**Limit cycle dynamics of the gymnastics longswing**

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**Abstract**

The purpose of the study was to examine the macroscopic dynamics of the longswing through a limit cycle analysis of the motion of the center of mass (CM) as a function of skill level. One elite international, five gymnasts able to perform release and regrasp skills, five gymnasts unable to perform release and regrasp skills, and five novice gymnasts each performed four consecutive longswings on a high bar. Kinematic data were collected to facilitate the calculation of the center of mass position of the performer during swinging. Poincare plots were used to explore the limit cycle dynamics of the center of mass movement. The attractor dynamic was very close to a one-dimensional limit cycle for the elite (D=1.18) but higher for the release and regrasp group (D = 1.35 ± 0.06) and non-release and regrasp group (D = 1.37 ± 0.07). The novice dynamic was characterized by a two-dimensional limit cycle (D = 2.49 ± 0.28) that also had more variability and lower determinism. In the frequency domain, Inharmonicity was lower and the Q factor higher as a function of increased skill level. The findings show that the dynamical degrees of freedom of the CM in the skilled performance were reduced compared to those of novices and represented a more efficient and predictive, rather than exploratory, technique.

**217 words**

**KEY WORDS:** dynamical systems, variability, motor control, collective variable, biomechanics, gymnastics

**1.0 Introduction**

The concepts of thermodynamics and self-organization have underpinned the introduction of dynamical systems approaches to further the understanding of movement coordination, control and skill (Kelso, 1995; Kugler & Turvey, 1987; Kugler, Kelso, & Turvey, 1980; Lipsitz & Goldberger, 1992). Dynamical approaches seek low dimensional solutions, such as limit cycles and attractors, to capture the global macroscopic movement dynamics of the complex multi-segment musculoskeletal system in action. Macroscopic coordination dynamics in motor control has typically been limited to the consideration of relations between two limb segment oscillators through measures of relative phase and vector coding (Haken, Kelso, & Bunz, 1985; van Emmerik, Ducharme, Amanda, & Hamill, 2016). However, Bernstein’s (1967) problem was to understand how the many degrees of freedom (DF) of the system are organized so as to master the redundancy of the system: a multivariate challenge that requires a theoretical and experimental strategy to decompose the contribution to system control of the many degrees of freedom (DF) in whole-body action.

Gel’fand and Tsetlin (1962), in an early systems approach, sought a determination of the qualitative structural relations by distinguishing essential variables from the nonessential variables, where the latter were hypothesized to play primarily a scaling role in a movement coordination pattern (see Kugler et al., 1980). The coordination dynamics framework has a macroscopic qualitative variable in movement dynamics fulfilled by an order parameter of relative phase in the bimanual task (Haken et al., 1985; Kelso, 1995).

In the coordination dynamics framework, it is assumed that mechanical DF of joint space are working to preserve (and are reciprocally preserved by) the dynamic characteristics of the collective variable in the context of a successful performance of the action (Kelso, 1995; Mitra, Amazeen, & Turvey, 1998; Schöner, Zanone, & Kelso, 1992). To date, investigation of the construct of collective variables for complex multi-variable human movement has been largely limited to the 2 DF bimanual task. The unpacking of the collective variable in whole-body motion DF tasks is a more difficult challenge. It has been approached through post hoc decomposition of a set of movement variables through principal component analysis (PCA; Daffertshofer, Lamoth, Meyer et al., 2004; Haken, 1996; Lamoth, Daffertshofer, Huys & Beek, 2009) to derive coordinated components, reduce dimensionality, and interpreted as providing a characterization of the functional or dynamical DF of the movement coordination pattern (Haken, 1995). In addition, multivariate techniques have been demonstrated by Delignières, Nourrit., Sioud, Leroyer, Zattara, and Micaleff (1998), Latash (2010), Nourrit, Delignières, Caillou, Deschamps, and Lauriot (2003), and Vereijken et al. (1992). Alternatively, the macroscopic state of the system has been investigated through the guess of a biomechanically relevant global dynamical variable. For example, Newell et al. (2015) provided evidence that the collective variable for postural control was the phase relation between CM and the center of pressure (CP). Segers, Aerts, Lenoir, and De Clercq (2007) suggested that for gait transitions, the order parameter was the phase relation between the potential and kinetic energy of the center of mass.

For the purpose of this paper, a macroscopic variable is a global variable, biomechanically associated with the performance of the skill (such as the CM which is the key outcome measure of the longswing (Hiley &Yeadon, 2016)), while a collective variable is a way to describe the relative motions of mechanical degrees of freedom, as demonstrated by relative phase measure or PCA techniques (Haken et al., 1985; Daffertshofer, Lamoth, Meijer and Beek, 2004).

Several studies have provided experimental support for the freezing and freeing of the joint space degrees of freedom with learning, as proposed by Bernstein (1967) (Newell, 1991; Newell, Broderick, Deutsch, & Slifkin, 2003; Vereijken, van Emmerik, Whiting, & Newell, 1992).  However, there does not appear to be a single strategy to change in learning as the emergent pathway in the adaptive organization of the torso and limbs in action is strongly influenced by the particulars of the task constraints (Newell & McDonald, 1994). In addition, research has shown that depending on the task constraints and intrinsic dynamics the functional dynamical DF of the attractor dynamic can either increase or decrease with learning (Newell & Vaillancourt, 2001; Newell et al., 2003). The confluence of these approaches has resulted in much debate as to what the DF of movement organization represent in motor control, for example mechanical or dynamical variables (Latash, 2010; Newell & Vaillancourt, 2001).

The longswing on the high bar (Figure 1; also referred to as the “horizontal bar”) involves the rotation of the performer about the high bar in the sagittal plane. The longswing is a fundamental skill that underpins the development of more complex high bar skills. The biomechanics of the joint motions, as well as the inter-joint/segment couplings, in the longswing as a function of skill level has been extensively studied (Busquets et al., 2011, 2013a,b, 2016; Williams et al., 2012; 2015a,b; 2016). However, from a dynamical perspective, the longswing has an implicit limit cycle demand in the task constraints, where consecutive rotations are sustained via the input of energy in each cycle. Specifically, a limit cycle is defined as an attractor with a closed trajectory in phase space. Further evidence to support the use of an angular position-angular velocity (phase space) representation of the skill is found in forward dynamics models. These consider the angular velocity of the center of mass (CM) of the performer about the bar as the key outcome measure (Hiley & Yeadon, 2016; 2014): suggesting that it is a macroscopic dynamical variable. Therefore, the dynamics of the CM could provide evidence of skill level in line with the dimensionality and deterministic properties of that limit cycle dynamic (see Goldfield, Kay & Warren (1993) for a related developmental oscillatory movement example).

———- Figure 1 ———-

The aim of this study was to investigate the limit cycle dynamics of the motion of CM during the longswing on the high bar as a function of the performer’s skill level. It was anticipated that oscillatory properties of the dynamics of the CM would provide a signature of the macroscopic structure of the longswing skill that was not forthcoming from the standard biomechanical kinematic, coupling, and kinetic analyses of joint motions (Busquets et al., 2011, 2013a,b, 2016; Hiley and Yeadon, 2016; Williams et al., 2012, 2015a,b, 2016). A central determination was the correlation dimension of the attractor dynamic of the CM to examine the proposition that the functional, dynamical, DF of a macroscopic variable of the system output is reduced with practice and skill level (Bernstein, 1967; Mitra et al., 1998). The phase plane characterization was supplemented with time (recurrent quantification analysis - RQA) and frequency (spectral and harmonic analysis) analysis of the motion of the CM to provide further insights to the macroscopic dynamics of the longswing. The analysis approach taken here with the set of dynamical analyses of CM is a mixture of descriptive and inferential statistics given the small and different number of participants in each skill group.

**2.0 Methods**

**2.1 Participants**

Prior to the onset of the study, approval was gained from the Cardiff Metropolitan University Ethics committee. Sixteen male participants: one elite international GB Squad gymnast (age: 23 years, mass: 71 kg and stature: 1.73 m), two intermediate groups, the first able to perform release and regrasp skills (n = 5, junior elite, mean ± SD age: 14 ± 2.5 years, mass: 44 ± 9 kg, stature: 1.55 ± 0.10 m), and those unable to perform release and regrasp skills (n = 5, junior national gymnasts, mean ± SD age: 12 ± 3.0 years, mass: 42 ± 10 kg, stature: 1.50 ± 0.10 m), and five novice gymnasts who had practiced for 7 weeks to perform the longswing (mean age: 24 ± 4 years, mass: 69 ± 4 kg, stature: 1.74 ± 0.10 m),gave voluntary informed written consent to take part in this study. Informed consent of the parent of legal guardian of participants under the age of 18 years was obtained. The longswing performance here of the novice was after 7 weeks of structured practice where they could complete a circle on the high bar. Anthropometric data were obtained using the digital image technique (Gittoes, Bezodis, & Wilson, 2009; Canon EOS400D SLR, Japan) facilitating the calculation of individual-specific body segment masses based on Yeadon’s (1990) geometric inertia model. Each participant performed a series of four longswings while looped to the high bar. Four was the maximum number of consecutive longswings that novice performers were able to complete. Unilateral kinematic data were collected using an automated 3D motion capture system (CODA) sampling at 100 Hz. Two CX1 CODA scanners (CODAmotion, Charnwood Dynamics Ltd, UK) provided a field of view exceeding 2.5 m around the centre of the bar. Active markers were placed on the lateral aspect of each participant’s right side at the estimated center of rotation of the shoulder and the elbow, at the mid forearm, greater trochanter, femoral condyle, lateral malleolus, fifth metatarsophalangeal and the center of the underside of the bar.

**2.2 Data Processing:**

Raw marker data in the horizontal and vertical directions were identified from 3D CODA output, and all subsequent analyses took place using customized code written in R (http://www.r-project.org). The angular orientation of the gymnast about the bar was described by the circle angle; defined by the mass center to bar vector with respect to the vertical rotating anti-clockwise 0° and 360° saw the CM of the performer above the bar (in handstand) (see Figure 1). Angular velocity of the center of mass about the bar is referred to as ωCM.

**2.3 Data Analysis**

**Poincare plots:** Poincare maps were used to represent the trajectory of the center of mass in the phase space, as a first step to investigate the existence of closed paths and attractors. Poincaré maps are computed taking the Takens' vectors as the continuous trajectory in the phase space (Kantz & Schrieber, 2004). Takens’ theorem states that reconstructed system is equivalent (there is a topological isomorphism) to the original dynamical system phase space that is generated by the observed time series. The n-th Takens' vector is defined as: T[n]=time.series[n],time.series[n+timeLag],...time.series[n+m\*timeLag]            (1)

The Takens' vectors were built from the angular velocity time series, time lagged by 18 points and embedded in a 3-dimensional space (Kennel, Brown & Abarbanel, 1992). The time lag was calculated using time delayed mutual information (TDMI), as suggested by Fraser and Swinney (1986). Unlike the autocorrelation function, the mutual information takes into account also nonlinear correlations, making it a suitable choice for non-linear dynamical systems.

**Correlation Dimension (CD)**: CD was used to evaluate the dimensionality of the ωCM limit cycle embedded in the phase space and it is the most common measure of fractal dimension, which provided an insight into the number of dynamic degrees of freedom of the collective variable. CD was estimated using the Grassberger and Procaccia algorithm (Grassberger & Procaccia, 1983a, b).

**Determinism (DET):**  Both Poincare and RQA analysis are tools to study the capacity of a system to return to the same status within a certain margin, revealing structures not easily detectable in the original state space, helping identify the existence of attractors, and revealing the self-similarity present in fractal systems (van Emmerik et al., 2016; van Mourik, Daffertshofer, & Beek).

Determinism (DET) was calculated over a length of 10 points and radius r = 0.1, to measure the proportion of recurrent points forming diagonal line structures in the RQA. The designation of determinism comes from repeating or deterministic patterns in the dynamic. Periodic signals (e.g. sine waves) will give very long diagonal lines, chaotic signals (e.g., Hénon attractor) will give very short diagonal lines, and stochastic signals (e.g. random numbers) will give no diagonal lines at all (unless parameter RADIUS is set too high).

**Phase space:** The CM movement was represented in an angle-angular velocity phase space in order to provide information about the variability of the ωCM relative to the circle angle, which provided an insight into the reproducibility of the macroscopic variable as a function of circle angle. Angular velocity of the center of mass (ωCM) was interpolated in 1° increments of rotation about the bar and presented as a polar plot (see Figure 3). For this analysis, kinematic data were filtered by way of a fourth-order low-pass Butterworth filter, cut-off frequency 10 Hz (Winter, 2005). Variability of ωCM over the four swings at each degree in the circle was calculated based on the standard deviation (SD). Average variability during the whole swing and in each quartile was calculated.

**Frequency analysis:** Frequency analysis were used in order to investigate the characteristics of the periodic, regular behavior of the oscillation, which provided an insight into the mechanical efficiency of the system. A frequency analysis was performed on the raw (not interpolated) ωCM data using the seewave package (http://rug.mnhn.fr/seewave/) to perform Fast Fourier Transform with 216 point moving Hanning window with no overlap. Discrete variables; base frequency, Q Factor, Inharmonicity Index and Simpson Entropy were calculated from the Fourier spectra. Base frequency was the fundamental frequency peak in the spectrum. The Q Factor was calculated based on the Q function in seewave and was the resonance quality factor at -3 dB level. Inharmonicity Index, calculated based as the average of the exponential of the deviation of a peak from a perfectly harmonic spectrum (Equation 2), is a quantitative description of how the spectrum was deviating from a perfect harmonic, where 0 is a purely harmonic spectrum as the overtones are integer multiples of a fundamental frequency.

(2)

where I is the Inharmonicity Index, N is the number of spectral components, *fi* is the frequency of the i-th peak in the spectrum, and *f1* is the fundamental frequency (Williams & Vicinanza, *minor revisions*).

Finally we can define a Spectral Structure Index as the ratio between the Q Factor and the Inharmonicity Index. Larger values of the Spectral Structure Index are associated to highly resonant oscillatory dynamics, smaller values represent less resonant dynamics with broader frequency ranges and more significant contributions from inharmonic components.

**3.1 Results**

**3.1.1 Poincare Plots and Correlation Dimension**

Poincare plots (Figure 2) show the limit cycle dynamics through a closed trajectory in the phase space. The cross section of this trajectory is almost one-dimensional for the elite (CD = 1.18), but two-dimensional for novices (Table 1). A significant difference was observed between the intermediate groups and the novice group (F(2,12) = 75.13, p = 1.64e-07, omega squared = 0.91). Post-hoc comparisons was carried out using Tukey’s HSD test, showing a significant difference between the no release regrasp/novices (p = 6e-07), and the release regrasp/novices (p = 5e-07). There was no significant difference between the two intermediate groups (release and regrasp/no release and regrasp, p = 0.970).  Position divergence can clearly be seen, particularly around the top right of the maps for novice and intermediates, that is relatively more stable for the elite gymnast. That the Poincare plot covers a larger area for the intermediate and novice participants implies that more energy is involved in the system.

-——— Insert Figure 2 around here ———-

-——— Insert Table 1 around here ———-

**3.1.2 Recurrence Plots and Determinism**

Recurrence plots revealed the non-linear variability, or recurrence, in the Poincare plots. Solid diagonal lines for the elite participant reflect high recurrence rate in the phase space over swings. Lower recurrence was shown for the novices through the ‘feathered’ appearance of the diagonal lines (Figure 3).

-——— Insert Figure 3 around here ———-

Determinism was higher for more skilled participants (Table 1). A significant difference was observed between the intermediate groups and the novice group (F(2,12) = 6.45, p = 0.013, omega squared = 0.42). Tukey’s HSD test showed a significant difference between no release regrasp/novices (p = 0.018), and release regrasp/novices (p=0.031). The difference between the two intermediate groups was not significant (p=0.949).

**3.1.3 Phase Space Plots**

Phase space plots (Figure 4) show that the ωCM was smoother, more symmetrical, and more consistent for the elite and intermediate gymnasts, when compared to the novice gymnasts. Maximum ωCM was higher for the novice gymnasts, suggesting a less efficient longswing.

-——— Insert Figure 4 around here ———-

Variability of the ωCM was on average higher during the whole swing and during all quadrants of the swing for the novice group, compared to the intermediate gymnast groups, where the elite gymnast had the lowest variability in ωCM between swings (see Figure 4; Table 2). In quartile 1, there was a significant difference between the novice and the intermediate groups (F(2,12) = 7.872, p = 0.007, omega squared = 0.48), with the novice group being the most variable and the release and regrasp group being the least (no release regrasp/novices: p = 0.006; release regrasp/novices: p = 0.042). The difference between the two intermediate groups was not significant (p = 0.549).

In quartile 2, there was a significant difference between the groups (F(2,12) = 4.647, p = 0.032, omega squared = 0.33). The post-hoc analysis showed that the no release regrasp group had significantly less variability than the novice group (F(2,12)=4.647, p = 0.039), while the difference between the two intermediate groups was not significant (p = 0.549) as the difference between release and regrasp and novices (p = 0.075).

No significant differences between groups were observed in quartiles 3 and 4. Quartile3: F(2,12)= 2.785, p = 0.102 (no release regrasp/release regrasp, p = 0.981; no release regrasp/novices, p = 0.169; release regrasp/novices, p = 0.125). Quartile4: F(2,12)=0.287, p = 0.756 (no release regrasp/release regrasp, p = 0.986; no release regrasp/novices, p = 0.842; release regrasp/novices: p = 0.756).

**3.1.4 Frequency Analysis**

The elite gymnast had the most harmonic spectrum for the ωCM, while all of the novices had the least harmonic spectrum (Table 3). Intermediate groups had significantly lower Inharmonicity than the novice group (F(2,12)=32.49, p = 0.000, omega squared = 0.81). Post-hoc analysis showed a significant difference between no release regrasp/novices (p= 0.000) and release and regrasp/novices (p = 0.000). No statistically significant difference was observed between the two intermediate groups (p = 0.493).

The Q factor was higher for elite and intermediate gymnasts compared to novices (F(2,12)= 32.08, p = 0.000, omega squared = 0.81). Post-hoc analysis showed a significant difference between no release regrasp/novices (p = 0.000) and between release regrasp/novices, p = 0.000). No statistically significant difference was observed between the two intermediate groups (p = 0.914)

The spectral structure index was significantly higher for the intermediate groups compared to the novices (F(2,12)=24.17, p = 0.000, omega squared = 0.76). In particular, post-hoc analysis highlighted a statistically significant difference between the two intermediate groups (release regrasp/no release regrasp, p = 0.010), between no release regrasp/novices (p = 0.015) and between release regrasp/novices (p = 0.000).

-——— Insert Figure 5, and Table 3 around here ———-

**4.0** **Discussion**

The aim of this study was to investigate the limit cycle dynamics of the motion of CM during the longswing on high bar as a function of skill level. Since the performers can all satisfy the biomechanical demands of the task, the goal is reproducibility and efficiency in the action, and we can assume that this is the product of practice. The limit cycle (based on the angular displacement and angular velocity of the CM) provided a dynamical signature of the macroscopic structure of the longswing skill that complements the standard biomechanical and inter-joint/segment coupling analysis of joint motions (Busquet et al., 2011, 2013a,b, 2016; Williams et al., 2012, 2015a,b, 2016) and provides evidence to support dynamical systems models of the DF problem in motor skill acquisition (Bernstein, 1967; Newell, 1985).

Overall, the structure of the limit cycle had a higher dimension, was less variable (quantified by SD), and was reflected by a less harmonic and resonant, less mechanically efficiency, oscillator for novice participants compared to intermediate and elite. The analysis confirmed limit cycle behaviour of the CM during the longswing via the closed trajectory in the state space. Extending previous work on the biomechanics and inter-joint/segment coupling of the longswing at different levels of performance (Busquets et al., 2011, 2013a,b, 2016; Hiley, Zuevsky and Yeadon, 2013; Williams et al., 2012; 2015a,b; 2016), the limit cycle is present due to the non-conservative gymnast-bar system that required a systemic injection of energy to preserve oscillations. If, for example, the frictionless gymnast moved about the bar as a stiff rod - a point attractor dynamic would have been observed.

Exploring the structure of this limit cycle, the correlation dimension provided evidence that the correlation dimension, representing the number of dynamical DF, underpinning skilled performance is lower than that of the less skilled and particularly the novices. Correlation dimension quantified an attractor close to a one-dimensional limit cycle for the elite (CD=1.15) and the intermediate (range CD=1.27-1.45) gymnasts, while the novice dynamic was in the order of 2 dimensions (range CD = 2.05-2.72). Previous studies have provided support for a reduction in dynamical DF with skill learning (Mitra et al., 1998; Newell & Vaillancourt, 2001; Newell et al., 2003; Vereijken et al., 1992) as well as breaks in recurrence (Eckmann, Kamphorst & Ruelle, 1987). However, it should be noted that lower dimensions in skilled or healthy states have been predicted for cyclic tasks, whereas the demands of a point attractor task have been associated with higher dimensional attractors in the less skilled (Vaillancourt and Newell, 2002).

While more variability and irregular behavior was present for novices, the limit cycle structure provides evidence given the relatively lower dimension (CD=2.05-2.72) than that of a random white noise profile (CD tending to infinite - uncorrelated random process). To explore further, Poincare plots and determinism through recurrence quantification analysis (RQA) were used to quantify self-similarity present in the system (stability and structure) within the series of repetitions in the phase space. Determinism (DET), the reproducibility of the signal, was higher for intermediate groups (release and regrasp 0.68 +- 0.05 %; non release and regrasp group 0.71 +- 0.7 %) compared to novices (0.43 +- 0.21%), suggesting that more of the data fell in the region of stochastics for novice participants. Since the novice DET is lower, it is suggested that the fewer shorter-range local correlation provide evidence for a ‘reactive’ rather than ‘predictive’ technique, where changes in one part of the circle are not associated with previous or future changes.

The elite gymnast on the other hand, shows a higher determinism, suggesting that predictive adjustments are made. The lower dimension of the limit cycle as a function of advanced skill level is consistent with the long held position on motor skill acquisition that with practice there is a shift in emphasis of control from peripheral to central sources (Pew, 1966).  In this view, the movement trajectory becomes more predictive and efficient with fewer perceptually based corrections of the movement trajectory as a function of practice; an interpretation that would also lead to a lower correlation dimension.  However, given the lack of experimental manipulations here beyond practice and skill level, we are not in a position to interpret the change in dimension of the limit cycle dynamics in this way, though as noted it is consistent with it.  Together these findings provide evidence to support performers reaching the learning stage of control (Newell, 1985) and reducing dynamical degrees of freedom (Bernstein, 1967) during learning. While it is acknowledged that there were differing ages and skill levels included in this analysis and while this is a limitation to generalizing the results to developing young gymnasts, this paper provides initial evidence of the changing non linear dynamics of a macroscopic variable as a function of skill level. This analysis also highlights the importance of future dynamical analysis of skill learning considering the notions of predictive and reactive phenomena in task dynamics.

**4.1 Variability**

Variability, and its functionality, is a notion being explored in human movement science (Newell & Corcos, 1993; Newell & McDonald, 1994). Evidence is being accrued that will enable greater understanding of the ‘level’ of variability associated with skilled and unskilled performance, and how this changes across different levels of the system. The current data provide evidence that variability of the macroscopic variable is on average lower during the longswing skill for the more skilled participants compared to novices. However, it is also clear that while the amount of variability is significantly lower in some quartiles of the circle (1st and 2nd), this difference is reduced in the latter half of the circle that does not provide statistically significant results for more skilled groups compared to novices.

Lower kinematic variability in the time domain for more skilled participants is consistent with the findings of previous literature, where a ‘U’ shaped association between variability in joint coordination variables and skill level (from novice, through intermediate, to elite) has been found (Busquets et al., 2016; Wilson et al., 2008). Hiley, Zuevsky & Yeadon (2013) identified that temporal and spatial (circle angle position) variability of hip and shoulder actions mechanically related to longswing performance was lower for elite gymnasts, compared to sub-elite gymnasts. However, the elite gymnasts had higher variability in actions performed near the top of the circle, that were later shown to be related to the control of the ωCM in Hiley and Yeadon (2016). Therefore, these papers provide a possible explanation for how the elite gymnast in this study was able to maintain a less variable ωCM throughout the swing compared to less skilled gymnasts, using the hip and shoulder angles in less mechanically important aspects of technique, and also support that the ωCM was most variable in what was referred to here as quadrant 4 of the circle, but captures the position between (2) and (3) in their representation (Hiley et al., 2013). However, it should be highlighted that the level and measure of the system will capture different aspects of variability. For example, for higher skill levels, variability is likely to be low in higher order variables that are associated with performance outcome, but may again be high in variables, or combinations of variables that are associated with underpinning mechanical degrees of freedom (Arutyunyan, Gurfinkel & Mirskii, 1968; Busquets et al., 2016; Wilson et al., 2006).

Busquets et al. (2016) explored inter-segmental coordination as a function of experience, and suggested that a reduction in arm-trunk variability was associated with completing the skill, while variability of the functional hip and shoulder events could provide adaptive flexibility to support more complex skills. Specifically, key variables P2H and P3S were characterized by a ‘U’ shaped function, with inexperienced and experienced participants showing higher inter-trial variability than intermediates. Therefore, Busquets et al. (2016) provided evidence that high inter-segmental coordination variability may support the lower ωCM variability shown in this study through (Busquets et al., 2016). Most other technique variables supported the current proposition that variability decreased as a function of experience of skill level (Busquest et al., 2016). Therefore, fundamental questions about the functional nature of variability for both novices, facilitating exploration of their environment, and skilled performers, maintaining overarching stability, are again highlighted at the joint/segment level, which relate to success. Longitudinal studies that capture both the performance, macroscopic and underpinning DF variability are required to further understand the association of performance outcome variability with underpinning technique variability (Busquets et al., 2016; Hiley & Yeadon, 2013; 2014; 2016).

**4.2 Frequency**

We used RQA and Poincare to introduced the concept of recurrence and periodicity. Extending the analysis to the frequency domain the periodic structure of the CM angular velocity behavior was investigated. From a quantitative perspective, the measures of Q and Inharmonicity provide a description of the profile of the spectrum, rather than a statistical breakdown of its content, showing that the movement of the ωCM was described by a largely harmonic and resonant oscillator. Specifically, elite swinging was described by a more harmonic combination of oscillations that has a stronger resonance than novice swinging.

A fundamentally harmonic structure was identified for the elite and skilled swings, suggesting that the macroscopic variable was dominated by changes that were at integer multiples of the swing frequency. Thus, the external drivers, the flexion and extension actions at the hips and shoulders, were occurring very close to harmonics of the resonant frequency in order to maximize the energy that the swing absorbs (Goldfield et al., 1993).  For novices, harmonics were significantly weaker (I = 0.25 +- 0.08) than for skilled participants (release and regrasp group I = 0.03 +- 0.01; non release and regrasp group I = 0.06 +- 0.02), suggesting that greater Inharmonicity is a characteristic of low skill level and, a less mechanically efficient swinging action.

The resonance quality (Q) was significantly lower for novices compared to skilled performers, and was highest for the elite gymnast, showing a lower dispersion of frequencies around peaks in the amplitude. Therefore, the novice technique was characterized by a more complex action not yet tuned to the task dynamic. Higher resonance suggests that, mechanically, the system is more ‘tuned’; phase coherent, efficient, and less susceptible to perturbations for skilled participants. The concept of stability is explored by van Emmerik et al (2016) who suggest that movement across an attractor landscape finds regions of stable states throughout the learning period.

From a motor control perspective, Inharmonicity can be considered a measure of complexity (Lipsitz & Goldberger, 1992); the closer to 0 the Inharmonicity, the more harmonic, hence simpler is the global dynamic. Inharmonic components suggest multiple oscillators that could be considered to be manifest through active degrees of freedom. Based on this proposition, the more inharmonic spectrum provides support for the greater dynamical degrees of freedom that characterize novice performance compared to skilled performance for the longswing task. Von Holst and Mittelstaedt (1950, 1973) suggested that integer or phase coherent frequencies were not the only functional relations that could exist in nature. However, oscillations independent of the base frequency suggest a less mechanically efficient action, which in line with previous biomechanical work on the mechanical efficiency of skill learning (Williams et al., 2015), that showed novices are less mechanically efficient that more skilled participants.

Efficiency is the last stage of motor learning models (Bernstein, 1967; Newell, 1985; Sparrow & Newell, 1998) and while an indirect measurement of bio-mechanical efficiency, the oscillatory properties of the center of mass velocity provide evidence that the elite followed by the intermediate gymnasts were more mechanically efficient than the novices. While this study is limited by the number of participants and trials included, overall, the frequency analysis of the phase plane dynamics highlights the concepts of complexity, stability and efficiency in a novel way.

**5.0 Concluding Comments**

The analyses of the CM motion have revealed its limit cycle dynamic structure as a function of skill level. The findings show that the dynamical degrees of freedom of the CM in the skilled performance were reduced compared to those of novices and represented a more efficient and predictive, rather than exploratory, technique. The motion of the CM is that of a macroscopic variable that acts in a manner consistent with the expectations of a collective variable (Kelso, 1995), the direct association of which requires further study.

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