; in particular, Chung et al. [14], optimized the frictional coefficient of a base isolation system. The more complex design of frictional base isolation system is proposed in [15], where the two conflicting objectives of first passage of probabilities of the device and inter-storey displacements are counterpointed to define the optimal friction coefficient of the device and the radius of curvature of the friction pendulum system.

Tuned Mass Dampers (TMD) are other widespread devices used to mitigate the structure vibrations. They achieve this mitigation without changing the structure characteristics like the base isolation and hysteretic devices. Since they had been first proposed and patented in 1909 by Frahm [16], TMD became widely used devices to control structural vibrations through a passive strategy. They were and are applied to both civil structures and mechanical systems.

In the first case, they are used mainly to mitigate vibrations due to wind and earthquakes. They are also applied to control man-induced vibrations, in particular to reduce traffic vibrations in bridges.

To maximize their efficiency in vibration control TMD mechanical parameters should be selected by optimum design. For this reason, many studies on this subject were proposed in the literature during the last century. However, the effectiveness of this kind of dampers in controlling vibrations induced in structures by ground motions is still a matter of discussion, as pointed out in [17] and [18]. The first investigation on TMD optimum design was carried out by Ormondroyd and Den Hartog in 1928, as reported in [19]. That pioneering study was followed by many others. A comprehensive survey about optimization methods for TMD proposed until the nineties was presented by Sun et al. [20]. A first attempt to optimal design of those devises was presented in [19]: it was based on formulas estimating the optimal values of the TMD characteristics in closed form. Some recent studies, such as those proposed in [21] and [22], adopted also such kind of approach. Other studies suggested numerical methods to estimate the TMD optimum design parameters [23]. In particular the researches presented in [24], [25], [26], [27] and [28] use nonlinear programming techniques to optimize the TMD characteristics to make effective the control of random vibrations of the main system. Complex systems composed by multiple masses mitigating the structure vibrations were studied. Those systems called Multiple Tuned Mass Dampers (MTMD) had been studied by several scientists in the past decades. For instance, Reed and Park [29] used a numerical method to assess the performance of uniformly and linearly distributed Multiple Tuned Mass Dampers (MTMDs). Other studies proposed methods to estimate the optimum design parameters of TMDs in case of uncertain design variables. Among the others, the study presented in [30] suggested a robust design-based method to optimize a TMD applied to a MDOF system. In it, the uncertainty affecting both main system and excitation characteristics was taken into account. Instead, in another study Adam and Furtmüller assessed the effect of the uncertainty of the main system parameters on the TMD performance by investigating the mistuning of the damper [17].

Besides the method applied, the studies about optimum design of TMDs are different because of the model of the mechanical system or structure to protect and the source of the vibrations to control. In [31] the optimum parameters of a single and multiple TMD were calculated through a parametric study. Their authors investigated the effect of a single TMD applied to a structure modelled as either a SDOF system or a Multi-Degrees of Freedom (MDOF) system to mitigate harmonic and seismic excitations. In other studies, the external excitation applied to the main system is either a harmonic force characterized by a well defined frequency [28-29] or random vibrations, such as White Noise (WN) [17, 23, 32], filtered WN [27] or seismic load [2]. In particular, in studies proposing optimum design of TMD mitigating earthquake induced vibrations in civil structures the dynamic excitation is defined through either artificial or real earthquake records. The second approach was chosen by a few researchers [17, 29, 34-35]; whilst the researchers, who opted for the first approach, chose either a stationary filtered WN model [23, 25, 26, 36] or an evolutionary filtered WN model [24]. Finally, it is worth of mention that some authors investigated also the optimal TMD parameters to mitigate the ground motion excitation including the Soil-Structure Interaction (SSI) effect [37], while others [38] probed the SSI effect on TMD performance by frequency domain analyses.

The main aim of this study is to investigate the influence of the time modulation of the earthquake ground motion signal on the optimum design of single TMDs. Secondly, the effect of the soil characteristics on the optimal design parameter of TMDs is studied. More in detail, those objectives are accomplished by modelling earthquake ground motion as stationary and non-stationary stochastic processes with different bandwidths. Moreover, to strength the conclusions drawn about the influence of time modulation of seismic excitation on TMD effectiveness the results obtained using two different modulation functions in the seismic load model are compared. As this study is directed to the optimal design of TMDs mitigating the vibrations of the dominant mode of a structure, the structure is modelled as a damped SDOF system and the TMD as a mass connected to it by a spring and a damper. Thus, the combined system acts as two degrees of freedom system. Such model is used to carry out numerical analyses, in which two different TMD performance indices are used as Objective Functions

(OF) of the optimization problem. The first one is the ratio of the displacement of the main system (i.e. the SDOF system modelling the structure) protected by the TMD and the displacement of the main system not protected by the TMD. The second one is the ratio of inertial acceleration of the main system protected by TMD and the acceleration of the main system in case it is not protected by the TMD. Finally, the study presents also the results of numerical analyses assessing the effects of the mass ratio on the optimum design of TMDs.

2 System dynamics equations and covariance analysis

2.1 Earthquake model

As said in the introduction, a TMD is composed by a mass connected to the system to protect by a spring and a damper (see Figure 1). It mitigates the vibrations that can ultimately damage the system by vibrating at almost the same frequency, but out of phase. In civil engineering, the systems to protect are mostly buildings, bridges, towers, offshore platforms, etc. Those structures can be modelled as MDOF systems characterized by several vibration modes. To mitigate their vibrations multiple TMDs can be used. This is the strategy adopted by Lee et al. in [23]. However, generally civil structures are characterized by a dominant vibration mode, so their model can be reduced to a SDOF system. In order to mitigate their vibrations caused by dynamic loads either simple TMDs or multiple TMDs can be used, as proposed by Park and Reed in [29]. Moreover, most of the civil structures have a non-linear hysteretic behaviour; therefore, some researchers took into account this aspect in the methods for TMD optimization they presented, as [28].

This study proposes an optimum design method for a linear TMD to reduce vibrations of the dominant mode of a structure excited by a dynamic non-stationary stochastic force. The structure is assumed to have a linear elastic behaviour, so a linear elastic SDOF system is used to model its dominant mode and the hysteretic behaviour of the structure is neglected. The optimal design of the TMD presented in this work is intended to minimize the structural dynamic response amplitude and avoid structural large displacements and inelastic deformations.

The dynamic equations of the system composed by the SDOF oscillator modelling the structure, also called main system in the following sections, and the single TMD (Figure 1) are:

$$\begin{cases} \ddot{x}_T(t) + 2\omega_T\xi_T(\dot{x}_T(t) - \dot{x}_S(t)) + \omega_T^2(x_T(t) - x_S(t)) = -\ddot{x}_g(t) \\ \ddot{x}_S(t) + \gamma_{TMD}2\xi_T\omega_T(\dot{x}_S(t) - \dot{x}_T(t)) + 2\omega_S\xi_S\dot{x}_S(t) - \gamma_{TMD}\omega_T^2(x_S(t) - x_T(t)) + \omega_S^2x_S(t) = -\ddot{x}_g(t) \end{cases}, \tag{1}$$

where $\omega_S = \sqrt{k_S/m_S}$ is the oscillator vibration frequency, $\xi_S = c_S/(2\sqrt{m_S k_S})$ its damping ratio, $\omega_T = \sqrt{k_T/m_T}$ the TMD vibration frequency and $\xi_T = c_T/(2\sqrt{m_T k_T})$ its damping ratio. Moreover, in equation (1) $\ddot{x}_S$, $\dot{x}_S$ and x_S are respectively acceleration, velocity and displacement of the main system, while $\ddot{x}_T$, $\dot{x}_T$ and x_T indicate acceleration, velocity and displacement of the TMD and γ_{TMD} is the mass ratio, i.e. the ratio of TMD mass m_T to the main system mass m_S ($\gamma_{TMD} = m_T/m_S$). Finally, $-\ddot{x}_g$ indicates the seismic acceleration and in this study is defined by filtering a zero-mean Gaussian WN process with a second order linear filter, known as Kanai-Tajimi (KT) filter [39-40]:

$$\begin{cases} \ddot{x}_f(t) + 2\xi_f\omega_f\dot{x}_f(t) + \omega_f^2 x_f(t) = -w(t)V(t) \\ \ddot{x}_g(t) = \ddot{x}_f(t) + w(t)V(t) = -(2\xi_f\omega_f\dot{x}_f(t) + \omega_f^2 x_f(t)) \end{cases}. \quad (2)$$

In this equation, x_f, $\dot{x}_f$ and $\ddot{x}_f$ are respectively the displacement, velocity and acceleration of the SDOF system defining the KT filter. This filter models the effects of the soil layer between the bedrock and the ground level. It is defined by four parameters. The first is $w(t)$, i.e. a stationary zero-mean Gaussian WN process representing the seismic excitation at the bedrock and characterized by Power Spectral Density S_0. The second and third parameters are ω_f and ξ_f that are respectively the KT filter frequency and damping ratio. They model the resonance bandwidth limitation effect due to the soil layer between the bedrock and the ground level. The fourth parameter is the modulation function $V(t)$ that defines the intensity variation of the random signal within its duration. It has constant unitary value for the stationary KT model, while it is a time dependent function for the non-stationary KT model. In this paper, two different modulation functions are applied: the deterministic modulation function proposed by Jennings et at. [41] and the deterministic exponential modulation function proposed by Hsu and Bernard [42]. The Jennings et al. modulation function is

$$V(t) = \begin{cases} \left(\frac{t}{t_1}\right)^2 & t < t_1 \\ 1 & t_1 \le t \le t_2 \\ e^{-\beta(t - t_2)} & t > t_2 \end{cases}, \quad (3)$$

where the parameters t_1 and t_2 indicate the beginning and the end of the “strong shaking” phase, whereas β controls the shape of the decaying end of the function [41]; whilst the exponential modulation function is

$$V(t) = \alpha t e^{-\beta t} \qquad \beta > 0, \quad (4)$$

Where $\beta = 1/t_m$ and $\alpha = e/t_m$ [42].

To compare the effect of the two modulation functions on the TMD optimal design their effect must be comparable through a parameter. For this study, the selected parameter is Arias intensity

$$I_A = \frac{\pi}{2g}\int_0^{t_f}\ddot{x}_g^2(t)dt, \quad (5)$$

as it is related to the ground motion energy. The expected value of the Arias intensity for the two non-stationary earthquake models must be the same for ground motions with the same total energy. The expected value of the Arias intensity is

$$\mu[I_A] = \frac{\pi}{2g}\int_0^{-t_f}\langle\ddot{x}_g^2(t)\rangle dt = \frac{\pi}{2g}\int_0^{t_f}V^2(t)dt\int_{-\infty}^{+\infty}S_{\ddot{x}_g}(\omega)d\omega, \quad (6)$$

where $S_{\ddot{x}_g}(\omega)$ is the power spectral density function of the ground acceleration [43] which is constant for the two non-stationary earthquake models. Thus, the integral of the squared modulation function is the element to estimate and compare to obtain earthquakes with the same energy by applying the two different modulation functions:

$$\int_0^{t_f}V_{Jen}^2(t)dt = \int_0^{t_f}{V_{exp}}^2(t)dt, \quad (7)$$

The values of the two square modulations functions were proposed in [43]; therefore, Equation (7) becomes

$$\frac{t_m e^2}{4} = \frac{t_1}{5} + t_d + \frac{1}{2\beta}, \quad (8)$$

where $t_d = t_2 - t_1$. Finally, from Equation 8 the value of the time at which the exponential modulation function exhibits the maximum value t_m is obtained

$$t_m = \frac{4}{e^2}\left(\frac{t_1}{5} + t_d + \frac{1}{2\beta}\right). \quad (9)$$

The two modulation functions are showed in Figure 2. It is clear that they spread the energy in different ways along the earthquake duration. Whilst the exponential function as a peak and a smooth decay after it; the Jennings et al,'s modulation function has a plateau and fast decay. During the plateau, the energy is distributed equally for a long time respect to the duration of the peak of the first modulation function: this indicates that such model has a stationary characteristic in that time interval. This affects the structure response and on the optimum design of the TMD.

2.2 System response covariance

Equations 1 and 2 are combined and expressed in state-space, therefore the motion equation of the system becomes

$$\dot{\mathbf{Z}}(\mathrm{t}) = \mathbf{AZ}(\mathrm{t}) + \mathbf{F}(\mathrm{t}). \tag{10}$$

In Equation (4) $\mathbf{Z}$ indicates the state-space vector

$$\mathbf{Z} = \begin{bmatrix} x_T & x_S & x_f & \dot{x}_T & \dot{x}_S & \dot{x}_f \end{bmatrix}^T, \tag{11}$$

$\mathbf{F}$ is the force vector

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -V(t)w(t) \end{bmatrix}^T, \tag{12}$$

and $\mathbf{A}$ is the state matrix

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{H}_K & \mathbf{H}_C \end{pmatrix}, \tag{13}$$

with

$$\mathbf{H}_K = \begin{pmatrix} -\omega_T^2 & \omega_T^2 & \omega_f^2 \\ \gamma_{TMD}\omega_T^2 & -\left(\gamma_{TMD}2\omega_T^2 + \omega_S^2\right) & \omega_f^2 \\ 0 & 0 & -\omega_f^2 \end{pmatrix} \tag{14}$$

and

$$\mathbf{H}_C = \begin{pmatrix} -2\xi_T\omega_T & 2\xi_T\omega_T & 2\xi_f\omega_f \\ \gamma_{TMD}2\xi_T\omega_T & -\left(\gamma_{TMD}2\xi_T\omega_T + 2\xi_S\omega_S\right) & 2\xi_f\omega_f \\ 0 & 0 & -2\xi_f\omega_f \end{pmatrix}. \tag{15}$$

In this study, all parameters of the main system and the TMD are assumed to be quantities deterministically defined, so the state-space covariance matrix $\mathbf{Q_{ZZ}} = E[\mathbf{ZZ}^\mathrm{T}]$ is obtained as solution of the Lyapunov equation:

$$\dot{\mathbf{Q}}_{\mathbf{ZZ}} = \mathbf{AQ_{ZZ}} + \mathbf{Q_{ZZ}A}^T + \mathbf{B}. \tag{16}$$

In Equation (10) $\mathbf{B}$ indicates the input matrix. It has all elements equal to zero, except the last one: $B_{66} = 2\pi S_0(V(t))^2$.

The vector of the inertial accelerations is $\ddot{\mathbf{Y}} = [\ddot{y}_T \quad \ddot{y}_S]^T$, so the system inertial acceleration covariance matrix is $\mathbf{Q}_{\ddot{Y}\ddot{Y}} = \langle \ddot{\mathbf{Y}}\ddot{\mathbf{Y}}^T \rangle = \begin{pmatrix} \sigma^2_{\ddot{y}_T} & E[\ddot{y}_T\ddot{y}_S] \\ E[\ddot{y}_S\ddot{y}_T] & \sigma^2_{\ddot{y}_S} \end{pmatrix}$. It is obtained from the equation

$$\mathbf{Q}_{\ddot{Y}\ddot{Y}} = \mathbf{D}\mathbf{Q}_{\overline{Z}\overline{Z}}\mathbf{D}^T, \qquad (17)$$

where

$$\mathbf{D} = \begin{pmatrix} -\omega_T^2 & \omega_T^2 & -2\xi_T\omega_T & 2\xi_T\omega_T \\ \gamma_{TMD}\omega_T^2 & -(\gamma_{TMD}2\omega_T^2 + \omega_S^2) & \gamma_{TMD}2\xi_T\omega_T & -(\gamma_{TMD}2\xi_T\omega_T + 2\xi_S\omega_S) \end{pmatrix}$$

and $\overline{\mathbf{Z}} = [x_T \quad x_S \quad \dot{x}_T \quad \dot{x}_S]^T$.

The response of the unprotected main system is also calculated by solving the respective Lyapunov equation in order to define the performance indices of the TMD optimum design.

For sake of clarity, the authors remind that the unprotected system is the SDOF system modelling the dominant vibration mode of the structure without TMD, while the protected system indicates the same SDOF system coupled with the TMD.

3 Statement of the optimization problem

A few researchers [44-47] developed methods for optimum design of TMDs. The damper performance in those optimum designs was defined in different ways: reduction of either displacement or inertial acceleration of the system to protect, maximization of the energy dissipation [27-30]. For flexible structures, generally, displacement is the dominant quantity that need to be controlled. On the contrary, for stiff structures the acceleration is of more concern, because it generates high inertia forces in structures that must be mitigated.

To reach one or more of those objectives a suitable set of values of TMD design variables are searched within their admissible domain $\mathbf{\Omega}$. Those variables are collected in a vector called Design Vector (DV) $\mathbf{b}$. Eventually constraint conditions must be also satisfied by the solution of the optimum design problem.

In this study an unconstrained optimization problem is defined to search the optimum value of a two-dimensional DV $\mathbf{b} = [\omega_T \quad \xi_T]^T$. The mass ratio γ_{TMD} is held constant owing to technological limits for civil structures. In fact, some studies [24-26] estimated also the optimum value of that parameter and it was found to be rather high for most of the technological solutions in civil engineering. Moreover, two different optimization criteria are considered here and for both the Objective Function (OF) is the ratio of a physical quantity of the main system response protected by a single TMD to the same quantity of unprotected main system response. In this way, a dimensionless value of the vibration mitigation produced by the TMD is assessed. The OF unitary value indicates the boundary between positive and negative effect of the TMD on reduction of the main system vibrations. The OF of the first optimization criterion is the ratio of the displacement standard deviation of the SDOF system protected by the vibration absorber and the displacement of the system without such device; therefore, the first optimization problem is so defined:

$$\min_{\mathbf{b}\in\mathbf{\Omega}} \left(\frac{\sigma_{x_S}(\mathbf{b})}{\sigma_{x_S^0}(\mathbf{b})}\right), \; with\, \mathbf{b} = [\omega_T \quad \xi_T]^T \qquad (18)$$

Thus, the displacement is used here as structure excitation or damage indicator. In the second optimization criterion the OF is the ratio of the standard deviation of the inertial acceleration of the main system equipped with a TMD to the one of the system unprotected by such damper; the second optimization problem results to be

$$\min_{\mathbf{b}\in\mathbf{\Omega}}\left(\frac{\sigma_{\ddot{y}_T}(\mathbf{b})}{\sigma_{\ddot{y}_T^0}(\mathbf{b})}\right),\ with\ \mathbf{b}=[\omega_T \quad \xi_T]^T \tag{19}$$

The inertial acceleration is used in this problem as structure excitation or damage indicator, as suggested also in [39].

4 Numerical analyses

In this section, the results of two optimization problems defined by Equations 18 and 19 are presented and discussed. In order to estimate the effect of the earthquake ground motion characteristics on TMD optimum design, non-stationarity and stationary artificial accelerograms with different bandwidths are considered. The optimization method proposed in this work uses the seismic signal parameters organized in the As said before, the bandwidth is defined by the KT filter parameters, therefore two different sets of their values are used in the numerical analyses: one for soft soil and the other for stiff soil (see Table 1). The power spectral density of the WN defining the seismic excitation at the bedrock is calculated through the formula proposed by Buchholdt [49]

$$S_0=\frac{0.141\xi_f\ddot{x}_{g\_peak}{}^2}{\omega_f\sqrt{(1+4\xi_f^2)}}. \tag{20}$$

In Equation 20 $\ddot{x}_{g\_peak}$ is the Peak Ground Acceleration (PGA) expected at the site where the structure to protect is located. The length of the seismic excitations used in the optimization is 30s and the sampling time interval is 0.01s. The use of artificial accelerograms has been preferred to real accelerograms in order to compare easily results obtained with stiff soil and soft soil conditions due to constant length of the excitation and intensity of the WN power spectral density. Indeed the selection of natural earthquakes with the same length and comparable power spectral density is an arduous task. Moreover, though natural accelerograms provide the information for the assessment of the structural response, they are strongly related to the geophysical and geological characteristics of the area where they were recorded, such as the fault mechanism, the epicentral distance and the depth of the focus, the geology and its variation on the path of waves. The most common tool used to overcome this challenge is the response spectra averaged from response spectra obtained from response of SDOF systems to a set of natural earthquakes. However, this tool has some limitations: it does not includes the uncertainty of the structural response due to the uncertainty of the real seismic input and it is not related to other energy dependent intensity measures. Thus, artificial accelerograms generated through a stochastic parametric function were preferred to natural recorded accelerograms.

Sensitivity analyses are also carried out to assess the efficiency of TMDs under different conditions. In particular, the characteristics of the main system are varied within ranges: the vibration period $T_S={}^{2\pi}/_{\omega_S}$ in the range [0.05s – 4s] and the damping ratio ξ_S in the range [0.02 - 0.10]. The range of the main system vibration period is chosen by taking into account the correspondence between the dominating frequency range of most of the earthquakes (1-10 Hz) and the range of the fundamental period of the most common structures [51]. Furthermore, the vibration period range of structures with a response spectrum independent from acceleration and displacement is from 0.05 s to 4 s. The constant value of the mass ratio used in the first numerical analyses is $\gamma_{TMD}={}^{m_T}/_{m_S}=0.05$, while also in the sensitivity analyses for mass ratio it is varied in the range [0.05 4].

The design parameters domain $\boldsymbol{\Omega}$ is defined by boundary values collected in Table 2. The lower boundary value of the TMD damping ratio indicates a TMD without any damping element, as it was firstly proposed by Frahm [16]. The upper boundary value of the TMD damping ratio is defined by taking into account that the vibration absorber could be a structural element with a damping ratio value depending on its construction material. The lower boundary value of the tuning ratio of the TMD and structure vibration frequency $\rho_T = {}^{\omega_T}/\omega_S$ does not include zero: it indicates that the TMD does not move. The upper limit value of this tuning ratio, i.e. TMD-structure frequency ratio, is defined only in order to limit the search domain of the optimum solution.

The optimization problems are solved by a hybrid method: a classical Genetic Algorithm (GA) [52] combined with an interior-point algorithm. The used GA setting parameters are uniform initial distribution of the design parameters over their domain $\boldsymbol{\Omega}$, elitism of the best two individuals, crossover fraction 0.6, population size of 150 individuals and 15 generations. The number of individuals and generations is taken relatively low, because this study is directed to estimate the correlations among the different problem parameters and not to design a real TMD.

In the following subsections, the results of the analyses performed with different values of the problem parameters are presented.

4.1 Analyses for soft soil conditions

Figures 3, 4 and 5 show the results of the analyses performed for soft soil conditions both stationary and non-stationary earthquake models. A comparison of the results calculated with artificial earthquake modulated by the function proposed by Jennings et al. [41] and the one modulated by the exponential function proposed by Hsu and Bernard [42] allows to highlight the effect of the stationary characteristics of the seismic excitation on the TMD optimal design. The Figures 3, 4 and 5 show respectively the minimum OF value and the respective design parameters values (i.e. TMD damping ξ_T and TMD frequency ω_T) obtained by solution of either optimization problems defined by Equations 18 and 19. Such values are showed as function of both the structure vibration period T_S, on the graphs lower abscissa axis, and the ratio of the structure dominant frequency to the KT filter frequency ω_S/ω_f (also called structure-KT tuning ratio or frequency ratio), on the upper horizontal axis. In order to make easy the comparison between the results they show the results for the main system characterized by two different values of its damping ratio ξ_S ([0.02, 0.10]). In particular, for structures with damping ratio $\xi_S = 0.02$ the solid black line indicates the results obtained for the analyses performed with the stationary KT model, the dash-dot red and solid magenta lines the results for the non-stationary KT model modulated by respectively the function proposed by Jennings et al. [41] and the exponential function proposed by Hsu and Bernard [42]. Instead, for structures with damping ratio $\xi_S = 0.1$ the dotted blue, dashed green and dashed grey lines show respectively the results for the stationary earthquake model, for the non-stationary model with Jennings et al.'s modulation function [41] and the non-stationary model with the exponential modulation function [42]. Those colours are used for all figures presenting the results of the optimization problems with constant mass ratio value ($\gamma_{TMD} = {}^{m_T}/m_S = 0.05$).

A first analysis of the results shown in Figure 3 reveals an increase of the TMD efficiency owing to the decrease of the structure damping ratio in case either the displacement optimization criterion (Equation 18) or the acceleration one (Equation 19) is applied: the magenta, red and black curves are below the green, blue and grey ones. This result falls in with the literature [26]. It is possible to notice the complete overlapping of the curves resulting from the analysis of structure with high damping ($\xi_S = 0.1$) excited by ground motion modelled either as stationary filtered WN (blue curve) or the two non-stationary filtered WN modulated by Jennings et al. function [41] (green curve). The grey curve presenting the results for structures excited by the non-stationary KT model modulated by the exponential function is partially overlapped to those blue and green curves. In particular, for structures more flexible than the soil, i.e. for $\omega_S/\omega_f < 1$, the effectiveness of a TMD optimized for a structure excited by artificial earthquake modulated by the exponential function is lower than those optimized to mitigate vibrations due to earthquake model with stationary characteristics, although these are limited to the plateau interval of Jennings et al. modulation function. Indeed the grey curve runs above those blue and green for $\omega_S/\omega_f < 1$. Therefore, the influence of restricted distribution of energy along the earthquake duration on the TMD effectiveness is clear. Instead, for structures with $\xi_S = 0.1$ and structure-KT model tuning ratio in a range between 2 and 1 the effectiveness of the TMD optimized using the non-stationary earthquake model modulated by the exponential function is slightly higher than those optimized by applying either stationary earthquake models or non-stationary model modulated by Jennings et al. function [41]. This brings to the conclusion that artificial earthquakes modulated in such way have high energy in the range of frequencies similar to the one of the structure. These results are true for both optimization criteria. By observing the upper horizontal axis, it is clear that the maximum positive effect of the TMD is reached in case of resonance of the structure with KT filter, i.e. $\omega_S/\omega_f = 1$, for all earthquake models.

The curves of the analyses performed for structures with damping ratio $\xi_S = 0.02$ show that the TMD effectiveness in mitigation of vibrations caused by the earthquake ground motion is function of the model used to define the latter. For such low damped structures characterized by structure-KT model frequency ratio $\omega_S/\omega_f > 2$, i.e. for structure stiffer than the soil, the effectiveness of TMD in mitigating the vibrations caused by a stationary artificial earthquakes or by non-stationary artificial earthquakes modulated by Jennings et al.'s function is the same. This result is independent of the applied optimization criterion. In fact, the solid black curve is overlapped to the dash-dotted red one in Figures 3a and 3b. The same figures show higher effectiveness of TMD optimized to mitigate vibrations produced by a stationary KT model than those produced by a non-stationary KT model in case of structure-KT filter frequency ratio $\omega_S/\omega_f < 2$. This is valid for both non-stationary KT models. However, the effectiveness of TMD design to mitigate vibrations due to the non-stationary earthquake model modulated by the exponential function is lower than the one of TMD designed for non-stationary model modulated by Jennings et al. function. Indeed the magenta curve has values higher that the red and black one. It is possible to conclude that the concentration of energy of the excitation in short time reduces the efficiency of the TMD in case it is designed to mitigate the vibrations of structures characterized by a low damping ratio ($\xi_S = 0.02$). These results are independent of the optimization criterion.

Figure 3a shows also the ineffectiveness of a TMD applied to structures with low vibration period ($T_S < \sim 0.3s$) when its design parameters are optimized through the displacement optimization criterion. This result was already clear in [26] for stationary KT model and in this study it is proved also for earthquake defined by non-stationary KT models. For such vibration period, structures on soft soil site are acceleration sensitive, so the displacement optimization criterion is not suitable. As consequence, for such vibration period range ($T_S < \sim 0.3s$) Figure 4a shows a sudden change of optimal value of the structure-TMD frequency ratio $\rho_T = \omega_T/\omega_S$ reaching the minimum value of ρ_T within the search domain $\mathbf{\Omega}$ (see Table 2); whilst Figure 5a presents a sharp decrease of the optimum value of TMD damping ratio ξ_T. It is possible to conclude that a change of the optimal TMD vibration frequency corresponds to an increase of its optimal damping ratio to guarantee that it does not amplify the structural vibrations during earthquakes. This behaviour is independent of the modulation in time domain of the earthquake signal, although the time modulation of the artificial earthquake influences the decrease of the optimum value of the TMD damping ratio.

By observing the results presented in Figure 4 for the optimal values of the TMD-structure frequency ratio ρ_T few further results can be abstracted. First, Figure 4a shows that in the case of the displacement criterion the optimum value of the TMD-structure tuning frequency is almost independent from the earthquake model applied both for $\xi_S = 0.02$ and $\xi_S = 0.10$ the few differences can be observed. For $1 < \omega_S/\omega_f < 2$ the optimum TMD frequency is different in case of non-stationary earthquake model modulated by the exponential function respect the other two earthquake models; for $\omega_s/\omega_f < 1$ the trend of curves is equal with value of ρ_T between 0.9 and 1. From the last observation, it is possible to conclude that the tuning of TMD and structure is critical for the vibration mitigation in case of displacement optimization criterion whatever is damping ratio the structure and the earthquake model applied.

Figure 4b presents the optimum values of the TMD-structure tuning ratio in case the acceleration optimization criterion is applied. In it, the curves highlight how TMD-structure tuning ratio depends on the earthquake model applied. For values of structure-KT filter frequency ratio $\omega_S/\omega_f > 1$, i.e. for flexible structure, the optimum values of the TMD-structure tuning ratio increases becoming higher than one in case the non-stationary model modulated by the exponential function is applied. It is possible to infer that the energy of the artificial earthquakes obtained with this model is concentrated in a short time respect to its whole duration and this requires a TMD more flexible than the structure. This result is equal for both structure damping ratio used in the analyses: $\xi_S = 0.02$ and $\xi_S = 0.1$. For structures excited by an earthquake model with stationary characteristics, i.e. a stationary model or a non-stationary modulated by Jennings et al. function, the optimum TMD-structure tuning ratio is around 1 if $\omega_S/\omega_f < 1$. Finally, figure 4b shows also that the minimum value of the optimal TMD-structure tuning ratio is obtained for structure-KT frequency ratio higher than one, in particular for $\omega_S/\omega_f = 1.33$. Such observation is independent of the structure damping ratio.

Figure 5a shows optimum values of the TMD damping ratio in case the displacement criterion is applied. First, it is possible to notice that for structures with damping ratio $\xi_S = 0.1$ the value of the TMD optimum damping ratio decreases abruptly for structure-KT filter frequency ratio higher than 0.2. The change occurs when the structure becomes more sensitive to displacement than to acceleration, as explained for the figures presenting TMD effectiveness and optimum TMD-structure tuning ratio. The value of the TMD damping ratio becomes zero in case the Hsu and Bernard exponential function [42] is used to modulate the artificial earthquake. This means that a TMD optimized to mitigate vibrations due to a dynamic load with energy concentrated in short time has no need of a TMD with a viscous damping, but an additional mass and stiffness is enough, as Frahm proposed in his first study [16]. In general, for all types of earthquake model the optimum value of the TMD damping ratio is lower than the structure damping ratio in case it is $\xi_S = 0.01$ for almost all values of ω_S/ω_f. The observation of Figure 5a shows a reduction of the optimum value of the TMD damping ratio also in case of low structure damping ratio ($\xi_S = 0.02$). Such reduction is higher for non-stationary earthquake models than for the stationary one. In particular, for the artificial earthquake modulated by the exponential function [42] the optimum value of TMD damping ratio reaches values close to zero for $\omega_S/\omega_f < 1$. This indicates that the TMD mitigating better vibrations due to artificial earthquakes with the energy concentrated in short interval of time does not need damping ratio. Moreover, by comparing the results shown in Figure 5a with those presented in Figure 4a it is clear that the optimal TMD is an additional mass and stiffness linked to the structure and for $\omega_S/\omega_f < 1$ and stationary earthquake model the TMD should have a damping ratio $\xi_T \cong 0.1$ and $\omega_T = \omega_S$ for the displacement optimization criterion.

Figure 5b presents the optimum values of TMD damping ratio in case the objective is to minimize the structure acceleration due to earthquakes. By observing this figure, it is possible to notice that also in this case the value of the optimum TMD damping ratio drops to zero with the increase of structure-KT filter frequency ratio in case the structure is excited by artificial earthquakes modulated by the exponential function. Comparing this result with the one presented for Figure 5a it is clear that this occurs for higher values of ω_s/ω_f respect to those in case of displacement optimization criterion. Figure 5b shows also that in case of acceleration optimization criterion and stationary earthquake model the minimum value of optimum TMD damping ratio occurs for $\omega_S/\omega_f = 1$. Finally, the optimum values of the TMD damping ratio obtained with the same criterion and the non-stationary earthquake model modulated by Jennings et al. function present a trend with a local minimum for $\omega_S/\omega_f = 1$ and a decrease for $\omega_S/\omega_f < 1$. Thus, few general results can be abstracted. The value of the optimum TMD damping ratio tends to decrease in case of artificial earthquake modulated in time, expecially for flexible structures ($\omega_S/\omega_f < 1$); the minimum value of the optimum parameter is obtained for structure in resonance with the soil ($\omega_S/\omega_f = 1$) in case the earthquake model is at least partially stationary.

4.2 Analyses for stiff soil conditions

Figures 6, 7 and 8 show the results of numerical analyses carried out with values of the KT filter parameters for stiff soil site (Table 1). The horizontal and vertical axes of those figures present the same physical quantities as in Figure 3, 4 and 5. The results shown in Figure 6 agree with those in Figure 3 about the TMD effectiveness. Also in this case the lowest minimum value of the OF is obtained under frequency resonant conditions of the structure and the KT filter, i.e. $\omega_S/\omega_f = 1$. For stiff soil site condition the acceleration optimization criterion results more efficient: the OF values of the Figure 6a are lower than the ones in Figure 6b. One can observe that the performance of the TMD connected to a structure with high damping ratio ($\xi_S = 0.1$) is independent from the earthquake ground motion model used in the analyses in case has a part with stationary characteristics, i.e. stationary KT model and non-stationary KT model modulated with the function proposed by Jennings et al []. The results obtained for the same structure, but the non-stationary KT model modulated by the exponential function present a reduction of the TMD effectiveness with lower values of the frequency ratio ω_S/ω_f starting from the resonant condition of structure and KT filter. In fact, the blue dotted line and the green dashed one are fully overlapped in Figure 6a and in Figure 6b whereas only the grey line moves away from previous one. Regarding the performance of TMD optimally designed to mitigate vibrations of a structure with low damping ratio ($\xi_S = 0.02$), no difference is observed for structure–KT filter frequency ratio $\omega_S/\omega_f > 1$ if the stationary or the non-stationary KT model modulated by the function proposed by Jennings et al. is applied. On the contrary, for structure-KT filter frequency ratio $\omega_S/\omega_f < 1$ the TMD performance is overestimated if the optimization problem is formulated with the stationary ground motion model instead of the non-stationary one with Jenning et al. modulation function [41]. The magenta curve showing the effectiveness of the TMD optimized for a structure excited by a non-stationary earthquake signal modulated by the exponential function is above those calculated by applying the other earthquake models, i.e. the red dash-dotted and black solid curves, independently of the value of ω_S/ω_f. These results are true for both displacement and acceleration optimization criterion. Therefore, the TMD optimization performed with a stationary earthquake model leads to an overestimation of the damper effectiveness and this is even more evident through the comparison of the results calculated with the stationary artificial earthquake and those with non-stationary artificial earthquake modulated by the exponential function, which has the energy concentrated in a short time interval. The comparison of Figure 6a and 6b shows that for structure with fundamental frequency higher than the one of the site where it is located ($\omega_S/\omega_f > 1$) the TMD optimized through the acceleration criterion has a better performance than the one optimized by the other criterion. This result was also presented in [26].
Figure 7 shows optimum values of the TMD-structure tuning frequency ratio. First, Figure 7a shows for structures with low damping ratio ($\xi_S = 0.02$) lack of dependence of the optimum TMD frequency ω_T from the earthquake model applied, especially if it is also only partially stationary, as the non-stationary KT earthquake model modulated by Jennings et al. function [41]. In case of structures with high damping ratio ($\xi_S = 0.1$) and extremely flexible, i.e. characterized by a vibration period $T_S > 2s$ according to the parameters used for this study, the optimum value of the TMD-structure tuning frequency ratio $\rho_T = \omega_T/\omega_S$ results higher if the non-stationary KT model modulated by the Jennings et al. function [41] is applied rather than the stationary one. Such difference increases with structure vibration period T_S, as the dashed green curve running above the dotted blue one show. On the contrary, for such structure the TMD-structure tuning frequency ratio $\rho_T = \omega_T/\omega_S$ obtained using artificial earthquakes

modulated by the exponential function decreases with the increase of the structure-earthquake model tuning ratio (ω_S/ω_f) and the dashed grey line run below the blue one.

Figure 7b proposes the results obtained for the acceleration optimization criterion. Those results show that the optimum values of the TMD-structure tuning ratio is in the range between 0.9 and 1 and it is independent of the structure damping ratio and the earthquake model, except in case it is non-stationary modulated by the exponential function [42]. Indeed the figure presents the solid magenta and dashed grey curves above all the others, which they touch in case the structure is in resonance with the soil ($\omega_S/\omega_f = 1$). For structures with low values of ω_S/ω_f and excited by non-stationary artificial earthquake modulated by the exponential function proposed by Hsu and Bernard [42] the TMD-structure tuning ratio ρ_T increases with higher structure-KT filter tuning ratio. This means that for earthquake excitation with energy concentrated in a short interval of time the TMD must be even more flexible than the structure to mitigate structure acceleration. However, this increase of damper flexibility is not justified by the reduction of its effectiveness shown in Figure 6b.

Figures 8a shows slightly higher optimal values of the TMD damping ratio ξ_T, independently of the applied earthquake ground motion model in case of structure stiffer than the soil ($\omega_S/\omega_f > 1$) and high structure damping ratio ($\xi_f = 0.10$). This result is not observed in case the acceleration criterion is applied (see Figure 8b). In Figure 8, it is possible to observe that for flexible structures ($\omega_S/\omega_f < 1$) the optimum value of the TMD damping ratio depends on the earthquake model and partially on the structure damping ratio, it does not on the optimization criterion applied. The values of the TMD damping ratio decreases with the increase of the flexibility of the structure respect to the soil if a non-stationary earthquake model is applied. In particular, Figures 8a and 8b shows that for $\omega_S/\omega_f < 0.2$, the blue dotted and black solid curves are fully overlapped, the red dash-dotted and green dotted curve run below the blue and black one, while the magenta solid and grey dashed curves drops to zero. This means that the concentration of energy of the earthquake in a short interval of time reduces the need of adding damping to the TMD to mitigate the vibrations induced by the earthquake for structure more flexible than the soil, but TMD stiffer than the structure. Therefore, the energy of artificial earthquake with a time modulation is dissipated through a stiffer TMD with low or null damping.

4.3 Study of the mass ratio influence on the TMD optimal design

Here a sensitivity study of the TMD optimum design parameters as function of the mass ratio $\gamma_{TMD} = m_T/m_S$ is presented. For sake of brevity, in this study only numerical analysis for stationary earthquake model and non-stationary earthquake model modulated using the function proposed by Jenning et al. [41] are presented and the optimum mass ratio is calculated only for a constant value of the structure vibration frequency $\omega_S = \pi$. Figures 9, 10 and 11 show the results of such study for soft soil parameters of the ground motion model and Figures 12, 13 and 14 for stiff soil parameters (Table 1). All these figures show the value of the mass ratio γ_{TMD} on the horizontal axis. On the vertical one Figures 9 and 12 show the OF values obtained from numerical analyses performed with the two different values of structure damping ratio, i.e. $\xi_S = [0.02,\ 0.1]$ and the two ground motion models, i.e. the stationary and the non-stationary filtered WN. Figures 10 and 13 show on the vertical axis the optimum values of TMD-structure frequency ratio $\rho_T = \omega_T/\omega_S$, while Figures 11 and 14 the optimum values of TMD damping ratio ξ_T. The colours and styles used in these figures are identical to those used to present the results of the numerical analyses carried out with constant value of the mass ratio.

The first interesting result of Figure 9 is the lower TMD efficiency for structures with high damping ratio, i.e. $\xi_S = 0.1$ than for those characterized by low damping ratio, i.e. $\xi_S = 0.02$. This is true for both displacement (Figure 9a) and acceleration (Figure 9b) optimization criteria and confirms the results presented in the previous subsections for constant value of the mass

ratio γ_{TMD} and variable structure vibration frequency. In case of analyses carried out with soft soil condition, the structure is tuned with the KT filter, i.e. $\omega_f = \pi$ and $\omega_S = \pi$. Furthermore, in that case the damping ratio of the KT filter is $\xi_f = 0.2$, so the filtered WN defining the seismic load has energy concentrated almost all around the filter frequency. For such conditions it is important to observe that the optimization criterion used influences the optimum value of the mass ratio γ_{TMD}. Figure 9a clearly presents a minimum of the OF for mass ratio $\gamma_{TMD} \cong 0.6$, whatever is the structure damping ratio or the model of ground motion. The acceleration optimization criterion produces a different result: the TMD does not reach its maximum efficiency in the range of mass ratio taken into account in the study, i.e. $\gamma_{TMD} = {}^{m_T}/{m_S} = [0.05 - 4]$. In Figure 9b the value of the OF decrease with the increase of the mass ratio γ_{TMD}. Few comments about the influence of the ground motion model on the efficiency of the TMD are worth of mention. Firstly for structures with high damping ratio ($\xi_S = 0.1$) the KT model used has not influence on the TMD efficiency, except for high mass ratio value, i.e. $\gamma_{TMD} = {}^{m_T}/{m_S} > \sim 2.5$. In that case, the TMD efficiency is lower if the stationary ground motion model is applied in the formulation of the optimum problem rather than the non-stationary one. In both figures, indeed, the green dashed and blue dotted lines are overlapped for $\gamma_{TMD} < 2.5$, but the first lines runs below the others for higher value of γ_{TMD}. For the acceleration criterion, this trend is barely perceptible. On the contrary, for structures with low damping ratio ($\xi_S = 0.02$) the TMD efficiency is always lower in case the earthquake model is non-stationary rather than stationary; therefore in Figures 9 the red dash-dotted curves run above the black solid ones.
Figures 10 present the optimum values of TMD-structure frequency ratio ρ_T as function of the mass ratio γ_{TMD}. When the displacement optimum criterion is applied (Figures 10a), the optimum of such ratio appears to be fully independent of the earthquake model, but not of the structure damping ratio ξ_S. In fact, the optimum TMD-structure frequency ratio is slightly higher for structures with low damping ratio ($\xi_S = 0.02$). Furthermore, such ratio decreases with the increase of the mass ratio γ_{TMD} for both values of structure damping ratio and both earthquake models. The optimal TMD must have frequency more tuned with the structure, whether it is very small respect to those of the structure. When the acceleration criterion is applied for the optimum design, (Figure 10b) the TMD-structure frequency ratio results to be unrelated to of the structure damping ratio ξ_s and the KT model used, whether the mass ratio is $\gamma_{TMD} < 2$. In the opposite case, i.e., for mass ratio $\gamma_{TMD} > 2$, it is related to both structure damping ration and ground motion model used: the optimum TMD frequency results to be more tuned with the structure one if the non-stationary earthquake model is used in the optimum design problem and still more for low structure damping ratio. Figures 11 present the optimum TMD damping ratio ξ_T. For the displacement optimum criterion the value of this TMD parameter increases with the mass ratio γ_{TMD}.It reaches the boundary of the solution existence domain $\mathbf{\Omega}$ (see Table 2) of the optimization problem for mass ratio $\gamma_{TMD} = \sim 0.5$. Such value is close to the one for which the TMD reaches its maximum performance (see Figure 8a). Thus, it is possible to conclude that the TMD effectiveness obtained in case of displacement optimization criterion is strictly dependent on the boundaries of the solution existence domain $\mathbf{\Omega}$ and a wider domain would produce different results in terms of TMD effectiveness. It is important to remind that those boundary values of the TMD parameters are defined according to technological limits. The dependence of the TMD damping ratio ξ_T from the mass ratio γ_{TMD} appears clearly both in Figure 11a and 11b. The latter presents the results obtained with the acceleration optimum criterion. In it the optimum value of the TMD damping ratio ξ_T increases till the TMD mass reaches the structure one ($\gamma_{TMD} = \sim 1$). For TMD with mass larger than the structure one, i.e. $\gamma_{TMD} > 1$, the optimum value of the TMD damping ratio ξ_T decreases. This trend is observed for structures with either high or low damping ratio and seismic excitation modelled as either stationary or non-stationary filtered WN. However, the optimum value of the damping ratio of TMDs protecting structures with low dissipating capacity (ξ_S=0.02) is higher than the one of

TMDs protecting structures with high dissipation capacity (ξ_S=0.1). Finally, for the main purpose of this study it is important to notice that the TMD damping ratio ξ_T obtained by solving the optimization problem formulated with the stationary earthquake ground motion model is higher the one obtained from that problem formulated with the non-stationary one.
Figures 12, 13 and 14 present respectively OF value and optimum value of the TMD design parameters obtained through application of either optimum design criteria and for stiff soil condition (Table 1). The earthquake excitation resulting for such soil condition is characterized by broad band with prevalent earthquake frequency $\omega_f = 6\pi$, therefore not tuned with the structure one ($\omega_S = \pi$). The remarks made for Figures 9, 10 and 11, i.e. for soft soil condition, are generally fitting to this other related to the stiff soil condition. However, in Figure 13b and in both the Figures 14 the boundaries of the solution existence domain of the optimization problem $\mathbf{\Omega}$ are reached. In particular, in first of these figures the lowest TMD-structure frequency ratio value possible within the search domain $\mathbf{\Omega}$ is reached ($\rho_T = \omega_T/\omega_S \geq 0.1$), while in second figure the highest value limiting the search domain of the TMD damping ratio ($\xi_T \leq 0.2$).

6 Conclusions

The study presented dealt with the optimum design of a single TMD applied to a structure excited by earthquake ground motion. Two different optimization criteria were defined and independently applied. The first one aims to mitigate the structure displacement produced by earthquake excitation, while the second one to reduce the structure acceleration caused by such excitation. Here the structure was modelled as a linear elastic SDOF and the ground motion excitation thought the KT filter and eventually modulated with functions proposed either by Jennings et al. [41] or Hsu and Bernard [42] to include the non-stationarity in time of real earthquake ground motion signal. Numerical analyses are performed to assess the TMD performance and the optimum values of its design parameters. Different sets KT filter parameters are used in them: one producing a broad band excitation and the other one a narrow band excitation. The first one is typical of earthquakes occurring on stiff soil, while the latter on soft soil. The TMD-structure mass ratio is taken constant in the first sets of numerical analyses. Their results are compared and few observations are made. They can be summarized as follow.

- The performance of TMDs in mitigating vibrations in structures with high damping ratio is lower than the one of those applied to structures with low damping ratio.
- For structures with low damping ratio the TMD performance is influenced also by the ground motion model used: it is lower in case a non-stationary filtered WN modulated by the exponential function [42] is used rather than a stationary one. This result is obtained for different values of the structure vibration period. It is true also in case TMD-structure mass ratio is varied while the structure frequency value is constant. This indicates that the amplitude modulation of artificial earthquakes can lead to lower values of the TMD effectiveness; hence the use of stationary earthquake model produces misleading results in case of real applications. This result brings to the conclusion that also the use of specific real earthquake records can produce non careful optimization of TMD design.
- The amplitude modulation in time of the ground motion excitation affects also the optimum values of the TMD damping ratio and vibration frequency. The first ones are lower than the value obtained for the stationary ground motion excitation, whereas the second ones are higher, but only in case the acceleration optimization criterion is applied and structure is more flexible than the soil. Such result is valid for mass ratio value equal to 0.5. For values of that ratio higher the one, no general remark can be easily made, also in case of variable value of the mass ratio and constant structure frequency.

References

[1] Karavasilis T.L., Kerawala S., Hale E., “Hysteretic model for steel energy dissipation devices and evaluation of a minimal-damage seismic design approach for steel buildings”, Journal of Constructional Steel Research, 2012, 70, 358-367.
[2] Apostolakis G., Dargush G.F., “Optimal seismic design of moment-resisting steel frames with hysteretic passive devices”, Earthquake Engineering and Structural Dynamics, 2010, 39(4), 355-376.
[3] Brando G., D'Agostino F., De Matteis G., “Seismic performance of MR frames protected by viscous or hysteretic dampers”, The Structural Design of Tall and Special Buildings, 2015, 24(9), 653– 671.
[4] Gluck, N., Reinhorn, A. M., Gluck, J., & Levy, R. “Design of Supplemental Dampers for Control of Structures”, Journal of Structural Engineering, 1996, 122(12), 1394–1399.
[5] Symans M. D., Charney F. A., Whittaker A. S., Constantinou M. C., Kircher C. A., Johnson M. W., McNamara R. J., “Energy Dissipation Systems for Seismic Applications: Current Practice and Recent Developments”, Journal of Structural Engineering, 2008, 134(1), 3–21.
[6] De Matteis G., Sarracco G., Brando G., “Experimental tests and optimization rules for steel perforated shear panels”, Journal of Constructional Steel Research, 2016, 123, 41-52.
[7] Brando G., D’Agostino F., De Matteis G., “Experimental tests of a new hysteretic damper made of buckling inhibited shear panels”, Materials and structures, 2013, 46(12), 2121-2133.
[8] Formisano A., Mazzolani F.M., “On the selection by MCDM methods of the optimal system for seismic retrofitting and vertical addition of existing buildings”, Computers & Structures, 2015, 159, 1-13
[9] Fahnestock L. A., Sause R., Ricles J. M., “Seismic Response and Performance of Buckling-Restrained Braced Frames”, Journal of Structural Engineering, 2007, 133(9), 1195–1204.
[10] Fahnestock L. A., Ricles J. M., Sause R., “Experimental Evaluation of a Large-Scale Buckling-Restrained Braced Frame”, 2007, Journal of Structural Engineering, 133(9), 1205–1214.
[11] Ghabraie K., Chan R., Huang X., Xie Y. M., “Shape optimization of metallic yielding devices for passive mitigation of seismic energy”, Engineering Structures, 2010, 32(8), 2258-2267.
[12] Levy R., Lavan O., “Fully stressed design of passive controllers in framed structures for seismic loadings”, Structural and Multidisciplinary Optimization, 2006, 32(6), 485-498.
[13] Baratta A., Corbi I., “Optimal design of base-isolators in multi-storey buildings”, Computers and Structures, 2004, 82, 2199-2209.
[14] Chung L.-L., Kao P.-S., Yang C.-Y., Wu L.-Y., Chen H.-M., „Optimal frictional coefficient of structural isolation system”, Journal of Vibration and Control, 2015, 21(3), 525-538.
[15] Bucher C., “Probability-based optimal design of friction-based seismic isolation devices”, Structural Safety, 2009, 31, 500-507.
[16] Frahm H. Device for damping vibrations of bodies, US-Patent 989958, 1911
[17] Adam C., Furtmüller T., “Seismic performance of Tuned Mass Dampers” In: Mechanics and Model Based Control of Smart Materials and Structures, Publisher: Springer, Vienna, Editors: Irschik, H., Krommer, M., Watanabe, K. ,2010, pp. 11–18.
[18] Iemura H., “Principles of TMD and TLD — Basic Principles and Design Procedure”. In book: Passive and Active Structural Vibration Control in Civil Engineering, Publisher: Springer, Vienna, Editors: Soong, T.T., and Costantinou, M.C., 1994; pp. 241-253.
[19] Den Hartog J.P. Mechanical vibrations, McGraw-Hill, New York, 1956.
[20] Sun J.Q., Jolly M.R., Norris M.A., “Passive, adaptive and active tuned vibration absorbers – A survey”, Transactions of the ASME, 1995; 117, 234-242.
[21] Anh N.D., Nguyen N.X., “Design of TMD for damped linear structures using the dual criterion of equivalent linearization method”, International Journal of Mechanical Sciences, 2013; 77, 164-170.

[22] Salvi J., Rizzi E., "Closed-form optimum tuning formulas for passive Tuned Mass Dampers under benchmark excitations", Smart Structures and Systems, 2016; 17(2), 231-256.
[23] Lee C.-L., Chen Y.-T., Chung L.L., Wang Y.-P., "Optimal design theories and applications of tuned mass dampers", Engineering Structures, 2006; 28, 43-53.
[24] Leung A. Y. T., Zhang H., Cheng C.C., Lee Y.Y., "Particle swarm optimization of TMD by non-stationary base excitation during earthquake", Earthquake Engineering & Structural Dynamics, 2008; 37(9), 1223-1246.
[25] Hoang N., Fujino, Y., Warnitchai P., "Optimal tuned mass damper for seismic applications and practical design formulas", Engineering Structures, 2008; 30(3), 707-715.
[26] Marano G.C., Greco R., Chiaia B., "A comparison between different optimization criteria for tuned mass dampers design", Journal of Sound and Vibration, 2010; 329(23), 4880-4890
[27] Marano G.C., Greco R., "Optimization criteria for Tuned Mass Dampers for structural vibration control under stochastic excitation", Journal of Vibration and Control, 2011; 17(5), 679-688.
[28] Zhang Z., Balendra T., "Passive control of bilinear hysteretic structures by tuned mass damper for narrow band seismic motions", Engineering Structures, 2013; 54, 103-111.
[29] Park J., Reed D., "Analysis of uniformly and linearly distributed mass dampers under harmonic and earthquake excitation", Engineering Structures, 2001; 23, 802–14.
[30] Greco R., Marano G.C., "Optimum design of tuned mass dampers by displacement and energy perspectives", Soil Dynamics and Earthquake Engineering, 2013; 49, 243-253.
[31] Rana R., Soong T.T., "Parametric study and simplified design of tuned mass dampers", Engineering Structures, 1998; 20(3), 193-204.
[32] Crandall S.H. and Mark W.D. Random vibration in mechanical systems, Academic Press, New York, 1973.
[33] Greco R., Lucchini A., Marano G.C., "Robust design of tuned mass dampers installed on multi-degree-of-freedom structures subjected to seismic action", Engineering Optimization, 2014; 47(8), 1009-1030.
[34] Bendaş G., Nignelim S. M., "Optimization of Tuned Mass Dampers for minimizing story drift of structures". In: Proceedings of 2nd European Conference on Earthquake Engineering and Seismology, Istanbul, 2014; Contribution 399.
[35] Bekdas G., Nigdeli S.M. "Mass ratio factor for optimum tuned mass damper strategies", International journal of mechanical sciences, 2013; 71, 68-84.
[36] Farshidianfar A. and Soheili S., "Ant colony optimization of tuned mass dampers for earthquake oscillations of high-rise structures including soil-structure interaction", Soil dynamics and earthquake engineering, 2013; 51, 14-22.
[37] Khatibinia M., Gholami H., Labafi S.F., "Multi-objective optimization of Tuned Mass Dampers considering soil-structure interaction", International Journal of Optimization in Civil Engineering, 2016; 6(4), 595-610.
[38] Ghosh A., Basu B., "Effect of soil interaction on the performance of tuned mass dampers for seismic applications", Journal of Sound and Vibration, 2004; 274(3–5), 1079-1090.
[39] Kanai K., "Semi-empirical formula for the seismic characteristics of the ground", Bulletin of Earthquake Research Institute, University of Tokyo, 1957; 35, 309-325
[40] Tajimi H., "A statistical method of determining the maximum response of a building structure during an earthquake", In: Proceedings of 2nd International Conference Earthquake Engineering, Tokyo, 1960; 2, 781-797.
[41] Jennings P.C., Housner G.W., Twai C., "Simulated earthquake motions for design purpose", In: Proceedings of 4th Word Conference on Earthquake Engineering, Santiago, 1968, A-1, 145-160.
[42] Hsu T.I., Bernard M.C., "A random process for earthquake simulation", Earthquake Engineering Structural Dynamics .1978, 6(4), 347–362
[43] Greco R., Fiore A., Marano G.C., "The role of modulation function in nonstationary stochastic earthquake model", The journal of Earthquake and Tsunami, 2014; 8(5), 450015.
[44] Özsarıyıldız Ş.S., Bozer A., "Finding optimal parameters of tuned mass dampers", The Structural Design of Tall and Special Buildings, 2015; 24(6), 461-475.

[45] Mohtasham M., Abdolreza J., ”Designing optimal tuned mass dampers for nonlinear frames by distributed genetic algorithms”, The Structural Design of Tall and Special Buildings, 2012; 21(1), 57-76.
[46] Ribakov Y., Agranovich G., “Control of structural seismic response by a limited set of active dampers”, The Structural Design of Tall and Special Buildings, 2011; 20, 594-611.
[47] Marano G.C., Greco R., Trentadue F., Chiaia B., “Constrained reliability-based optimization of linear tuned mass dampers for seismic control“, International Journal of Solids and Structures, 2007; 44(22), 7370-7388.
[48] Motavalli M., Feltrin G., Gsell D., Meyer J. ”Instrumentation of the Indoor Cable Stayed Bridge at Empa”, In book: Sensing Issues in Civil Structural Health Monitoring, Netherlands, Publisher: Springer, Editors: Ansari F., 2005.
[49] Buchholdt H. Structural dynamics for engineering, Thomas Teldfort, London, 1997
[50] Der Kiureghian A., “A Coherency model for Spatial Varying Ground Motions“, Earthquake Engineering and Structural Dynamics, 1996; 25, 99-111.
[51] Iemura H., Jain S.K., Pradomo M. H., “Seismic base isolation and vibration control”, In book: Vibration monitoring, testing and instrumentation, Publisher: CRC Press, Editors: De Silva C.W., 2005
[52] Srinivas N., Deb K., “Multi-objective optimization using non dominated sorting in Genetic Algorithms”, Journal of Evolutionary Computation, 1994; 2(3), 221-248.

		Stiff Soil	Soft soil
Ground motion intensity	$\ddot{x}_{g\_peak}$ (PGA) [g]	0.2	0.2
Kanai-Tajimi filter parameters	ω_f *[rad/s]* [41]	6π	π
	ξ_f [41]	0.6	0.2
White Noise	S_0 [cm^2/s^3] [40]	110.6	320.8
Modulation function proposed by Jennings et al. [32]	t_1 [s]	5	5
	t_2 [s]	10	10
	β	0.4	0.4
Exponential modulation function [33], parameter calculated according equation (9)	t_m	3.92	3.92

Table 1: Values of the Kanai-Tajimi filter parameters [41], modulation function [33, 34] and WN intensity [40].

	Lower boundary	Upper boundary
ξ_T	≥ 0	≤ 0.2
$\rho_T = \omega_T/\omega_S$	≥ 0.1	≤ 1.5

Table 2: Boundaries of the optimization problem search domain.

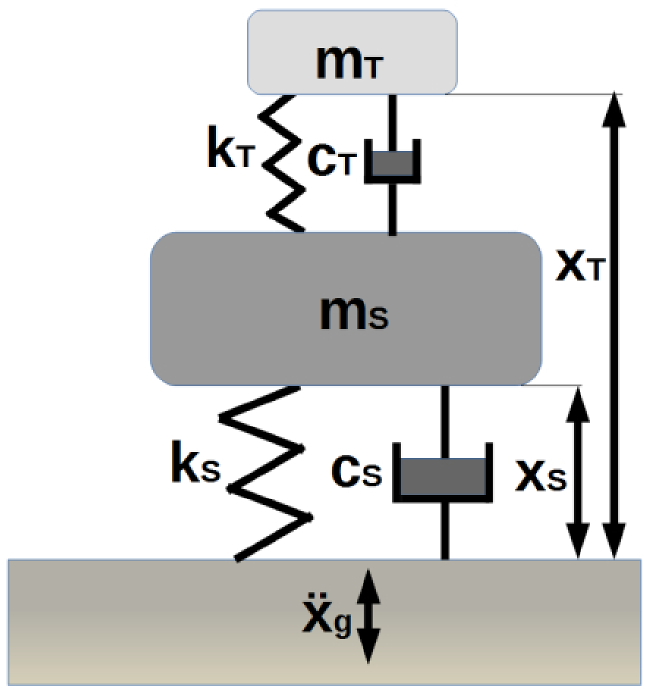

Figure 1: Structure modelled as a SDOF and protected by a single TMD.

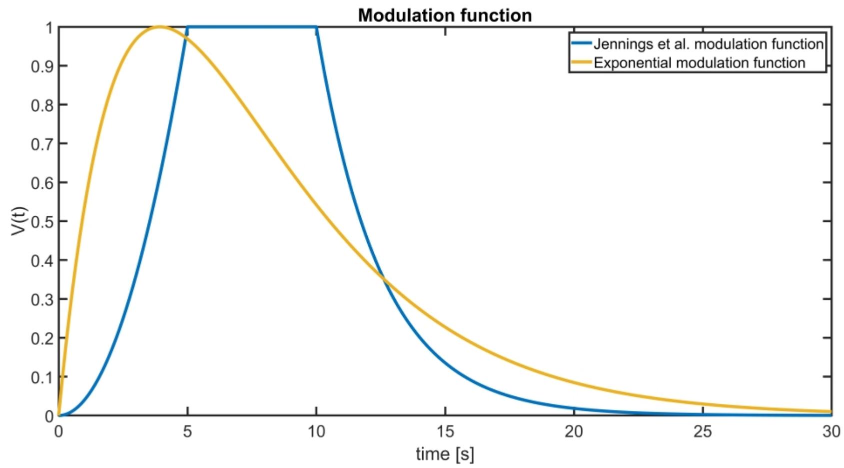

Figure 2: Modulations functions.

508x242mm (300 x 300 DPI)

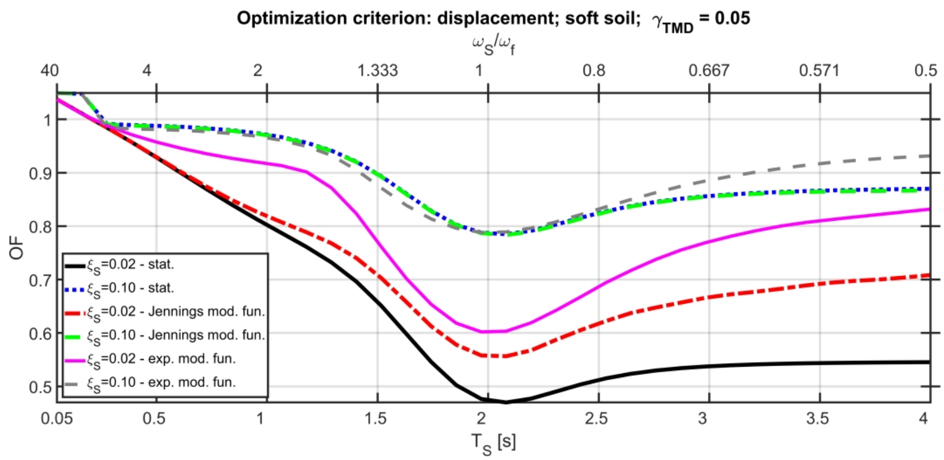

Figure 3: OF value of the displacement (a) and acceleration (b) optimum criteria as function of the structure vibration period T_S in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

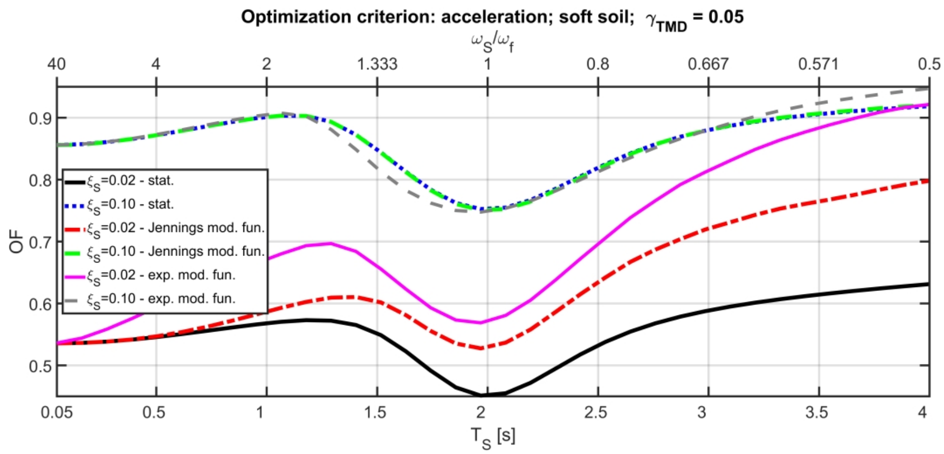

Figure 3: OF value of the displacement (a) and acceleration (b) optimum criteria as function of the structure vibration period T_S in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

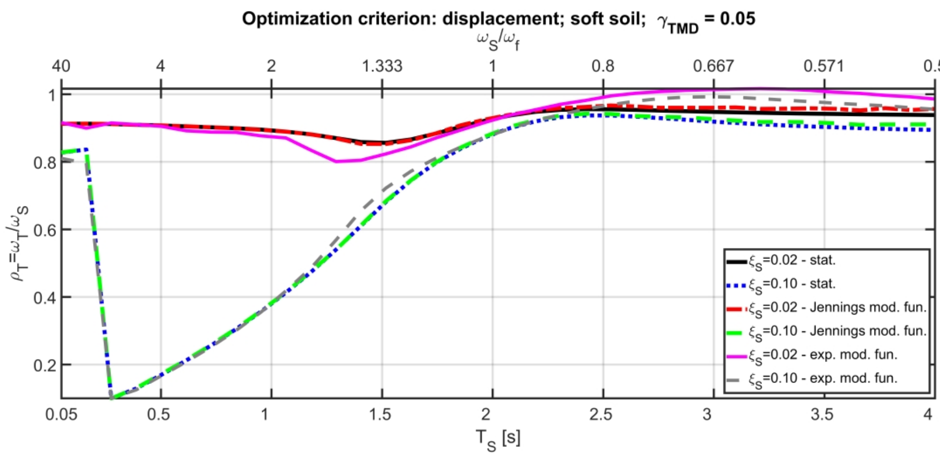

Figure 4: Optimum values of ρ_T=ω_T/ω_S as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

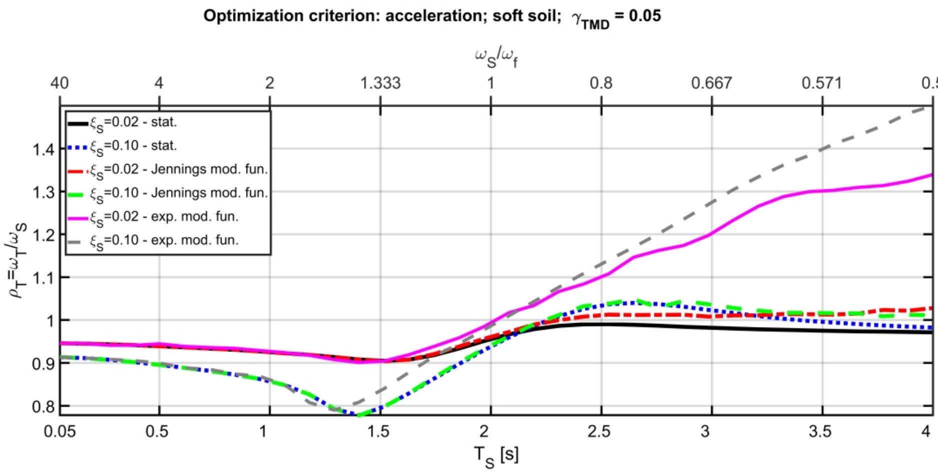

Figure 4: Optimum values of ρ_T=ω_T/ω_S as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

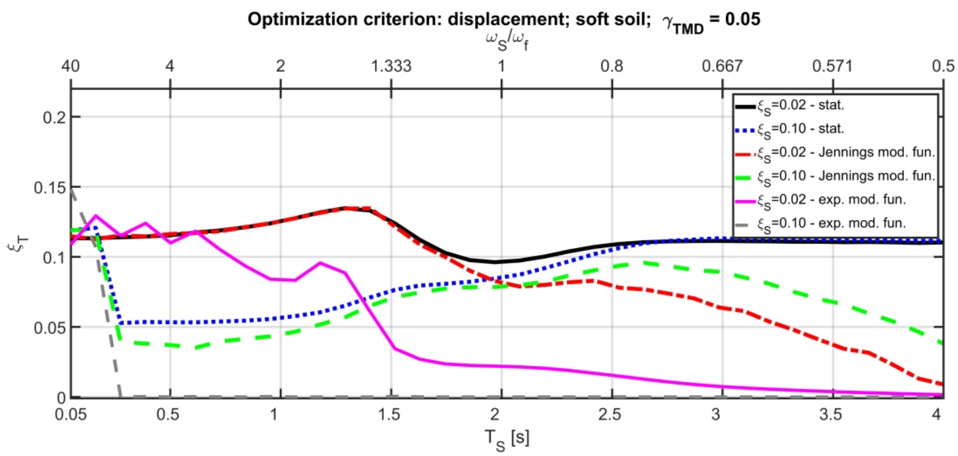

Figure 5: Optimum values of ξ_T as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

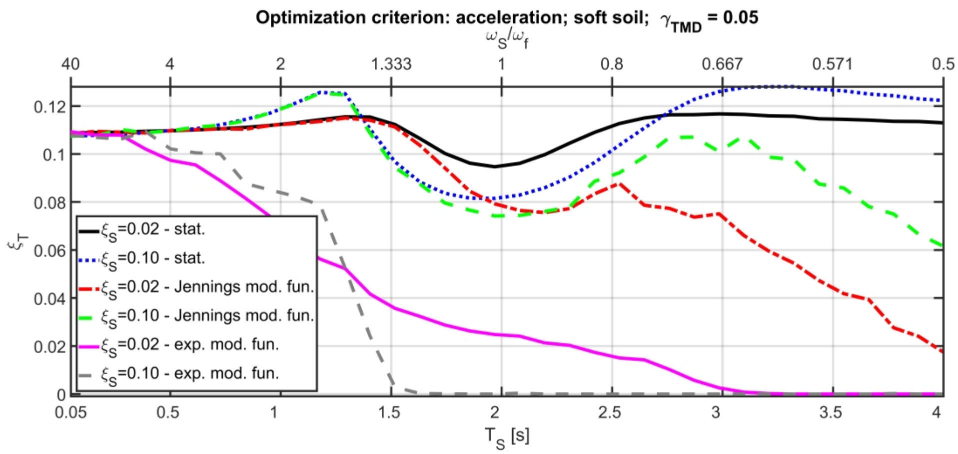

Figure 5: Optimum values of ξ_T as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

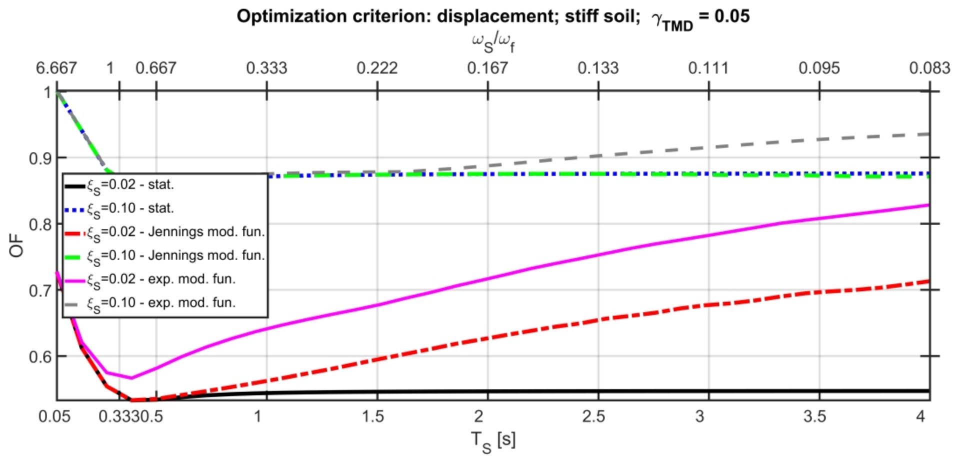

Figure 6: OF value of the displacement (a) and acceleration (b) optimum criteria as function of the structure vibration period T_S in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

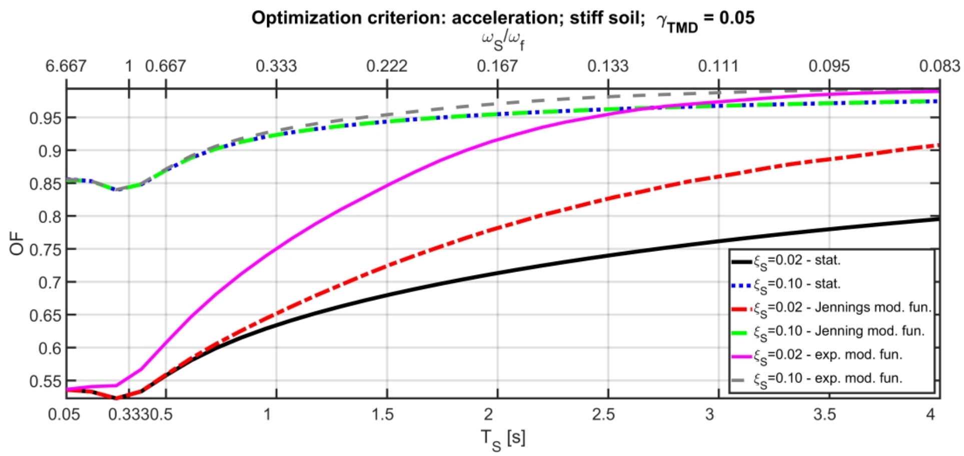

Figure 6: OF value of the displacement (a) and acceleration (b) optimum criteria as function of the structure vibration period T_S in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

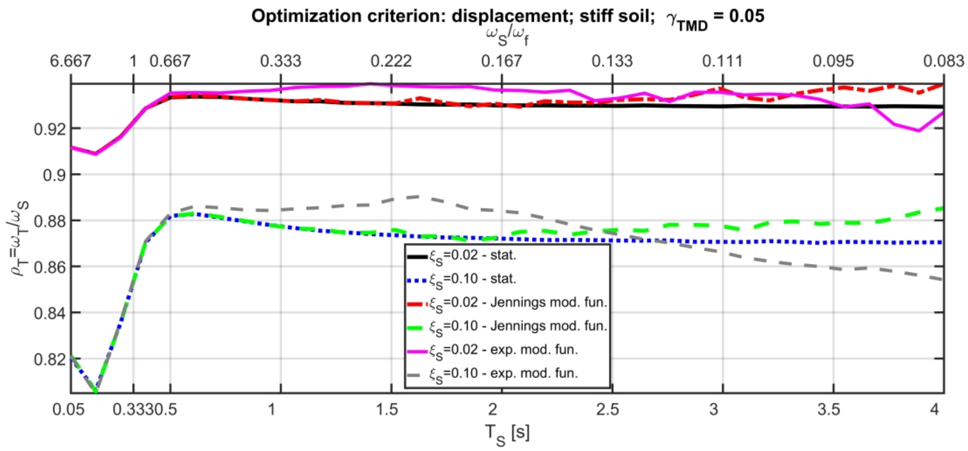

Figure 7: Optimum values of ξ_T as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

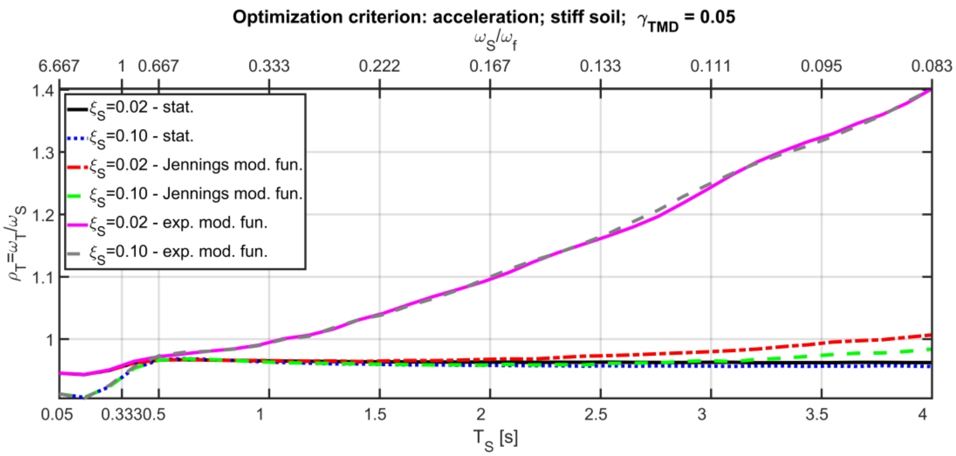

Figure 7: Optimum values of ξ_T as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

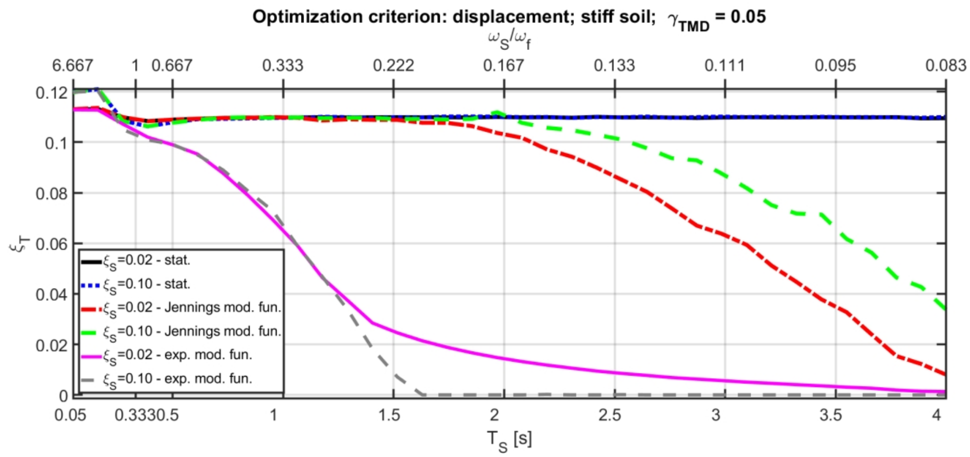

Figure 8: Optimum values of ξ_T as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (96 x 96 DPI)

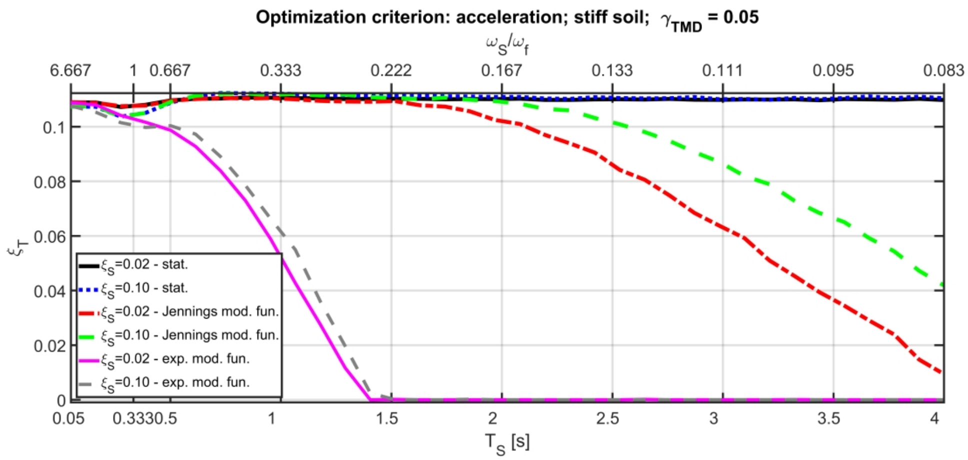

Figure 8: Optimum values of ξ_T as function of the structure vibration period T_S obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

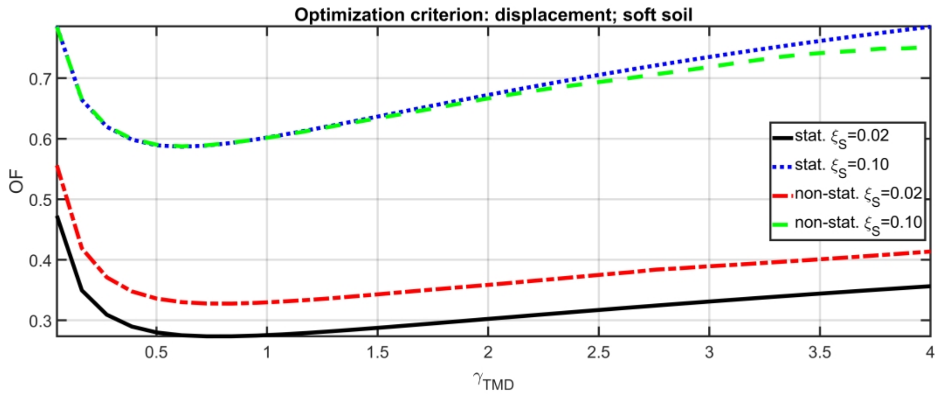

Figure 9: OF value of the displacement (a) and acceleration (b) optimum criteria as function of the mass ratio in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

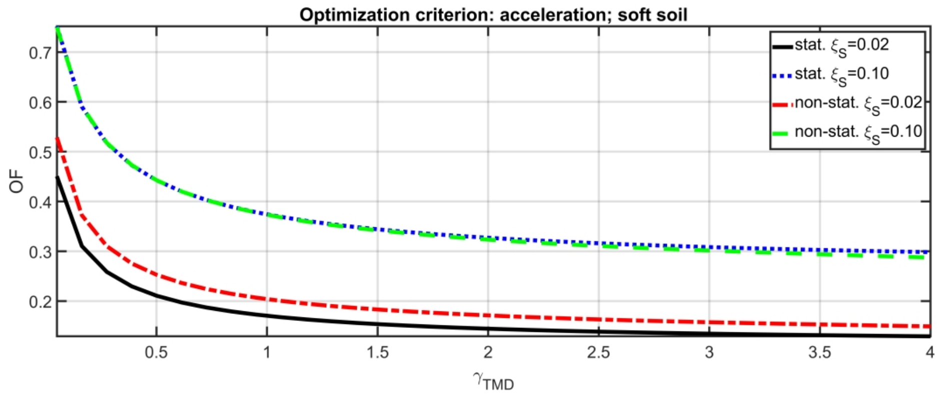

Figure 9: OF value of the displacement (a) and acceleration (b) optimum criteria as function of the mass ratio in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

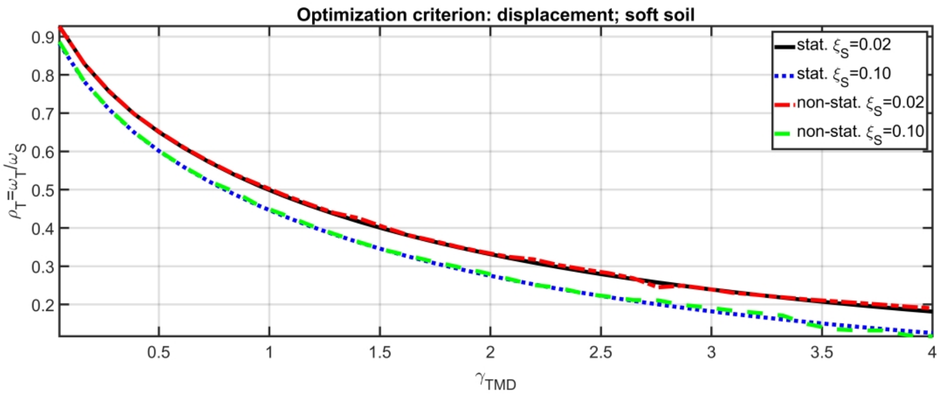

Figure 10: Optimum values of ρ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

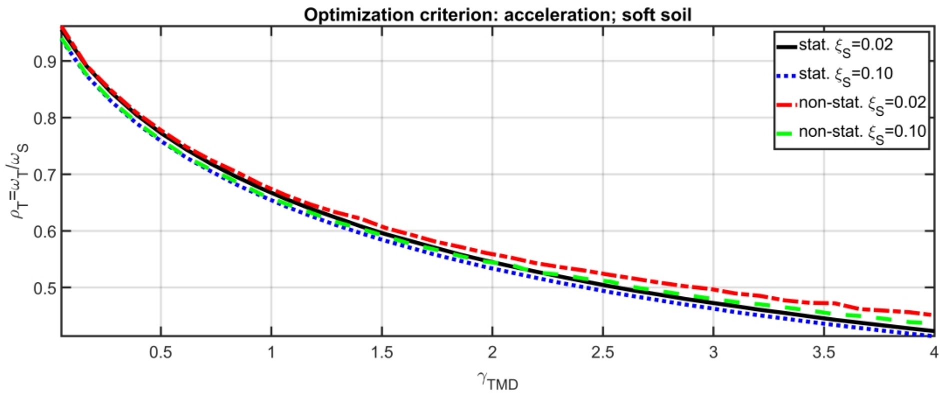

Figure 10: Optimum values of ρ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

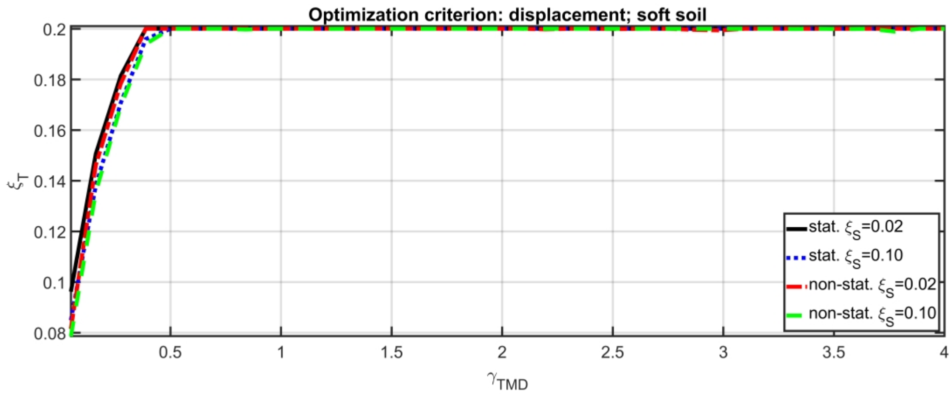

Figure 11: Optimum values of ξ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

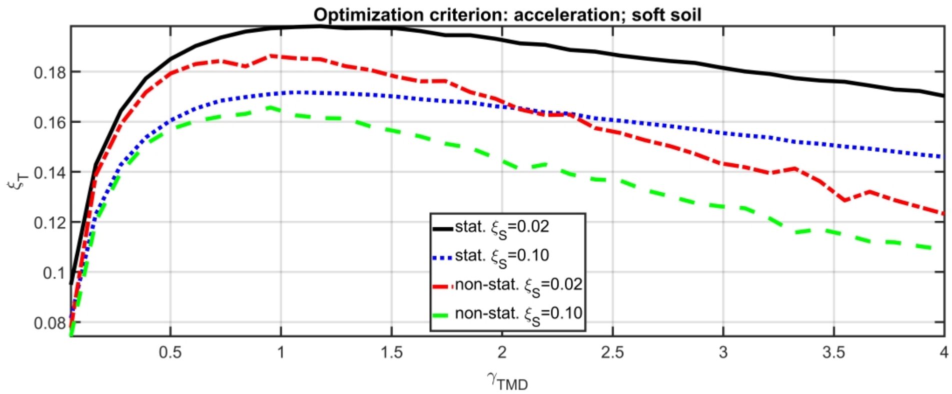

Figure 11: Optimum values of ξ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of soft soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

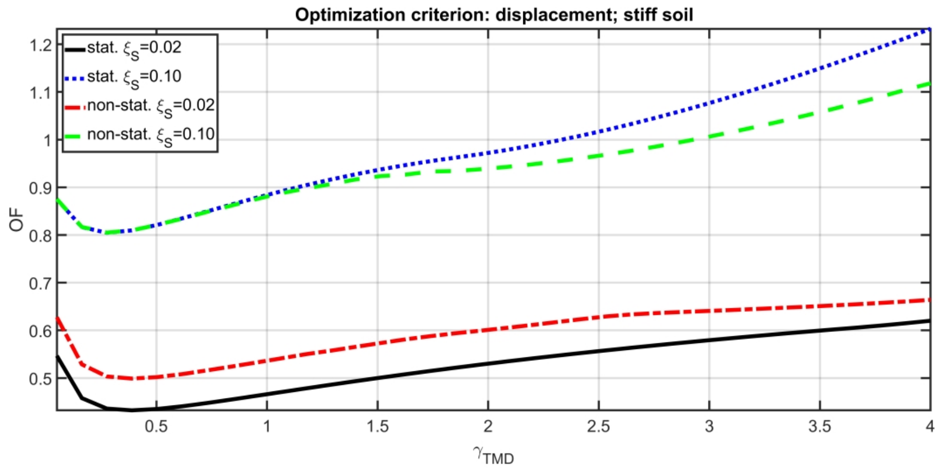

Figure 12: Optimum values of ρ_T=ω_T/ω_S as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

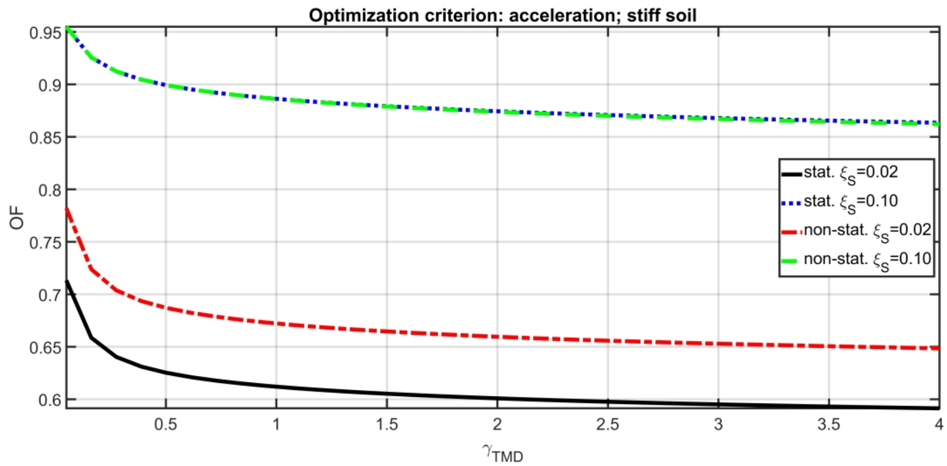

Figure 12: Optimum values of ρ_T=ω_T/ω_S as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

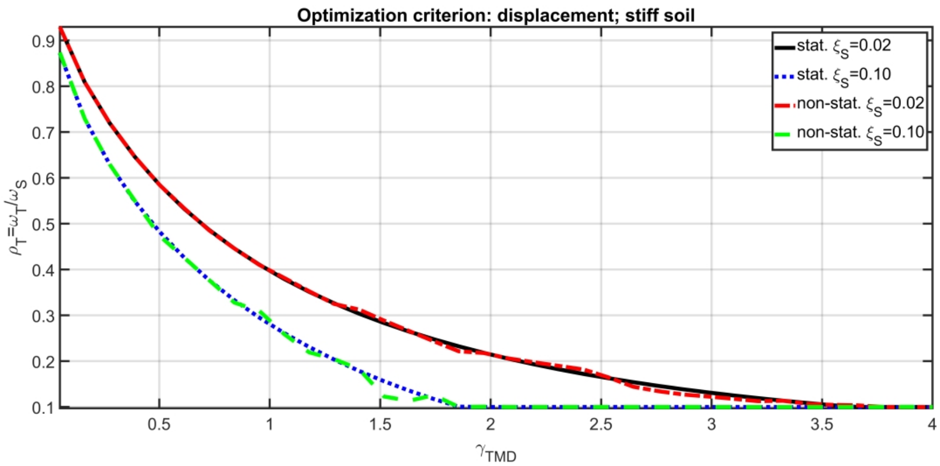

Figure 13: Optimum values of ρ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

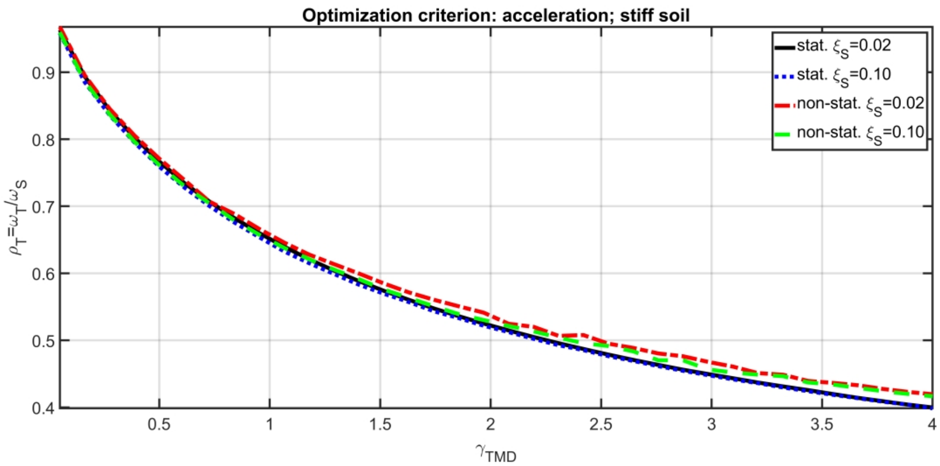

Figure 13: Optimum values of ρ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

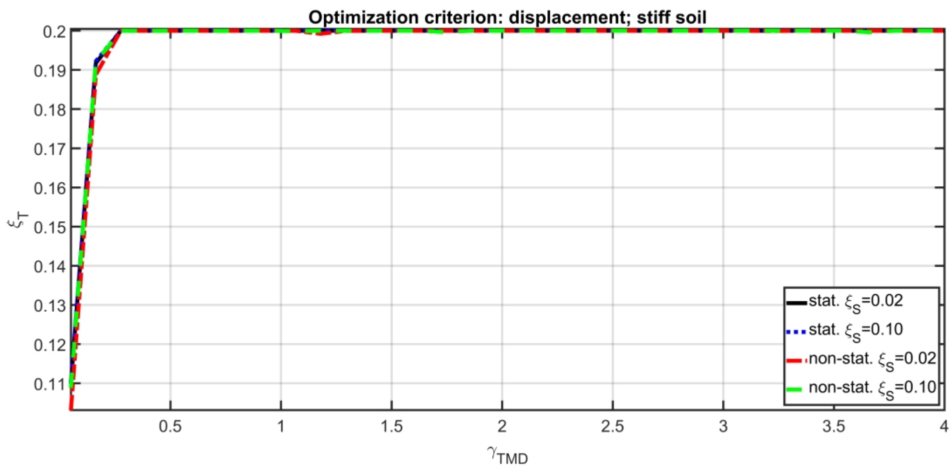

Figure 14: Optimum values of ξ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)

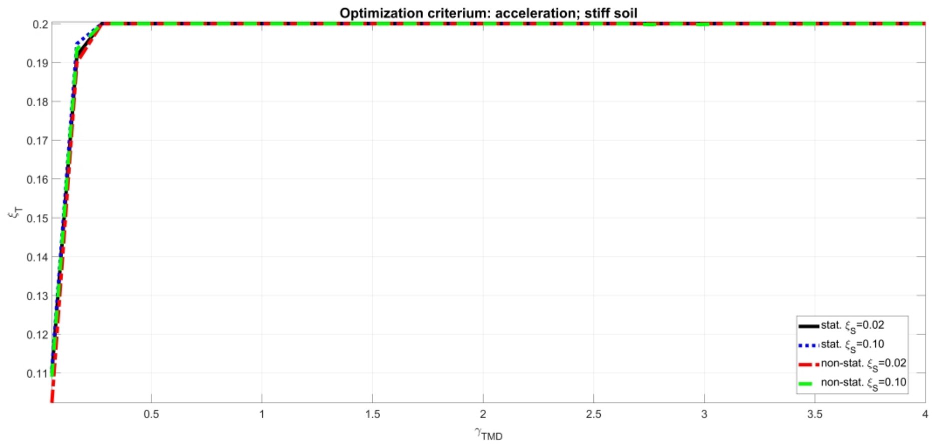

Figure 14: Optimum values of ξ_T as function of the mass ratio obtained through displacement (a) and acceleration (b) optimum criteria in case of stiff soil earthquake ground motion model.

508x242mm (300 x 300 DPI)