ANGLIA RUSKIN UNIVERSITY

SHAPING PLATO:
INTERPRETING PLATO'S PHILOSOPHY THROUGH HIS GEOMETRY

JUDITH BAILEY

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ANGLIA RUSKIN UNIVERSITY

ABSTRACT

FACULTY OF ARTS, LAW AND SOCIAL SCIENCES

DOCTOR OF PHILOSOPHY

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'Let no one who is not a geometer enter' was allegedly displayed outside Plato's Academy and his love of geometry is revealed by the many geometrical references occurring throughout his work. They are often ignored but I demonstrate their importance by showing that Plato's use of geometry can help to shape our understanding of his philosophy.

Although I mention other examples I focus on two geometrical references. I examine Plato's exploration of apparently diminutive distant objects, whose real size can be found using geometrical calculations, and his employment of *diagramma*, which implies a diagram associated with theorems. Both are repeated and I track them across several dialogues to analyse, from Plato's perspective, the validity of the associated philosophical arguments, and the significance of the repetitions. I also use my findings to critique, from a modern viewpoint, some theories of Platonic interpretation.

I found that Plato associates distant objects with pleasures, which can also appear deceptively small, and *diagramma* with the acquisition of eternal and unchanging knowledge. Examining each individual account leaves unresolved issues, but I show that Plato's repetition of the geometrical examples allows him to continue discussions aimed at philosophers across several dialogues, rectifying omissions and demonstrating the value of writing as a reminder. Analysis of his use of geometry also supports various interpretations of the dialogues, relating to their reception, the role of Socrates, implied references to the Forms, and the spuriousness of the *Epinomis*.

From my examination of Plato's repetitive use of geometry I deduce that it is incorrect to read each dialogue as an autonomous text. I conclude that the extended discussions have a value for Plato, as he advances his views for the edification of philosophers, and also for us, as we use them to gain insights into aspects of the dialogues beyond those envisaged by Plato.

Key words: Plato, geometry, philosophy, pleasure, knowledge, interpretation.

Table of contents

Introduction 1

Chapter One. Plato's geometry 13

1.1 Geometry in ancient Greece 14
1.2 Geometry in Plato's dialogues 22
Summary of Chapter One 30

Chapter Two. The size of distant objects and pleasures 32

2.1 The *Protagoras* 33
2.2 Analysis of the application of the analogy 40
2.3 The *Republic* 48
Summary of Chapter Two 57

Chapter Three. More on the distant object analogy and pleasures 59

3.1 The *Philebus* 60
3.2 Analysis across the *Protagoras, Republic* and *Philebus* 69
3.3 Insights given by the analogy and its geometry 76
Summary of Chapter Three 82

Chapter Four. *Diagramma* and the acquisition of knowledge 84

4.1 Plato's theory of knowledge 86
4.2 The *Lesser Hippias* and lies 93
4.3 The *Cratylus* and errors 100
4.4 The *Theaetetus* and false theories 107
Summary of Chapter Four 114

Chapter Five. Further implications of *diagramma* 116

5.1 The *Euthydemus* and discovery 117
5.2 The *Phaedo* and recollection 124
5.3 The *Republic* and reason 131
5.4 The value of *diagramma* in Plato's dialogues 136
Summary of Chapter Five 143

Chapter Six. The geometrical references applied to Platonic interpretation 145

6.1 Cross-dialogue connections 146
6.2 The reception of the dialogues 153
6.3 Form and content 161
Summary of Chapter Six 168

Conclusion 171

Bibliography 176

Appendix: Mathematical expressions and calculations 196

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Introduction

Αγεωμετρητος μηδεις εισιτω[1]

In the fourth century CE an anonymous scholiast, in the annotation of a manuscript by Aelius Aristides, wrote that the above words were inscribed at the front of Plato's school. Fowler (1987, pp.200-201) gives the translation as 'Let no one who is not a geometer enter', noting that sacred places often had signs saying 'Let no unfair or unjust person enter', so that *ageōmetrētos,* ungeometrical*,* replaces unfair or unjust. The focus of modern debate is on whether the sign ever existed but for me the vital significance lies in the wording. The legend of the necessity for an understanding of geometry for entrants to the Academy indicates that, in antiquity, it must have appeared that geometry was extremely important to Plato. This is supported by his many references to geometry in the dialogues and his insistence, in the *Republic*, Book 7, that all would-be philosophers must be trained for several years in geometry.

Nowadays philosophers and classicists are not necessarily required to have a love of any mathematics and many of those who study Plato's work overlook the geometry that is scattered throughout the dialogues. A search on any database for articles or books on 'Plato' and 'geometry' reveals very few hits[2]. The main purpose of my work, therefore, is to show that Plato's geometry should not be ignored. I consider the question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how? My hypothesis is that geometry, for Plato, was more than an illustrative tool to clarify a particular point in a dialogue, since his repetition of geometrical examples reveals further aspects of the associated discussions which he can aim at philosophers. In addition, our comprehension of Plato's philosophy is shaped by our interpretive skills and the geometrical examples have a role to play in promoting further insights. The research necessary to demonstrate both these points allows me to make several contributions to Platonic scholarship through a topic which is rarely investigated.

In taking this approach I fall between two major hermeneutical points of view. As Connolly and Keutner (1988, p.26) observe, before the twentieth century hermeneutical theory

[1] *Ageōmetrētos mēdeis eisitō*. All later Greek will be given only in transliteration.

[2] Forexample, on 22 May 2015, Web of Science (an online database searching over 55 million records) had 10 152 hits for 'Plato', with only 43 hits for 'Plato and geometry'.

declared that 'interpretations are decidable, and they are so either by virtue of the author's intention, or by virtue of the text-intention'. They continue by noting that in the twentieth century it was claimed that 'interpretations are not decidable' because 'every interpretation gives the text a new reading within the framework constructed by the pre-judgments of the reader's time, epoch, or culture'. When I assess Plato's geometrical references I argue for the feasibility of surmising what his intentions were but I also give 'the text a new reading' based on my assessment of his use of geometry, seen from a twenty-first century perspective.

I look to Gadamer (1975) for support for this approach. He proposes the notion of a horizon which is 'the range of vision that includes everything that can be seen from a particular vantage point' (p.269). He then asks whether there are 'two different horizons here, the horizon in which the person seeking to understand lives, and the particular historical horizon within which he places himself' (pp.270-271). He concludes (p.273):

> There is no more an isolated horizon of the present than there are historical horizons. Understanding, rather, is always the fusion of these horizons which we imagine to exist by themselves. ... it is part of the hermeneutic approach to project a historical horizon that is different from the horizon of the present. ... In the process of understanding there takes place a real fusing of horizons, which means that as the historical horizon is projected, it is simultaneously removed.

Gadamer's concept of the fusing of horizons has not been universally accepted and Gadamer cites, for example, the criticisms of Pannenberg, which he refutes (1977, pp.36 ff). Other authors apply the expression to various situations, with Malpas considering a 'fusion of horizons' with regard to the ideas of Gadamer and Davidson (2002, p.195). I apply a 'fusing of horizons' as I start with the texts themselves, in order to let Plato's voice be heard, but then move to the present in order to interpret the dialogues from a modern perspective. Thus I allow Plato to enter into a dialogue with the reader, demonstrating a further point made by Gadamer (1991a, p.19): 'the Platonic dialogue will awaken a living discussion now as before, and will achieve the fertile fusion of all horizons in which, questioning and searching, we must find our way in our own world'.

The method I adopt is to track key geometrical references across numerous dialogues but although I use a linguistic feature to explore links between the texts I also consider the background context, which offers the possibility of new interpretative insights. The geometry is embedded within passages on philosophical topics and I show that there is a continuity and development of these discussions, allowing a more complete analysis of some aspects of Plato's philosophy. In so doing I draw upon several authors who examine the possibility that Plato spread his arguments over more than one dialogue. I give a brief survey of some of

these works before explaining how tracing the exploration of geometry adds to the value of Platonic interpretation more than using other topics.

Miller (1986, pp.168 ff) considers Plato's assessment of the nature of the soul in the *Parmenides* and concludes that it can be seen as a critical re-appropriation and epilogue of the *Republic* and he also suggests that it 'introduces several of the central new teachings of the *Sophist* and the *Statesman*' (p.168). To do justice to such connections will, he claims, 'require several whole new studies in the future' (p.168). Later, Kahn (1996) combines passages from three dialogues to examine one philosophical topic. He suggests that he is making 'controversial claims' when he states: 'the three dialogues, *Laches-Euthyphro-Meno*, form a unified, continuous, exposition of the logic of definition' (p.149). His stated aim is to demonstrate that 'they were planned as, and intended to be read as, an interlocking group' (p.170). Kahn later (2003) tracks another philosophical subject across three dialogues when he explores *anamnesis* in the *Meno*, *Phaedo*, and *Phaedrus*. He concludes: 'the obscurities and omissions in each version of Recollection can be clarified and completed by reading the three dialogues together' (p.312).

My initial conclusions are similar to those found by Kahn but I claim that further insights are obtained by using non-philosophical examples. He employs the occurrences of the topics themselves to track their development across the dialogues. My geometrical references, associated with the passages which I examine, are more distinctive and stand out in a philosophical work. This allows me to stress their repetition, in order to make a connection with Plato's views on the use of writing as a reminder, which he gives at *Phaedrus* 275a. Schofield (2010, p.xxv) states of Plato, 'Repetition is never just repetition'[3] and I claim that the repetitions can serve as reminders of previous debates, allowing the presentation of more complete philosophical discussions than are found in any one dialogue. Geometry also has worth in that it bridges the gap between the thoughts of 'the majority of people' (*Republic*, 479d) and the enhanced understanding of philosophers. I claim that the expertise required to thoroughly understand the principles of geometry allows Plato to draw a distinction between philosophers, who have studied geometry, and non-philosophers, who have not studied the subject. This means that he can restrict some of his advice to philosophers. I also consider some further repercussions of this, as I move outside the text to

[3] Schofield is comparing the list of characters, the milieu and some of the thematic elements in the *Protagoras* and *Symposium* but the statement could apply equally well to the repetitions of geometrical examples.

assess some aspects of interpretation of the dialogues. In doing this I am keeping in mind Gadamer's notion of the 'fusion of horizons'. I look at the possibility of inferring references to the Forms in dialogues where they are not explicitly mentioned, and offer support for the spuriousness of the dialogue known as *Epinomis*. Thus, I provide several examples of *how* our understanding of Plato's philosophy can be enhanced by noting his use of geometry and this allows me to answer both parts of my question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how? Many different geometrical references appear in the dialogues but I limit myself to two, in order to provide a thorough exploration of each. As I describe them, I offer specific evidence of the lack of scholarly attention given to Plato's geometry.

The first one which I examine is the analogy of an apparently diminished distant object, found at *Protagoras* 356c, *Republic* 602c and *Philebus* 41e-42a. Geometry is needed to find the true height of the object and Plato suggests that the true size of distant, or future, pleasures and pains should also be calculated. The reference to the estimation of distant objects is very distinctive and I am not the first person to notice it. Some translations of one dialogue use a footnote to direct the reader to one or both of the other passages, but writers who examine these texts tend not to analyse the analogy. It is noticeable that even books purporting to give line by line commentaries on the dialogues often omit to explore the geometrical analogy. Examples can be given for each dialogue. Denyer (2008), in his commentary on the *Protagoras*, mentions Euclid's *Optics* and propositions which refer to the measurement of height (p.191) but he does not explore the topic further. Several other authors ignore the reference completely. For the *Republic*, Waterfield (1994) lists all sections, but in the one for 602c-e he mentions only 'misled by colouring' (p.448). Frede (1993), in her notes on the *Philebus* section 36c-50e, makes no mention of the distant object analogy (pp.xlv-liii). These examples are representative of many others and the lack of attention is longstanding, as a couple of references from older commentaries demonstrate. Adam (1902, p.407) mentions only 'images in water' for *Republic*, 602c-e, and Wayte (1880, pp.148-9) quotes part of the relevant passage in his notes on the *Protagoras* text, but concentrates on the Greek grammar. A similar lack of interest in the geometrical reference is found in some more general articles about the dialogues. It appears that many authors do not believe that there is any value in noting Plato's use of the distant object analogy, a belief which my work is designed to overturn.

The second use of geometry which I have chosen is the word *diagramma*. As I explain later, the translation of this word is not as easy as its similarity to the English word 'diagram' would

suggest. My understanding of the word is that it denotes a geometrical diagram whose purpose is to demonstrate a particular construction or theorem. Rather than use such a lengthy description as a translation, and unwilling to correlate it simply with a diagram, I refer throughout to *diagramma*. It occurs in seven dialogues, at *Cratylus* 436d, *Lesser Hippias* 367d and e, *Theaetetus* 169a, *Euthydemus* 290c, *Phaedo* 73b, *Republic* 529e and *Epinomis* 991e. In the first six of these it is associated with advice about the acquisition of knowledge. As with the distant object analogy, there is a lack of interest in this geometrical reference amongst Platonic scholars. I offer a few examples, representative of many others across all the dialogues listed.

In an account of the *Cratylus*, given by Gonzalez (1998, p.85), a discussion of lines 437a-c follows that of line 436b, omitting any mention of 436d. In his commentary on the *Theaetetus*, Stern (2008, pp.147-8) discusses lines 169a, b, and c, omitting any comment on the word *diagramma* in line 169a. For the same dialogue, Polansky (1992, p.127) covers lines 168c2-169c7 but does not mention *diagramma* at 169a. Bostok (1988, p.84) jumps from the defence of Protagoras in lines 166a-168c to the refutation starting at line 169d, omitting any reference to line 169a. Cooper (1990, pp.72-77), among others, examines the same grouping of lines and Chappell (2004, pp.108-117) examines lines 168c-171c but skips over 169a without mentioning *diagramma*. Many commentaries on the *Phaedo* skip line 73b1, where the word occurs. Examples of such an omission occur in Bostock (2000, p.63), who mentions sections 72e-73a and then goes to lines 73b3-4, and Gallop (1993, pp.88-89), who analyses section 73a and then skips to 73d. Rowe (1993, p.164), does not list 73b1 but when analysing 73a10 he suggests that 'taking them to diagrams' is connected to 'the case of the slave-boy in *Meno*'. Articles and books which discuss text which contains *diagramma* in any of the listed dialogues rarely include any reference to this unusual word which, I claim, means that worthwhile insights into Plato's philosophy are missed.

Research into any aspect of Plato's work has several associated problems, and some create limitations which must be incorporated into this thesis. One problem could be that of varying translations of the text. Fortunately my hypothesis is not dependent on any particular translation. When there are significant discrepancies across different translations I give the alternative versions, especially with regard to the term *diagramma.* Secondly, there are copious books and articles about each dialogue and any philosophical topic which Plato considers. Associated with the geometrical examples which I examine are Plato's views on pleasure and knowledge, both of which have extensive secondary literature. I impose a limit on my use of this and consider mainly that produced in approximately the last fifty years, in

English. Earlier landmark work, and that in other languages, is usually re-examined by more recent authors and translated into English where necessary. However, I refer to a few early texts when these are cited repeatedly in later works. I have kept in mind, throughout my research, that my interest is in Plato's use of geometry. I limit my analysis of pleasure and knowledge, therefore, to those aspects which are relevant to Plato's geometrical examples. However, within each of the dialogues under consideration, I examine several aspects of the background, for use when I assess the value of Plato's geometry.

My approach to analysing the geometrical references

Eco (1979, p.21) suggests that 'no text is read independently of the reader's experience of other texts'. This is a particular problem for readers of Plato's texts. We have over two thousand years of philosophical writing to give us a different attitude, concerning some of the philosophical problems and notions that are discussed in the dialogues, than the original audience would have had. Heidegger (1975, p.85) was particularly concerned about such a point:

> All depends on whether the dialogue we have undertaken first of all and continually allows itself to respond to the questioning address of early thinking, or whether it simply closes itself off to such an address and cloaks early thought with the mantle of more recent doctrines.

In an attempt to overcome some of the problems which are caused by distance from the original texts, there has been a move in recent years to examine every aspect of Plato's dialogues. In this way it is hoped to expose some of the 'early thought' associated with the philosophy. Michelini (2003, p.3) explains that 'recent developments in Platonic scholarship have emphasized the grounding of Plato's texts in the culture from which they emerged'. Although such statements do not envisage any examination of the geometry of 'the culture', my examination of the geometrical references could be seen as part of such a method of approach. However, I claim that there is no necessity to remain within the boundaries set by the knowledge of ancient Greece. Gadamer (1977, p.122) suggests that 'understanding can also go beyond the author's subjective act of meaning, and perhaps even necessarily and always goes beyond it'. My aim is to analyse the geometrical references found in Plato's dialogues both as he used them to further his philosophical insights and as we can use them to enhance our interpretation of his work.

In addition to the geometry, I will consider other aspects of the dialogues beyond the purely philosophical topics. Dillon (1999, pp.207 ff), when analysing the Neoplatonic approach, shows that there was a dispute in ancient times about the importance, or otherwise, of the

non-philosophical content of the dialogues. He concentrates on the prologues and notes that, although some ancient authors ignored them, others, such as Iamblichus, commentated on the whole dialogue, including the setting. Proclus (*Commentary on Alcibiades*, 18.15 ff) says: 'the introductions to the dialogues of Plato accord with their overall aims and have not been invented by Plato for the sake of dramatic charm'. The same could apply to any background detail in the dialogues. Scott (2007, p.x) notes that although we may now distinguish between philosophical components and literary or dramatic ones, such categories would not have occurred in Plato's time. This implies that, for Plato, all aspects of the dialogues were important and I adopt this view, adding geometrical references to the elements which are listed by modern interpreters.

The non-philosophical aspects, which authors such as Press (2007, pp.189-190) suggest should be covered, include the characters, dramatic date, physical setting and relation to historical events. I assess each of these in turn. It should be noted that the chronology of the dialogues, once the subject of copious investigations, is now considered unlikely to be resolved with any accuracy. From many possible sources, I cite one author's support for this view about the lack of a decisive chronology. Howland (1991) devotes his article to assessing the validity of several methods of determining the chronology of the dialogues before concluding (p.214): 'the most influential and widely accepted arguments pertaining to the chronology of the dialogues have proved unable to withstand close critical scrutiny'. Fortunately, although my work will cross several dialogues, the order in which the particular dialogues were written does not affect most of my arguments. Any reference to the chronology will only be made with regard to dialogues which are accepted as being written some years apart.

I consider first the importance of the characters. This was recognised by some ancient commentators. Howland (1991, p.193) refers to the *Anonymous Prolegomena to Platonic Philosophy* (15.2-13) and notes that it declares that we must take note of all the characters in the dialogues, 'the questioners and the questioned'. Plato uses different characters in different dialogues, with very few, apart from Socrates, appearing in more than one. Arieti (1991, p.201) observes that 'the cast of characters is frequently a clue to Plato's intent'. He explains that some of the characters are notorious and we should be careful how we accept what they say, as Plato's contemporaries would have known to do. I include a brief assessment of the characters present in each dialogue which I examine, and state what we know about the reputation of the interlocutors and any silent observers of the discussions held by Socrates. I also assess the possible geometrical and philosophical understanding of

the main interlocutors. In this I am greatly helped by Nail's reference book (2002), which lists all the characters of the Platonic dialogues, with any biographical detail that is known.

The other aspects listed above as worthy of coverage are more problematic since Plato does not always make them clear. Halperin (1992, p.121) considers a question which he claims is unanswerable: 'whether Plato intends his Dialogues to be read as works of philosophy or as works of literature'. Wolfsdorf (1998, p.127) makes a similar point, but looks at it from the point of view of the reader. He states, of the dialogues, that one problem is 'how elements we perceive to be dramatic and elements we perceive to be philosophical correlate'. Another problem is stated by Nails (1995, p.33) who denies that we can know why Plato chose the subjects, settings and characters which he did. The implication is that we should not consider them. However, Rowe (2007, p.11) notes that the characterisations and dramatic action are so obtrusive in many dialogues that the notion that they are ornamental should be dismissed. While agreeing with Nails, that we cannot know why Plato made the choices which he did, I maintain that Plato had his reasons and, even though we may not know how, the non-philosophical elements must have some bearing on the philosophical analysis of each dialogue. I will, therefore, examine the literary aspects of the dialogues under scrutiny.

For each dialogue under consideration I examine first what is known about the characters, setting, dramatic date and relation to historical events. I then give a summary of the relevant philosophical content found in the text before the geometrical reference is used, in order to put the geometry in context. I finally assess Plato's use of geometry and how it can enhance our understanding of his philosophy. An enhanced understanding would come from a more insightful Platonic interpretation, which is no easy task. Griswold (1988, p.1) writes of 'the hermeneutic problem of textual interpretation' which leads to several queries such as 'What assumptions about the text do we make when we open a Platonic dialogue?' and 'How would one go about defending them?'. I consider these throughout my work. Griswold later (1988, p.6) mentions being 'prompted by the text'. I use the geometrical references in the dialogues to provide such a prompt, revealing a more thorough analysis of the estimation of pleasures and the acquisition of knowledge, respectively, as the distant object analogy and the use of *diagramma* are explored. Some authors are critical of looking beyond Plato's philosophy and Notomi (2006, p.198) warns that some literary readings avoid the philosophical parts of the dialogues. However, Press (2007, p.191) recommends what he calls an 'integrative reading' which merges both these methods of approach. Cooper (1997, p.xiv) would endorse this, stating that readers of Plato's dialogues should 'concentrate on the literary and philosophical content of the works'. This is the approach which I adopt, with 'literary' including references

to geometry. Within this overall scheme, each chapter has a different purpose, as I explain below.

Synopsis of chapters

Before analysing some of the geometrical references found in Plato's dialogues, I examine the concept of geometry in the ancient world. Chapter One begins with an assessment, using early sources, of the geometry which was known in Athens, in the fourth century BCE. This is not necessarily the geometry to which Plato had access so I then explore how knowledge was disseminated in order to determine, as much as it is possible, the geometry which Plato knew. I deem it necessary to understand the limits of Plato's geometrical knowledge, since otherwise we might expect to see references to topics such as trigonometry, which are familiar to us but unknown to him. I explore how education in geometry can separate philosophers and non-philosophers, so that passages containing the geometrical references can include advice which is specifically useful for philosophers. I move on to an overview of the types of geometry found in the dialogues and a description of the geometrical facts which form the basis of my thesis.

In Chapter Two I start with an examination of the *Protagoras* and the example of practical geometry found in it. Plato uses the need for measurements to determine the true size of distant objects, to support a philosophical argument about the best way to choose pleasures and pains. Although he mentions several different measurements, I defend my decision to focus on the geometrically orientated one. In my analysis of this I reveal some problems with measuring and choosing the pleasures and pains. Plato leaves these unresolved in the *Protagoras* but I note that the same analogy is used in the *Republic* and *Philebus*, where the choice of pleasures and pains is also discussed. I check for any other passages which include similar geometrical references, which could provide further debate about the size of pleasures and pains, and show that there are none. I examine, therefore, how the distant object analogy is used in the *Republic*, where Plato continues his discussion on choosing pleasures and pains. Unfortunately, the passages in the *Republic* do not repair all the omissions detected in the *Protagoras* account.

Chapter Three completes my exploration into the use of the analogy. I examine the *Philebus* and show that the associated passages cover the remaining omissions in the debate about the choice of pleasures and pains. Tracking the analogy across several dialogues has allowed a more complete discussion, offering support for my claim that the geometrical references have value when looking at Plato's philosophy. I suggest that the use of the analogy has a

different significance for different people and I look at some possible targets for the discussions, before considering alternative interpretations which could be made. I end with an overview of my work so far. I claim that it offers support for the importance of noting the presence of the analogy and I assess how the geometry enhances our understanding of the philosophy connected with the choice of pleasures and pains.

In Chapter Four I commence my examination of the second geometrical reference, the single word *diagramma*, found in discussions about how to acquire knowledge. I start with a brief account of Plato's theory of knowledge, approaching it from the perspective of his views about the separation of philosophers and non-philosophers. I claim that the term *diagramma* would only be fully understood by philosophers since it is a specialized geometrical term. I offer support for this proposal by considering the *Meno*, where there are many references to geometrical diagrams but the word *diagramma* is never used. I continue by showing that in some references to *diagramma* Plato gives warnings about how knowledge may be presented falsely. In the *Lesser Hippias* he considers deliberate lies and in the *Cratylus* he looks at unintentional errors. In the *Theaetetus* he proceeds to assess the work of renowned philosophers and decides that this may contain false theories, since even philosophers can be mistaken. The content of all these passages appears to offer no hope for a budding philosopher who wants to obtain advice about how to gain knowledge.

I continue the examination of *diagramma* in Chapter Five, where I present more positive views about the acquisition of knowledge. In the *Euthydemus* Plato claims that there is a possibility that knowledge can be discovered and the notion of recollection, described in the *Phaedo*, gives a way that this can be achieved. Finally, in the *Republic*, philosophers are told to look within themselves in order to find knowledge by reason. The only remaining use of *diagramma* occurs in the possibly spurious *Epinomis*. Before examining this dialogue, I assess my work on *diagramma* so far by summarising how it has been used in relation to the philosophy discussed. When I examine the use of *diagramma* in the *Epinomis* I show that my work demonstrates support for the spuriousness of the *Epinomis*, with some insight into the possible identity of the author. I then assess more fully the meaning of *diagramma* and address the problem of why it is often ignored by writers on Plato. I offer an explanation based on the widespread use of a word similar to *diagramma* in many modern languages. I end with an overview of the value of *diagramma*.

Having claimed that Plato's geometry has an importance for his philosophy I examine, in Chapter Six, the implications of my findings for some of the various theories of interpretation which are applied to Plato's work. In each case, I give a brief outline of the theory being

tested and show how it has gained backing from some scholars, but criticism from others. I use the results, from my assessment of both the chosen examples, to critique these theories. I look first at some opposing theories about Platonic cross-dialogue connections and refer to several examples where cross-dialogue connections have been explored, such as that of Kahn (2003), mentioned above. After using the repetition of the geometrical examples to claim that Plato is providing reminders to aid his readers to find the relevant discussions, I assess the nature of Plato's audience and how the dialogues were perceived by his contemporaries. I demonstrate further roles for the geometrical examples and address the question of why Plato wrote in the dialogue form and the part that Socrates plays, showing that my work can throw some light on these perennial problems. Although I propose that two examples are sufficient to demonstrate that taking note of Plato's use of geometry enhances our understanding of his philosophy I end with a brief account of other geometrical references found in the dialogues. These could provide the basis for further investigations.

The content of each chapter allows me to build from an initial investigation of Plato's repetition of the distant object analogy, through his use of *diagramma*, to an assessment of present-day interpretations of the dialogues. Thus the way that I put into practice Gadamer's 'fusion of horizons' is by starting with the value of Plato's use of geometry within the dialogues and gradually moving to the significance of the geometrical content of the dialogues for modern Platonic interpreters.

Referencing

There are some points which need to be clarified about how I reference the texts. In general, I use phrases such as 'Socrates says' when referring to particular words that are expressed by his character in a dialogue. When talking about an overall view I usually refer to Plato. In addition, the modern care taken to avoid sexism was not a concern in Plato's time, when reference was made to 'man' almost exclusively. I consider it would be anachronistic to use 'she' or even '(s)he' when discussing examples given by Plato. Therefore, the use of the masculine descriptor is maintained throughout, when close reference to Plato's work is made. Since most of the dates given in the thesis are BCE this is omitted, in future, when it is clear that this is what is meant, so that 323 BCE is written as 323. Dates which come after the birth of Christ are designated as CE. Ancient texts are cited in the accepted way, using page number and section from Stephanus' 1578 edition for Plato, and other similar standard line numbering for other authors, such as Bekker's page number, column and line for Aristotle. For Plato's work, I use the translations, by various scholars, found in Cooper's *Plato: complete works* (1997), except on the rare occasions where a comparison is made with a different

translation. These are cited by translator in the Bibliography. The translations of other Greek authors are listed under 'Ancient Texts' in the Bibliography. All passages from Greek texts are given only as English translations, with the exception of a few individual Greek words which are given as transliterations.

Abbreviations

TLG The online Thesaurus Linguae Graecae. A Special Research Project at the University of California, Irvine. It contains digitized versions of most texts written in Greek from Homer (8c. BCE) to the fall of Byzantium in CE 1453, and beyond, with more than 105 million words from over 12 000 works associated with 4 000 authors.

Chapter One

Plato's geometry

1.1 Geometry in ancient Greece 14

Sources of information about ancient geometry 14

The spread of geometrical ideas 16

The study of geometry 20

1.2 Geometry in Plato's dialogues 22

Practical and theoretical geometry 22

The height of a distant object 24

Diagrams and *diagramma* 27

Summary of Chapter One 30

When Plato makes geometrical references he does not give any explanations about why he uses them. We may be aware of more appropriate examples which were unknown to him so in order to assess the value of his geometrical references it is important to try to be clear about what geometrical knowledge was, and was not, available to him. Unfortunately, we cannot be sure about this because there are no extant mathematical texts before Euclid's books, which were written approximately fifty years after Plato's death. My aim in this chapter is to show that, although the original sources are often lost, corrupted or inaccurate, it is possible to assess approximately how much geometry Plato knew. In order to do this it is necessary to decide when certain concepts became known, using the histories of mathematics written by later scholars of antiquity. The availability and reliability of these ancient sources will be examined, although extracting from them the mathematical discoveries which were made does not necessarily tell us what Plato could have known. An image of Plato receiving books of the latest geometrical treatises to read at home is full of anachronisms, so I investigate how knowledge was disseminated in his time. I end the first section of this chapter with an assessment of how the study of geometry could separate philosophers from non-philosophers and I indicate some of the consequences of this claim, which will be explored more fully in the following chapters.

Next, I examine the geometrical references which are found in Plato's dialogues. I study their nature and choose two representative examples in order to give an in-depth examination of each. I take one from practical geometry, the measurement of the size of an apparently diminished distant object, and one from theoretical geometry, the term *diagramma*. I give some background to each of these examples but any associated calculations are given in the Appendix[4].

1.1 Geometry in ancient Greece

Although much of the geometry which we use today originated in ancient Greece, there is uncertainty about how the subject first came to be studied. Our information comes from several sources and I examine some of the problems associated with these. After a brief survey of its possible origins I give an account of the geometry likely to be available to Plato and how he could become familiar with it. I end this section with a claim that the geometrical references can be used by Plato to separate philosophers from non-philosophers.

Sources of information about ancient geometry

Since this thesis is concerned with Plato's choice of geometrical examples, some background work is necessary to give an indication of what the term 'geometry' meant in his time. It might appear logical to confine the enquiry to his account of the subject. In some support of this Fauvel and Gray (1987, p.61) suggest that Plato's own words 'provide some of our best evidence (in lieu of much else) for the mathematical concerns and activities of the early fourth century' but, they continue, 'the evidence provided is not always easy to interpret unequivocally' and they cite several passages to support this view. Plato never made any attempt to write a mathematical treatise so that the background mathematics which he draws on for his views is never explicitly described. It is, therefore, necessary to look at other sources which do describe the mathematical knowledge, in particular the geometry, of his day. The first extant specialized geometrical work, after Plato's death, was by Euclid, who wrote the *Elements* in around 300.

We are fortunate to have so much of Euclid's work extant. It does not appear to have suffered the corruption that many ancient texts did, when they were copied repeatedly over many centuries, possibly because of its mathematical nature. Euclid is thought to have included most of the geometrical knowledge up to his time so it might be assumed that a

[4] My work can be followed without accessing the Appendix but I consider that the content should be available to support my assertions about the knowledge available to Plato.

rough guide to geometry in Plato's Athens could be gained from Euclid's work, but there are several problems associated with this approach. Not all of Euclid's works have survived and, since mathematical knowledge was increasing rapidly in the fourth century, it is difficult to estimate how much was done between Plato's death and the writing of Euclid's books.

The only sources available which might clarify the situation are the works of later scholars who commentated on earlier mathematical texts which were available to them, although they are lost to us. These commentaries date to many centuries after the original texts were written and, unfortunately, the reliability of each of the main sources has been questioned by modern scholars. One example is Fowler (1987, p.8), who says that the writings of Iamblichus, in the third century CE, provide us with comments on the mathematics of Pythagoras and the Pythagoreans but many of the accounts are 'fragmentary, anonymous, secondary and tertiary'. In the introduction to her translation of Iamblichus' *On the Pythagorean life,* Clark (1989, p.xix) says that the situation is complicated by the existence of several 'pseudepigrapha', works which were ascribed to Pythagoras or his followers, written between the third and first centuries BCE. The situation with Iamblichus demonstrates the problems found when relying on ancient texts for information about early geometry.

Proclus is the source of much of our knowledge about Greek mathematics, although he wrote in the fifth century CE. It is generally thought that within his commentary on Book One of Euclid's *Elements* there is a summary of the *History of Geometry* by Eudemus of Rhodes, written about 320 and lost to us. However, Fowler (1987, p298) suggests that some of the work may not have been in Eudemus' original text but could be 'almost certainly a neo–Platonic gloss, perhaps by Proclus himself'. Knorr (1975, p.5) goes further and suggests that 'any late source which ascribes any mathematical discovery to Plato or to Pythagoras or to unnamed Pythagoreans is suspect *per se*'. Unfortunately, as Blumenthal and Clark (1993, p.2) state, much of the work of Proclus is now seen as being heavily dependent on the work of Iamblichus. This implies that the uncertainties in the texts of Iamblichus have been transferred to the texts of Proclus. O'Meara (1989, p.157) also thinks that much of Proclus' text corresponds to that of Iamblichus, but suggests that this is because they both used the same source. Thus it can be seen that there is no agreement and we must exercise caution when citing Proclus' texts.

Following Proclus, Simplicius gives information about early geometry. He quotes from Archytas, a renowned mathematician in Plato's Athens, but in doing so he is accepting material from the person who is now denoted as pseudo-Archytas (Blumenthal, 1997, p.12). Some modern scholars believe that there was no such person, others that he was a

Pythagorean with some of the works ascribed to him being fictitious. Simplicius is generally accepted as being conscientious in his use of earlier works but it appears from his dubious use of pseudo-Archytas that he cannot be completely relied upon.

There remains the problem of some modern interpretations of ancient texts being overturned because advances in scholarship lead to new insights. One example is sufficient to demonstrate this. Knorr (1975, p.93) reports that Archytas, in fragment 4, says that 'arithmetic completes the proofs of geometry'. Huffman (2005, pp.225-6) claims that the translation of this fragment is 'problematic' and also notes that of the fragments of Archytas accepted as genuine, this one is the one about which there is least confidence. Knorr's statement can, therefore, be criticized, possibly because new discoveries have rendered his work somewhat questionable. However, other authors may criticize Huffman's analysis. One way to ensure that the latest theories are incorporated is to refer only to the latest works written, but this is not a feasible approach when dealing with ancient Greek texts. This is because of the texts themselves and also because some standard commentaries, which are still used by the latest scholars, were written many years ago, often in the 1920s. It is therefore important to maintain a critical approach to all such works.

The investigations for this section have shown that we have no totally reliable ancient accounts of the geometry up to Plato's time. However, an examination of the available sources does give an indication of several geometrical discoveries which can be ascribed to individual mathematicians with some degree of certainty. This is not a treatise on textual transmission and so the texts generally considered to be fairly accurate will be accepted at face value, without repeated warnings of possible inaccuracies. It should now be possible to give an account of the geometry discovered by Plato's time, although this is not necessarily what Plato knew.

The spread of geometrical ideas

Burton (2011, p.53), and others, record that there is a tradition that geometry started in Egypt with land measurement, when farms were surveyed after the regular Nile floods for tax purposes, since the areas of land would have changed. Burton notes that Herodotus proposed this as the origin of geometry and he offers some support for the view (p.54). However, Fowler (1987, pp.284-5) rejects this because, he says, there was confusion about whether size should be measured in terms of area or perimeter. To justify his claim he notes that Herodotus (*Histories* i, 170) gave Sardinia as the biggest island in the world although Sicily has a greater area, with a smaller perimeter. A criticism of Fowler's view is that any

possible confusion between area and perimeter need not mean that geometry was not used for land measurement. It might simply imply that it was used incorrectly.

There is then the question of how geometry was brought to Greece. Several sources mention Thales, for example Proclus (*A commentary on the first book of Euclid's Elements*, 65) states that 'Thales, who had travelled to Egypt, was the first to introduce this science [geometry] into Greece'. Heath (1921, pp.128-139) gives many extracts which list the geometrical facts supposedly known by Thales. These include theorems about circles and triangles, such as the fact that the base angles of an isosceles triangle are equal. Heath notes that we have no knowledge of the progress of geometry between the work credited to Thales and the time of Pythagoras and there is confusion about whether Thales or Pythagoras made certain discoveries. One example is the fact that the angles of a triangle add up to two right angles. What is clear is that when work is attributed to Pythagoras it could be the work of his followers. However, it is likely that such work would be known, over a century after Pythagoras, in Plato's time.

The word '*geōmetria*', found in Plato's dialogues, is recognisable as the origin of our word 'geometry' but this does not necessarily imply that the meaning is the same. In modern times, geometry encompasses several different topics: plane geometry, including work on circles, triangles and other shapes, and solid geometry, covering three-dimensional objects such as cubes and spheres. *Geōmetria,* in ancient times, also included these topics but, in addition, it incorporated ideas which we would now express algebraically. Fauvel and Gray (1987, p.142) explain how Zeuthen coined the phrase 'geometric algebra' to describe the results, obtained geometrically, which we would write algebraically. There are several concepts which could be termed 'geometric algebra' in Plato's work, such as the square of a number which would be seen as a square drawn on a line equal in length to the number. We retain this terminology when we recite Pythagoras' theorem as 'the square on the hypotenuse of a right angled triangle is equal to the sum of the squares on the other two sides', although we use the algebraic form of $a^2 = b^2 + c^2$. However, I consider that our understanding of the term 'geometry' is close enough to Plato's use of the word that there is no confusion in applying it without further explanation.

Although Plato refers to many geometrical concepts within his dialogues it is not obvious how he could have become familiar with all the latest ideas in his day. One problem was the scarcity of mathematicians. Netz (1999, p.291) conducted a thorough analysis of all the people mentioned in antiquity as being mathematicians and estimated that each year produced the birth, on average, of one mathematician, plus a few passively interested in

mathematics. He concluded that in every generation there would only be a few dozen active mathematicians who had to discover each other and, being thinly spread across the Mediterranean, were 'doubly isolated, in time and in space'. This would make the dissemination of any mathematics, including geometry, very difficult. Modern scholars have several ways in which to keep abreast of the latest ideas in their fields. Books or articles give the latest theories while personal correspondence can be used for an exchange of information, with letters now being replaced by e-mails. Face-to-face meetings give the opportunity for presentations, questions and discussion. If the reference to electronic communication is omitted, all of these methods for disseminating information would have been available in ancient Greece and I examine each briefly.

I look first at written dissemination of knowledge. We know that books could be bought in Plato's time, with some evidence coming from the dialogues, for example at *Phaedo,* 98b, where the books of Anaxagoras are mentioned. However, Thomas (1992, p.126) makes the point that in a world where only a few copies were made of any text it would be unwise to rely on books alone to ensure survival of any works. They could be lost at sea, copied badly, or otherwise damaged. Personal letters do not have all of these disadvantages but much of the evidence for letters written in Plato's time is dubious. Of the thirteen letters once attributed to Plato modern experts have thrown doubt on the veracity of most of them (Bury, 1966, p.391). However, since letters were more likely to be thrown away than books, the lack of extant copies does not prohibit the existence of a flourishing exchange of letters, between Plato and various mathematicians. Unfortunately, delays between the writing of a letter and its delivery, and further delays for a reply to be received, mean that this was not an ideal way to convey information on the latest geometrical theories. When new theories are postulated, discussion is probably the most beneficial way to explore their ramifications.

Face-to face meetings allow discussion of the latest theories and these could have taken place in Plato's Academy, which he established in the 380s (Scott, 2009, p.271). There is evidence that the Academy was famous for containing the greatest mathematical minds of the age, as the story of the Delian altar illustrates. Riginos (1976, pp.141-2) attributes the original version to Eratosthenes of Cyrene in the third century BCE. When the Delians were told by their god that they must double the volume of their altar they went to Plato at the Academy to ask how this could be done. Some versions of the story say that Plato told them the god was punishing the Greeks for neglecting a study of geometry, but others say that Plato set some geometers at the Academy to work on the problem. Regardless of Plato's response, this story shows that the Academy had a reputation for geometry, and the second

version indicates that Plato could call on several mathematicians in an attempt to solve the problem. The reputation of this centre of learning could thus ensure that Plato kept abreast of the latest geometrical discoveries.

There are several other famous problems which were being researched in the fourth century, with attempts at solutions attributed to mathematicians known to have visited the Academy. These include the trisection of an angle, which sources suggest was solved by Hippias of Elis (Heath, 1921, p.2). Perhaps the most famous problem of Plato's time was the attempt to 'square the circle' which meant constructing a square with the same area as a circle. A reference in Aristophanes' *Birds* (lines 1001-5) shows that the problem was known outside the world of mathematics. However, in the play, the circle is not squared but it is cut into quadrants by two diameters which are at right angles to each other, possibly a comic reference to the problem with which the mathematicians were wrestling. It is likely that the geometers at the Academy would have discussed the latest theories about this, and other problems, allowing Plato to keep abreast of 'cutting edge' geometry.

What is not so certain is how skilled Plato was in using geometry. A search of the dialogues for innovative work is difficult, partly because of the problems mentioned earlier concerning the dating of new geometrical discoveries. However, there are references by ancient authors about Plato's role in inspiring others to great geometrical achievements. Proclus in his *Commentary on the first book of Euclid's* Elements, for example, states:

> Plato.... greatly advanced mathematics in general and geometry in particular because of his zeal for these studies. It is well known that his writings are thickly sprinkled with mathematical terms and that he everywhere tries to arouse admiration for mathematics among students of philosophy. 66

Proclus then mentions Eudoxus of Crudus, 'a member of Plato's group'. He lists several more mathematicians and states that 'these men lived together in the Academy, making their inquiries in common' (67). Lasserre (1964, p.85) calls Eudoxus 'the most celebrated mathematician among Plato's contemporaries'. His work on proportion provides a foundation for the calculations necessary to find the height of a distant object, as discussed later in this chapter.

It appears that Plato could have an exposure to the latest geometry, through books, letters and discussions in the Academy. This means that, whether as an interested observer or a participant in some discoveries, he had access to a good range of geometrical ideas which he could incorporate into his dialogues. How much geometry was known by his fellow Athenians is uncertain but some conclusions can be drawn, as I explain next.

The study of geometry

Plato's use of many geometrical references indicates his love of geometry but there would be no point in including them unless people with access to the dialogues had some mathematical understanding. We know little about the education of fourth century Athenians but Plato indicates, in several dialogues, some subjects which should be taught. In the *Republic* (536d) it is written that 'calculation, geometry, and all the preliminary education required for dialectic must be offered to the future rulers in childhood'. This sheds some light on what was *not* taught in Plato's day, since if geometry was taught in the schools of Plato's Athens such a statement would be redundant, as a modern example demonstrates. No one in Britain today is likely to write an article insisting that children are taught mathematics in school because that is part of the curriculum in all schools. However, there are often comments suggesting that children should learn a foreign language, because that is no longer compulsory. The fact that Plato thought it necessary to declare that geometry must be included in the curriculum indicates that the subject was not taught to every educated Athenian. Presumably, therefore, many people would not understand the geometrical references found in the dialogues, and would not be able to aspire to be 'future rulers', or philosophers. When Plato writes of philosophers studying geometry he must be discussing the training of future philosophers since, if the schools of his day did not, in general, teach geometry, the schools in Plato's youth would not have done so either. However, philosophers who attended the Academy had the advantage of discussions with the geometers of the day, as did Plato. I assume, therefore, that Plato expected philosophers, of his time and the future, to understand his geometrical references.

People trained in geometry would include both geometers and philosophers but Plato does not equate these two groups of people so some explanation is needed. Of the people who are taught geometry, not all progress to become philosophers. Only those who 'can achieve a unified vision' and also 'remain steadfast in their studies' (*Republic,* 537c) should be chosen for consideration for the role of philosopher. Not all of these will necessarily become philosophers because the chosen ones must then be tested 'to discover which of them can relinquish his eyes and other senses, going on with the help of truth to that which by itself is' (*Republic*, 537d). Philosophers must study dialectic, and require much training to progress from knowledge of geometry to the state of being a philosopher. Those for whom geometry is an educational 'ceiling' are not able to continue with the higher levels of learning needed for philosophers, since only the most competent are chosen to proceed. These geometers

would be able to understand the geometry of the dialogues but, never having progressed to philosophical knowledge, would not find the relevance in it that philosophers would.

There is evidence that some people who could progress from knowledge of geometry to become philosophers decide that they do not wish to do so. This is demonstrated by Plato's representation of Theodorus in the *Theaetetus.* Theodorus attempts to persuade Socrates to have his philosophical discussion with Theaetetus rather than himself because, he says, 'I am not used to this kind of discussion' (146b). Since Theodorus is a renowned mathematician the implication is that he is not interested in philosophical discussions. This is borne out by his comment at 165a when he says 'I very soon inclined away from abstract discussion to geometry'. Miller (1980, p4) observes that thoughts such as those of Theodorus conflict with Plato's views on geometry and philosophy as given in the *Republic*, where geometry is given as an essential part of the curriculum for philosophers. I offer a different interpretation. Although Plato insists that all philosophers need a knowledge of geometry and geometrical methods he is showing here that the opposite is not true. Geometers do not need, or want, a knowledge of philosophical methods.

I suggest that the geometrical references are designed to restrict a full understanding of the texts to philosophers. To use geometry to separate out philosophers from everyone else requires that geometers do not take note of the associated philosophy, and I have indicated that this can be the case, since they have no interest in the philosophical content of the dialogues. This leaves only philosophers and those who, having completed the study of geometry, are trainee philosophers who aspire to become philosophers. These will all fully appreciate the geometrical references. I assume that all of these people continue to search for greater understanding and that Plato is addressing every one of them when he writes about acquiring philosophical knowledge. In future, when I mention non-philosophers and philosophers I include geometers in the former group, although I claim that it is geometrical knowledge which is used to separate philosophers from everyone else. The people with a training in geometry who are progressing towards greater philosophical knowledge will appreciate both Plato's use of geometry and the associated philosophical insights. However, since there are too many geometrical allusions for me to give an adequate account of them all, I present an overview of the type of geometry which Plato uses before choosing only two examples.

1.2 Geometry in Plato's dialogues

The word *geōmetria* [geometry] appears over fifty times in the dialogues and, in addition, there are comments on various aspects of geometry, such as three dimensional and plane, and allusions to the constituents of diagrams, including lines, curves and angles. Since there are many references, some form of categorisation is needed. I have chosen a broad one which separates practical from theoretical geometry, following a partition which dates from ancient times. In order to give an in-depth account of Plato's use of geometry, and the value that it gives in our understanding of his philosophy, I concentrate on two examples which Plato uses, taking one from each of the divisions into practical and theoretical geometry.

My first example is an analogy about the apparent diminution in the size of a distant object which Plato relates to distant pleasures and pains. The estimation of the actual size of the object is an exercise in practical geometry. My second example of a geometrical reference is concerned with diagrams. I examine what we know about the use of diagrams in Plato's Greece, before choosing a particular reference, the term *diagramma*, which is used in explanations about how knowledge should be acquired. The meaning of *diagramma* is not clear but I show that it can be considered to be part of theoretical geometry, because it is associated with theorems and constructions. My work thus covers both a practical aspect of geometry, about which Plato is generally dismissive, and theoretical geometry which, for Plato, has more worth. I give some background to both examples.

Practical and theoretical geometry

Lloyd (2012, p.298) declares that, in the classical period, there was a division between those who argued that mathematics should be studied for its practical use and those who saw it as an 'intellectual, theoretical discipline'. In support of this view he cites Xenophon. In *Memorabilia* (4.7.2-5) Xenophon states that Socrates promoted practical geometry for the purposes of measuring land, 'and this knowledge was so easy to acquire'. However, Socrates 'was against taking the study of geometry so far as to include the more complicated figures, on the ground that he could not see the use of them...'. This is interesting for two reasons. Firstly, Xenophon says that Socrates thinks that while practical geometry is easy to understand, the theory is more demanding and, secondly, he claims that Socrates sees no point in studying theoretical geometry. As Lloyd comments, Plato shows in his dialogues that he does not agree with this, irrespective of any views that were possibly held by the historical Socrates. In addition, Plato applies the distinction between practical and theoretical to all knowledge, as is shown by some lines from the *Statesman*. The visitor suggests that we

should ‘divide all cases of knowledge in this way, calling the one sort practical knowledge, the other purely theoretical’ (258e). He later compares ‘the theoretical sort of knowledge’, such as a statesman or king may have, with ‘the manual or generally practical sort’ (259d).

There is evidence that Plato thought that the truth was not given by any practical observations. In support of this assertion I offer one example. In the *Republic* (529a ff) the motion of the heavenly bodies is discussed, before a conclusion is drawn that what we observe is not what is actually happening:

> We should consider the decorations in the sky to be the most beautiful and most exact of visible things, seeing that they’re embroidered on a visible surface. But we should consider their motions to fall far short of the true ones ... these, of course, must be grasped by reason and thought, not by sight. 529c-d

Lloyd (1970, p.82) records that credit for a ‘mathematical account of the movements of the heavenly bodies belongs to Eudoxus of Cnidus, a younger contemporary of Plato and an associate of his in the Academy’. Mueller (1992, p.173) quotes from Philodemus’ history of the Academy, written in the first century BCE, where he talks of Plato setting problems for the mathematicians to solve including the anomalous motions of the planet. The implication is that practical, or observational, geometry, which is concerned with the things of this world, contains errors. Hence, theoretical geometry is needed to show the true orbits.

However, Plato realises the value of practical geometry for some people, although they may need only a little knowledge of the subject. He gives as an example soldiers, since: ‘setting up camp, occupying a region, concentrating troops, deploying them, or with regard to any of the other formations an army adopts in battle or on the march, it makes all the difference whether someone is a geometer or not’ (*Republic* 526d). Elsewhere, Plato is scathing about the value of ‘conjecture and the training of our senses through experience and routine’ (*Philebus* 55e). He encompasses many areas within his criticism, including arts and crafts in general and, more specifically, music, medicine, agriculture and navigation. However, building is stated to be a superior craft, with regard to both shipbuilding and house building, because ‘it employs straightedge and compass, as well as a mason’s rule, a line, and an ingenious gadget called a carpenter’s square’ (*Philebus*, 56b-c). Some of these are instruments used in geometry but this is practical geometry. Socrates soon distinguishes between ‘the art of calculating and measuring as builders and merchants use them and the geometry and calculations practiced by philosophers’ (*Philebus*, 56e). The latter is presumably a theoretical geometry as opposed to the practical geometry he had discussed earlier.

Diagrams are used in theoretical geometry, for example to prove theorems, and Plato recognises that diagrams can be used to show geometrical objects. However, he insists that these do not represent an ultimate understanding of the objects depicted. He mentions that students of geometry use 'various figures, the three kinds of angle, and other things akin to these in each of their investigations, as if they knew them' (*Republic* 510c). He then explains that:

> ... their thought isn't directed to them but to those other things that they are like. They make their claims for the sake of the square itself and the diagonal itself, not the diagonal they draw, and similarly with the others. These figures that they make and draw, of which shadows and reflections in water are images, they now in turn use as images, in seeking to see those others themselves that one cannot see except by means of thought 510d-e

It appears that, in addition to practical geometry, which is only useful for menial tasks, Plato is dismissive of too much value being given to diagrams. I show that, in spite of such assertions, Plato uses both practical geometry and diagrams to illustrate some of his philosophical discussions.

The height of a distant object

Plato uses the example of distant objects to reinforce the point he wishes to make about how to choose pleasures and pains wisely, based on their relative sizes. Distant objects appear smaller than when they are close. It could be argued that all objects we see are at some distance away from us so that a true size is difficult to define but Plato declares that measurement can reveal it. By Plato's time such a measurement could be made in widely accepted units since, as Ridgeway and Pryce (1931, pp.530-2) record, Solon is reported to have fixed standards for measures about two hundred years earlier. Since the physical measurement of an object is involved this can be considered to be practical geometry. Unfortunately, the simplicity of the statement that measurements can be made masks the difficulty of the task of actually undertaking the measurement. The objects under consideration by Plato are not close enough to apply any type of measuring device, otherwise the illusion would not occur. If the distant object is approached in order to apply a measuring stick to it then it is no longer distant, and its true size would be obvious. It is possible that Plato imagined sending someone to measure the distant object, and report back the actual size, but this is only feasible for objects which are fairly close.

Nowadays we would use trigonometry to find the height of a distant object, in a simple method which can be understood by present day school children. However, trigonometry, in the strict sense of our use, is believed to have been founded by Hipparchus, in the second

century BCE, when he produced tables of 'chords'. Toomer (1973, p.6) records that evidence for this comes from Theon of Alexandria, but it is assumed that the tables were similar to those given by Ptolemy (*Almagest* 1.11) which are effectively values for the sines of an angle. Plato would not have thought in such terms. It must also be kept in mind that the rearrangement of equations was not a simple matter, without the concept of algebra. The absence of trigonometry and algebraic manipulation make finding the height of a distant object a difficult task but I show that geometers in fourth century Athens could achieve it.

The problem of measuring inaccessible objects had been considered before Plato's time. Heath (1921, pp.129-130) relates stories from antiquity telling how Thales measured the height of a pyramid. He cites Diogenes Laertius, who quotes Hieronymus, a pupil of Aristotle, as saying that Thales observed the length of the shadow of the pyramid at the particular moment when the shadow of an object is the same length as the object. Later, Plutarch says that Thales set up a stick at the end of the shadow and used two similar triangles to argue that the height of the pyramid is to the length of the stick as the pyramid shadow is to the shadow of the stick. A problem with Plutarch's account is that the geometry necessary to apply the calculations, using similar triangles, was unlikely to have been known at the time of Thales. However, they are not needed since for the pyramid height it is only necessary to check that at a particular moment all objects have shadows equal to their height. Plutarch's account relates to geometry dating from fourth century Greece, centuries after Thales but available to Plato.

The objects which Plato mentions are not, like the pyramids, too high to measure directly. Rather, they are too far away and, without trigonometry, both similar triangles and proportion are needed to perform the required calculation which Plato needs. Although Katz (1998, p.82) notes that references to similarity date back to the fifth century BCE, the foundation of the idea of similarity was originally based on the notion that all quantities could be thought of as integers. When incommensurability was discovered this threw the Pythagorean dependence on integers into disarray and work on similarity must have stopped. It was not until around the mid fourth century BCE that proportion work was put onto a firmer foundation and this led to further work on similarity, which would have been 'cutting edge' mathematics then. Book 5 of Euclid's *Elements* is devoted to the theory of proportion[5].

[5] Some expressions which are relevant to finding the height of a distant object are given in the Appendix.

A scholiast says this is due to Eudoxus, a contemporary of Plato's and a member of the Academy. It is thus feasible that Plato knew about the theory.

Euclid proves several theorems about similar geometrical figures, using the concept of proportional magnitudes, and these could be applied to the problem of finding the height of a distant object. Proposition 2 of *Elements* Book 6 states that if a straight line is drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally. Proposition 12 of *Elements* Book 6 builds on this to state that in such a triangle the ratio of the corresponding sides can be compared. This work is implied in the theories of Book 5 and therefore follows directly from the work of Eudoxus. It should be noted that, like Euclid, the geometers of Plato's time would have used the ratios of the sides geometrically, whereas we use them algebraically. Katz (1998, p.68) notes that Euclid never multiplied two arbitrary lengths together, as we would in algebra, because he had no way of defining such a process. However, modern calculations reflect the content of the ancient ones sufficiently closely that we can understand how the required result, the height of a distant object, could be obtained using geometry known in Plato's time. The question then arises of how the necessary physical arrangement was envisaged.

In his account of perspective, in *Optics,* Euclid explains that things appear to look differently at different distances. I refer to the Arabic version of the *Optics* since this is readily available in translation and Kheirandish (1999, p.xix) claims, in the Introduction to this work, that the text corresponds closely to the earliest Greek versions which we know. Euclid's account of vision involves rays going from the eye to the object (*Definitions*, Arabic version of the *Optics*) and is similar to that given in the *Timaeus* (45b). The fact that, in reality, light rays come from the object into the eye does not affect the geometry. The rays are straight and form a cone with the vertex at the eye and the base at the object. Euclid states, in Proposition 56 (Arabic version of the *Optics*), that things seen with a greater angle appear larger, those with a smaller angle appear smaller. This explains the apparent diminution in the size of a distant object, a fact which he gives in Proposition 5. Euclid also states that the apparent size of two equal objects at different distances is not inversely proportional to their distance from the eye (Proposition 9). This is true, although we would use trigonometry and say that the distances are inversely proportional to the ratio of the tangents of the angles subtended at the eye by the objects. This fact means that the calculation of the height of a distant object is not simple.

Euclid wrote more on perspective and, since it is based on the geometry of Eudoxus, Plato would have been familiar with this work. His assertion that if two objects of different size

have the same visual angle, at the eye, they appear to be the same size, suggests a method for finding the size of a distant object. Proposition 19 (Arabic version of the *Optics*) mentions covering a distant object, visually, with a nearer object and, from similar triangles, the calculation of the height of the distant object is achieved. Euclid assumes that the distance of the distant object is known. The distant object could be covered by holding a stick of appropriate size at arm's length, for example, so that it exactly covers the height of the distant object. According to Lloyd (1991, p.309): 'the use of simple sighting aids such as the gnomen or upright rod goes back ... from the time of Eudoxus, at least'. Proportion applied to similar triangles would then give the required size. In Proposition 19 the shadow of the object is used, in the method later mentioned by Plutarch with regard to Thales and the pyramid. However, the diagram is the same as that required to find a height using a stick, without recourse to a shadow. If the distance to the distant object is not known the calculation is more complicated, but it still relies only on similar triangles and proportion. I have found no evidence in ancient works for these precise calculations, but Euclid's references to using a stick and its shadow, and the work on similar triangles, could form the basis of calculations to find a distant height[6].

I have indicated that, although the notion of trigonometry was not available in fourth century Athens, it would be possible, using similar triangles and the work of Eudoxus on proportion, to calculate the size of a distant object. It is feasible, therefore, that Plato could know how to do such a calculation, once Eudoxus had produced his theories. However, I suggest that it is also possible that Plato was convinced that a method existed, perhaps after discussion with some of the Academy's geometers, and did not pursue the actual mathematics necessary. Either scenario would allow him to declare that the heights of distant objects could be found and correlate this fact with the measurement of distant pleasures and pains.

Diagrams and *diagramma*

For geometrical constructions it is assumed that diagrams were used, and therefore I examine what we know of ancient diagrams, before choosing one aspect of the topic as an example of the theoretical use of geometry. The problem of how diagrams were drawn has not been satisfactorily solved and various media have been proposed. Riginos (1976, pp.146-7) cites Cicero who tells the story of Plato being shipwrecked and being cheered by the sight of some geometric forms drawn in the sand. In Aristophanes' *Clouds* (177) ashes are sprinkled on a surface. Such diagrams would have been very transitory but Netz (1999,

[6] See Appendix for my versions of these.

pp.15-16) notes that Aristotle mentions *anatomai*, books with anatomical diagrams for students to consult, and surmises that these must have been fairly permanent. Netz suggests that it is possible that Aristotle's drawings were on papyrus. It is, therefore, possible that Plato also used papyrus for geometrical diagrams.

In order to draw geometrical diagrams instruments are needed and Netz (1999, p.17) gives evidence from vase paintings that rulers, or straightedges, and compasses were used some time before Plato. Plato confirms his knowledge of these (*Philebus* 56b-c) and such knowledge was not confined to geometers. This is shown by a reference in Aristophanes' *Clouds* (lines 200-202) which mentions instruments used for geometry, although what they are is not specified.

We do not know how widespread diagrams were. In Aristophanes' *Clouds* (205-215) there is a demonstration that a map 'of the entire world [*gēs periodos*]' preserves shapes but not distances. The main point of the passage is that diagrammatic representations are only understood by some of the audience. References in Plato confirm that geometrical diagrams existed but, as Fowler (1987, p.67) notes, the surviving manuscripts of Plato's dialogues do not contain any of the diagrams on which his mathematical arguments depend, and perhaps never did. Several dialogues, notably the *Meno,* contain detailed information which should allow diagrams to be constructed but the descriptions, for example at *Meno* 86e-87b, are not clear and many different geometrical reconstructions have been proposed. Lloyd (1992, p.167) says of this passage that there are many 'ambiguities or obscurities' and his article is then devoted to an attempt to provide a worthwhile interpretation.

Whether or not Plato's texts contained drawn figures, he often refers to diagrams, using a variety of terms. I list a few, with the translation usually used. Each term appears many times but, for demonstration purposes, I give only a few for each. *Chōrion*, figure, is used throughout when diagrams are discussed, as at *Meno* 82c: 'a square then is a figure [*chōrion*]' and *Meno* 87a:'it is deficient by a figure [*chōrion*]'. *Schēma*, figure or shape, is sometimes used, as at *Meno* 74e: 'the round is as much a shape [*schēma*] as the straight'. It also appears at *Meno*, 76a: 'a shape [*schēma*] is the limit of a solid'. Other translators use different phrases but generally give the same translation of the words under consideration, for example Sharples (2004) gives 'shape' at 74e and 76a. In the *Theaetetus*, in Cooper (1997), Levett gives 'figure' at 148a, where either figure or shape could be used: 'a number of this kind we compared to an oblong figure [*schēma*]'. *Graphē* has several meanings in ancient Greek but it is also used for a figure or a shape, for example at *Republic* 510e: 'These figures [*graphousin*] that they make and draw ...'. Plato also uses *katagraphē* for a representation

using lines, for example at *Symposium* 193a. The *Meno* is the dialogue for which commentators frequently provide diagrams but one word which does not appear in this dialogue is *diagramma*. As I now explain, the translation of this requires more than the obvious 'diagram'.

In both ancient and modern times *diagramma* has been translated in a variety of ways. While the choice of the translator may play a part in some passages, there are other texts where the context indicates that something more than 'diagram' is indicated. There is only space to give a few examples which are representative of many others. These provide a foundation for a study of *diagramma* in Plato, as an example of theoretical geometry. After considering several ancient uses of the word I end with a few remarks about Plato's use of *diagramma*, but a more thorough analysis is left until the passages are examined in Chapters Four and Five.

I start with a summary of some translations of texts originating much later than Plato. Proclus uses *diagramma* in several books and it is translated in different ways, sometimes by the same translator. This is demonstrated in *A commentary on the first book of Euclid's* Elements, where the translator (Morrow, 1970) gives 'constructions' for *diagrammatōn* (213,7) but 'diagrams' for *diagrammatōn* (223,1). Netz (1999, p.37) says that Proclus never uses *diagramma* for just a diagram, for that he usually uses *katagraphē*. He also notes that Pappus uses *diagramma* as we use 'proposition'. The use of a diagram to provide information is demonstrated in the translation (Charlton, 2000), of Stephanus' *On Aristotle on interpretation*: 'let us go through the problem that was disclosed to us in the diagram about the implication of propositions [*diagramma*]' (47,19). Alexander, in *On Aristotle, Metaphysics 5*, 355.19, is translated (Dooley, 1993) as 'He [Aristotle] says that the elements of geometrical proofs [*diagramma*] ...'. Ross (1966, p.234), when commenting on his translation of Aristotle's *Metaphysics,* gives a statement by Asclepius that by *diagramma* Aristotle means geometrical propositions rather than figures. Aristotle uses *diagramma* thirteen times and the variation in translations is demonstrated in one version of *On the Heavens* (Guthrie, 1960). He uses 'diagrams drawn by mathematicians [*diagrammata*]' (279b34), 'diagram [*diagramma*]' (280a2), and 'geometrical figures [*diagrammatōn*]' (280a3). Knorr (1975, p.72) examines Aristotle's use of *diagramma* and concludes: 'so close was this association of theorem and diagram that the two terms might be used as synonyms'.

Modern authors rarely comment on Plato's geometry outside the *Meno*, but there are a few investigations into his use of *diagramma*. They reinforce the view which I have presented. Fowler (1987, p.33), for example, notes that *diagramma* in both Plato and Aristotle seems to

refer ambiguously to either a geometrical figure or a proof. Lloyd (1992, p.175) offers an explanation for differences in the use of some words by Plato, which could apply to *diagramma*. He suggests that problems might arise 'because mathematical terminology, in Plato's day, was still undeveloped and imprecise'. Netz (1999, p.35-6) says that *diagramma* can be defined as a 'figure marked out by lines' but although this is 'etymologically correct' he claims that the actual Greek usage was more complex. He notes that Plato is one of the first extant authors to use the term and claims that he uses it either for mathematical proofs or as the '*de rigueur* accompaniment of mathematics'. It is interesting that he cites where, in Plato, it can be found but although he gives three correct references, at *Euthydemus* 290c, *Theaetetus* 169a and [pseudo] *Epinomis* 991e, he wrongly gives *Republic* 510c and omits *Republic* 529e, *Cratylus* 436d and *Lesser Hippias* 367d,e. He also, mistakenly, gives 73b as occurring in the *Phaedrus* instead of the *Phaedo*.

The use of a single word would not normally have any great significance but *diagramma* is rarely found in extant ancient Greek texts and the problems of translation add to its interest. The rarity of its use in Plato, as listed above, supports the notion that it is a specialized word and, as Netz (1999, p.276) claims, Plato allowed his mathematical passages to be filled with what looks like 'jargon'. This is not intended as a derogatory term and Netz continues: 'Greek mathematics is the product of Greek elite members addressing other elite members' (pp.305-6). This means that an ancient author had to show that his writings were meant for highly educated people and 'Greek mathematics assumes a readership of initiates'. I will return to the notion of *diagramma* being aimed at initiates when I explore its use in Plato's advice about acquiring knowledge, in Chapters Four and Five.

Summary of Chapter One

Although there is uncertainty concerning the sources of our information about early geometry I have shown that we can have some understanding of which topics were known in Plato's time. I have also indicated that we can apply the term 'geometry' to Plato's works and its meaning is close enough to our understanding of the term to avoid any confusion. However, we must keep in mind the fact that some aspects of mathematics which are familiar to us would have been unknown to Plato. This will affect how he uses his geometrical references. If this were a thesis analysing Plato's geometry, the lack of certainty about Plato's knowledge of the subject could be a severe stumbling block to obtaining a worthwhile conclusion. However, my aim is to analyse Plato's use of geometry to enhance his philosophical discussions. I propose that for such an investigation it is only necessary to

have a general understanding of the main areas of geometrical knowledge which existed in Plato's time. This chapter has provided such an understanding and the amount of uncertainty which is present should not affect my analysis.

I concluded that contemporary geometrical knowledge was available to Plato, through the Academy and by other means, much as knowledge is spread today amongst academics. When I looked at how Plato wished geometry to be taught I suggested that, although geometers who were not philosophers could gain geometrical knowledge, only philosophers would be interested in the philosophical associations which Plato provides with his geometrical examples. I examined the geometry which can be found in Plato's dialogues and characterised it as either practical or theoretical, choosing one example from each of these. I described the geometry connected with the calculation of the size of a distant object, and showed that this knowledge was available to Plato. This allows me to correlate his accounts of it with the measurements of distant pleasures and pains. I then considered the use of diagrams in Plato's time. Although our knowledge of this is uncertain I showed that there is evidence that Plato was familiar with different types of diagrams and I chose, as the subject of my further exploration, the term *diagramma.* The translation of this is problematic but I conclude that there is much valuable analysis that can be done with regard to Plato's use of the term.

Chapter Two

The size of distant objects and pleasures

2.1 The *Protagoras* 33

The background to the dialogue 33

The analogy of distant objects 37

Other measurements 38

2.2 Analysis of the application of the analogy 40

The estimation of pleasures and pains 41

The choice of correct pleasures and pains 43

Other references to the size of objects 45

2.3 The *Republic* 48

The background to the dialogue 48

The use of the analogy in the *Republic* 51

The *Republic* argument combined with that in the *Protagoras* 54

Summary of Chapter Two 57

The purpose of this chapter is to begin the analysis of the distant object analogy, which appears at *Protagoras* 356c, *Republic* 602c and *Philebus* 41e-42a, as Plato examines finding the size of pleasures and pains. Since I am tracking the discussions associated with this geometrical reference across several passages, it is important to present a full account of each dialogue in which it appears. I assess each text separately, examining the first two in this chapter and leaving the *Philebus* until later. There is little agreement about the chronology of Plato's dialogues but it would not appear to be controversial to state that the *Protagoras* was written before the *Republic.* For example, Irwin (2008, p.79) gives a list, based on several considerations, in which the *Protagoras* comes before the *Republic* and he states that he is quoting a 'standard view' (p.80). Consequently, the *Protagoras* will be studied first. In this dialogue Socrates correlates the apparent diminution in the size of distant objects with that of distant, or future, pleasures and pains. After giving some

background to the dialogue, the argument associated with the distant object analogy is examined. Other types of measurement are mentioned but I show that these are not as appropriate as the distant object analogy, justifying the worth of concentrating on the use of geometry.

In the second section of the chapter an assessment will be made of any problems found in the *Protagoras* account. These relate to both the estimation of the size of pleasures and pains and the choice eventually made between them. Before moving to an analysis of the passage in the *Republic* which contains the distant object analogy, a survey of other dialogues will be carried out to check for any further relevant references to the measurements of objects. In the final section, some background to the *Republic* is given and, after the *Republic* has been examined, there is an investigation into whether any of the problems found in the *Protagoras* are solved in the *Republic*. This allows an evaluation to be made about the possible enhancement of Socrates' argument if both texts are interpreted together. Although a full assessment of the value of the geometry is left until the completion of the investigation, when the *Philebus* has been examined, my research here should indicate an answer to my question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how? I point out, throughout this chapter, ways in which the geometry aids our understanding of the philosophy.

2.1 The *Protagoras*

This section introduces the use of the distant object analogy. Before considering the argument connected with the analogy I give some background for the dialogue. This includes a brief account of the participants in the philosophical discussions, the setting and some reference to the dramatic date and the date of writing. Following a brief synopsis of the discussions which take place in the lines before those where the distant object is mentioned, the argument which is supported by the analogy will be described in detail. Finally, I examine the other measurements which Socrates mentions, in order to justify my exclusion of these from further analysis.

The background to the dialogue

The *Protagoras* is a reported dialogue, where Socrates recounts some events and discussions to an anonymous companion. There are, therefore, two audiences, the person to whom Socrates speaks and the reader of the text. The dialogue is set in the house of Callias, son of Hipponicus, where Protagoras, a sophist, is a guest. In the *Apology* (20a) Socrates mentions

the same Callias as someone who has spent more money on sophists than anyone else. Other people present, who make some comments in the ensuing conversation, are Hippias, Prodicus, Alcibiades and Critias. In addition, Plato lists the largest collection of personae found in any of his dialogues (Wolfsdorf, 1998, p.130)[7]. Socrates is leading a philosophical discussion on the choice of pleasures and pains and so it is of value to explore the nature of the people present.

Several of those listed were sophists. Protagoras was a renowned sophist but also recognised as a worthy philosopher and Plato imagines conjuring him up from the dead, in *Theaetetus* 166a-168c, to give advice on how to philosophize. Antimoerus is said to be Protagoras' best pupil (315a) and Prodicus of Ceos is mentioned in several of Plato's dialogues. Nails (2002, p.255) summarizes his many qualities, described by Xenophon, and others, in addition to Plato. His special skill was 'the exact use of words' and he was well-respected in Athens. Of the others a surprising number were involved in the scandal of 415, related to the Eleusian mysteries. Nails (2002, pp.17-18) records that Adeimantus the son of Leucolophides, Alcibiades, Charmides, Critias, Eryximachus and Phaedrus were all accused of 'the profanation, i.e. parody, or performance without the proper priests etc., of the Eleusinian mysteries'. Some were also accused of the mutilation of the city's herms, statues of the face and phallus of Hermes, which marked several boundaries throughout Attica. Some of those involved fled into exile but most returned at some stage, although their property had been confiscated. We know the names of those involved because they were recorded on *stelae* erected in the Eleusinium, and recently excavated (Nails, 2002, p.19). Frede (1992a, p.xv) suggests that the setting and characters of the dialogue should be examined within the background of the social and political situation in Athens at the time. Examining what we know of the characters has revealed that most of them appear to be an unlikely choice for participants in a discussion about pleasures and pains from a philosophical standpoint. I return to this point later. Frede's reference to the situation in Athens implies that the dramatic date of the dialogue is known but this is not the case.

There is little agreement between scholars as to the date in which the dialogue is set. It is obviously on an occasion when Protagoras (c.490-420) was visiting Athens and Walsh (1984, p.101-2) gives various possible dates, from the 440s to the 420s, although the main consensus is for a date in the 430s. Since the Peloponnesian War started in 431 this means

[7] Hippocrates, Paralus, Charmides, Xanthippus, Philippides, Antimoerus, Eryximachus, Phaedrus, Andron, Hipponicus, Pausanias, Agathon, Adeimantus son of Cepis and Adeimantus son of Leucolophides.

that for some of Protagoras' life Athens was at war and it may be that scholars see the dialogue as occurring before the start of the war. Unfortunately, this leads to several anachronisms, mostly associated with the ages of some of the people supposedly present or mentioned in the dialogue. To overcome this, Walsh suggests that Plato drew on action and discussion which occurred over more than one of Protagoras' visits. What is not in dispute is that Protagoras was a historical character, although it is likely that the Protagoras of the dialogue is an invention with 'an illusion of "authenticity"' (Schofield, 2010, p.xxvi). This type of representation of a historical character and events in his or her life has become very popular recently in television and films, and is described as faction, a blending of fact and fiction. One example of this is the film '*Alexander the Great'* (2004. Directed by Oliver Stone. USA: Warner) which, like all productions of this genre, causes disputes between those who favour historical accuracy and those who want a good story based loosely on a character who once existed. In the case of the *Protagoras* we can accept that Plato is making several philosophical points, while being aware that these may not all be accurately portrayed with regard to the setting of the dialogue. It is interesting to speculate about the views of Plato's contemporaries with regard to any inaccuracies.

With regard to the probable date when Plato wrote the dialogue, there is some consensus of opinion amongst scholars. The fact that it is one of several reported dialogues has allowed an agreement between scholars about its place in Plato's early writing career although which dialogue it follows is under dispute. Taylor (1963, pp.19-20) makes a case for it not being one of the earliest, placing it after the *Gorgias*, and more recently Schofield (2010, p.xxiv) argues that the *Protagoras* was written after the *Symposium*, probably in the late 380s.

The *Protagoras* text covers Stephanus pages 309a-362a. Most of the dialogue, therefore, comes before the reference to measurements at 356c and, to put the use of geometry into context, a brief synopsis of the preceding pages is given. Socrates first tells how, on the way to Callias' house, he discussed with Hippocrates the problem of what services a sophist offers. Having met Protagoras and his various companions, the stage is set for Socrates to start questioning them about whether the excellence which the sophists profess to teach is capable of being taught (319a-b). This leads on to an argument about the unity, or otherwise, of the virtues and there is a digression while a poem of Simonides is discussed. Socrates then asks for confirmation that they have been discussing whether the virtues of wisdom, temperance, courage, justice and piety are five names for the same thing or whether each is separate (349b).

Socrates introduces into the debate a question about what it means to say that a man lives well (351b). An apparent agreement between Socrates and Protagoras, that a pleasant life is good and an unpleasant life is bad, is broken when Protagoras insists on a proviso that one must take pleasure in honourable things (351c). Socrates compares this, scathingly, with what 'most people' believe; that some pleasant things are bad and some painful things are good. Protagoras insists that this can be the case and that there are also some things which are neutral, neither good nor bad (351d). There is then a discussion (352d-e) about why people do not always choose correctly when faced with various pleasures and pains. 'Correctly' is taken to mean choosing the greatest pleasure or the least pain which, with the equivalence which Socrates has set up, means choosing the greatest good or the least evil. Socrates says that the majority think that people who make a wrong choice do so because they are 'overcome by pleasure or pain or are being ruled by one of the things I referred to just now' (352e). These 'things' are anger, love and fear (352b).

After further discussion they reach an agreement. Several examples are given to show that a pain, for example, would only be called good if it brought about eventual good. One instance is that of medical treatment which can be painful but results in eventual health, which is good (354a-b). The term 'pleasure' is now understood as an abbreviation for the predominance of pleasure over pain. Pleasure is good if it does not lead to a situation whereby pain is so much greater that it outweighs the initial pleasure (354c-d). Socrates relates this to what he calls an absurd view whereby 'a man, knowing the bad to be bad, nevertheless does that very thing, when he is able not to do it, having been driven and overwhelmed by pleasure' (355b). If pleasure equals good then 'overcome by pleasure' would mean 'overcome by good' and someone who knows something to be evil would not choose it because he is 'overcome by good' (355c). This argument has produced much discussion amongst scholars and Wolfsdorf (2006, pp.114-6) lists the views of several commentators. It cannot be disputed that the argument depends on the use of an equality between pleasant and good and James Allen (2006, pp.24-5) is not alone when he queries whether Socrates really commits himself to this view. Reginald Allen (1996, p.125) voices the opinion of many writers when he notes that Socrates never offers a proof for the equivalence of pleasure and goodness. There is no space here to enter into the controversy about Socrates and hedonism since some scholars claim that Socrates endorses hedonism and others claim equally strongly that he does not. Several, such as Dimas (2008, pp.253-4), argue that Socrates contradicts himself about hedonism, both in this dialogue and across other texts. The important point for this thesis is, as Goldberg (1983, p.265) notes, that Socrates' examples show how we continually make

comparisons, for example judging that we will suffer greater pain if we do not take medical treatment.

The discussion moves on to a consideration of the size of pleasures and pains which are being chosen (356a). Socrates now uses the analogy involving measurement to make his argument clear. As Socrates says, when considering how the immediate pleasure, or pain, differs from a subsequent pleasure or pain, the only distinction that matters is the difference between pleasure and pain (356b). If two courses of action are compared, the more pleasant must be chosen, or the one with more pleasant elements compared with the number of painful elements, irrespective of whether they are near or far (356b). In other words, the distance should be irrelevant, and Socrates recommends finding the sizes of the pleasures and pains so that the deceptiveness of the appearance is overturned.

The analogy of distant objects

It is assumed that anyone would prefer greater over lesser pleasures and lesser over greater pains. The problem is how to assess the relative magnitudes and this is more difficult if one pleasure, for example, is imminent and one is a long way in the future. Socrates uses the analogy to clarify the point he makes:

> Do things of the same size appear to you larger when seen near at hand, and smaller when seen from a distance, or not? ....While the power of appearance often makes us wander all over the place in confusion, often changing our minds about the same things and regretting our actions and choices with respect to things large and small, the art of measurement in contrast, would make the appearances lose their power by showing us the truth, ...Since it has turned out that our salvation in life depends on the right choice of pleasures and pains, be they more or fewer, greater or lesser, farther or nearer, doesn't our salvation seem, first of all, to be measurement...
>
> 356c-357b

Some non-geometrical ways to compare near and far pleasures and pains are recorded and I assess their worth later. As Taylor (1976, p.191) points out, Socrates does not say that it is impossible to judge sizes without the use of measurement, but to make the right decision regularly, if life depended on it, observations alone would be insufficient. It is not critical in life to judge the size of objects but, Socrates says, it is necessary for our salvation to make the right choice of pleasures and pains (357b). We must, therefore, be able to measure pleasures and pains and the implication is that doing so is only possible if we have the correct knowledge. We can use this knowledge even if one pleasure is in the present and one is in the future, just as we can compare two objects by estimating their sizes if one is close and the other is far away.

If 'overcome by pleasure' is absurd then Socrates must explain why people sometimes make a wrong choice. Using the geometrical analogy, he has suggested that pleasures and pains can be chosen correctly using knowledge. A wrong choice is now seen to result from a lack of knowledge about how to measure the pleasures and pains which are available. An example of this could be someone choosing a present pleasure, such as having an extra drink at a party, because the future pain, of a hangover, is underestimated. This means that knowledge has not been used to determine the size of the future pain compared with the present pleasure. Unfortunately, as I explained in Chapter One, the estimation of the size of a distant object would not have been straightforward, and the measurement of pleasures and pains would not be any easier. It should also be noted that measurement alone, which is all that Socrates mentions, could not give the height of a distant object since calculations are needed[8].

It is interesting, as Weiss (1989, p.524) points out, that philosophers are not mentioned in the *Protagoras* and she suggests that this is because in this argument there is nothing to distinguish the character of the philosopher from everyone else. If no one does wrong, knowing it to be wrong, then the only difference between people is a proficiency in estimating the relative worth of different pleasures and pains. Although they are not mentioned, I claim that the role of philosophers is significant in this discussion. They are the ones who would be proficient in correctly finding the size of a distant object since they have been taught geometry. The implication is that they would be similarly skilled in correctly estimating the size of distant pleasures and pains.

I have presented the discussion associated with the distant object analogy without offering a critical analysis of it. There are several problems with Socrates' account and I consider these in the next section. Before doing this, I examine the other examples which he uses in his attempts to clarify the need to measure pleasures and pains. My purpose is to track the use of the geometrical analogy across several dialogues, but if there are other analogies, which are better for demonstrating the measurement of pleasures and pains, the value of my research work here could be undermined.

Other measurements

I show that the non-geometrical examples which Socrates uses, when he discusses the measurement of pleasures and pains which are observed incorrectly, are not as appropriate

[8] As shown in the Appendix.

as the size measurement. This gives support to my decision to confine my analysis to Plato's use of geometry. The first of these other measurement methods is weighing with scales: 'you put the pleasures together and the pains together, both the near and the remote, on the balance scale, and then say which of the two is more' (356b). Weighing was done, originally, for precious metals and the weights used were either naturally occurring small objects such as barley-corn, or coins such as drachma. Solon attempted to standardise weights (Ridgeway and Mattingly, 1931, pp.534) so that, by Plato's time, objects could be weighed with some degree of accuracy and with some standardisation. It is easy, therefore, to picture finding the 'weight' of some pleasures and pains by putting them on one side of a balance and some standard 'weights' on the other side, to obtain a quantitative result. Alternatively one pleasure could be placed on one side of the balance and another on the other side to check which is the greatest. Although Socrates explains the mechanics of the weighing, his statement is ambiguous with regard to present and future, or remote, pleasures and pains. It is not clear whether 'which of the two is more' means checking whether the total pleasures outweigh the total pains, present and future, or whether the present pleasures and pains together outweigh the future pleasures and pains. In either case, the usefulness of the weighing example can be questioned since the notion of placing present and future pleasures and pains together on one balance overturns the premise that our view of the future ones is in some way distorted.

The other examples, which are given without comment by Socrates, also appear to be inappropriate. He mentions 'thicknesses and pluralities' (356c) but I take thickness to be equivalent to size. Plurality could demonstrate a problem but several objects which can be distinguished close by may also be discernible as several objects in the distance. Other factors, such as a person's eyesight, will decide the result and Socrates gives no details that enable us to understand what he means by this example. Something which is affected by distance is volume of sound, and Socrates also refers to this. He reminds us that 'equal sounds seem louder when near at hand, softer when farther away' (356c). Although distance is taken into account this fails to satisfy another criteria which Plato needs; numerical values. These had been given to the harmonic ratios of sound by the Pythagoreans, but there was no method for assigning a value to the loudness of a sound, as we do now using a decibel scale. This means that the amount by which the distant sound has been reduced could not be measured in Plato's time. This analogy would, therefore, have some drawbacks for the discussion of measuring distant pleasures and pains, since Plato wants to suggest that the correct knowledge would enable us to measure them quantitatively. Neither weighing nor

listening to sounds would provide a good basis for a comparison with the measurement of pleasures and pains, especially those that are distant.

Although I claim that the distant object analogy is most apt, all the examples have some significance for non-philosophers, who can be made aware that they should examine pleasures and pains carefully, although they cannot measure them. Everyone, from before Plato's time until the present day, is familiar with the notion of weighing objects and observing 'pluralities'. The measurement of sounds would be specialized but, since no one in Plato's time could give a numerical value for the amount by which a distant sound is diminished, there is no group of people who could claim privileged understanding of this analogy. The notion of a distant object being apparently diminished can also be appreciated by everyone. However, to find the true size requires geometry, setting those with knowledge of geometry apart from the rest. This includes philosophers, as I have explained, and I return to this point when I assess the value of the analogy after completing my examination of its use in the three dialogues.

I concentrated, initially, on the geometrical example, using size measurements, because I wish to investigate Plato's use of geometry. I consider that this brief account of the other measurements which Plato mentions justifies my ignoring them in my analysis, since the non-geometrical examples of measurements do not allow for the relative sizes of near and far pleasures or do not give quantitative results. On the other hand, the analogy of the apparent diminution in size of a distant object does have a relevance for the quantitative measurement of distant pleasures and pains since numerical values could be given to lengths of objects. The use of measurements of objects, near and far, to illustrate the measurement of the relative sizes of present and future pleasures and pains therefore appears to be most apt. However, there are many problems with the analogy as Socrates uses it, as I explain next.

2.2 Analysis of the application of the analogy

My analysis of the *Protagoras* account of the distant object analogy now moves on to consider the connection with the estimation of size of pleasures and pains. Several criticisms can be made about the approach taken in the *Protagoras.* These fall into two main groups. The first relates to concerns about how an estimate of the size of distant objects and pleasures and pains can be made. I analysed the problem of finding the height of a distant object in Chapter One and I show, in the Appendix, how geometry can produce a value using various lengths which can be found. Here, I consider the difficulties encountered in finding the relative sizes of pleasures and pains. Socrates recommends an estimation of these so

that a correct choice can be made of pleasure over pain. The next set of problems relates to how such a choice can be made. I show that Plato has ignored many aspects of the pleasure/pain discussion with regard to both the estimation and the choice of pleasures and pains. My approach here is critical and I leave an analysis of the value of the analogy to the next chapter, when I have examined all the relevant dialogues. However, I check whether there are other passages with geometrical content which might overcome the deficiencies found in the *Protagoras* account, but find none except those in the *Republic* and *Philebus*.

The estimation of pleasures and pains

Socrates gives no hint about how an estimation of the relative sizes of pleasures and pains can be done or what type of knowledge would be needed to carry out the measurements. He says that he will discuss it later, although he does not do so in this dialogue, and adds: 'that it is knowledge of some sort is enough for the demonstration which Protagoras and I have to give' (357b). As Strauss (1999, p.186) remarks, scholars from Plato's time up to the present day have seen the need for a theory of measurement for pleasures and pains, but no one has been able to find one. I suggest that the necessary knowledge must comprise two parts, as the comparison with the measurement of lengths demonstrates. In order to compare the lengths of two objects I would obviously need to be able to use, correctly, a measuring device. This could be any type of measuring rod, a ruler in modern times, and it is possible to use a ruler incorrectly, for example by not lining up the start of the scale with the edge of the object. However, prior to the measurement I would need to know how to choose the correct measuring device. Someone who decided to compare the lengths of two objects by weighing them on scales could be successful if the objects had equal cross sections and densities but such a method would generally fail. However, with the correct choice and use of a measuring device, such as a ruler, the measurement and comparison of two lengths is possible.

I now examine more fully the situation with regard to pleasures and pains. Although we often speak of one pleasure being greater than another, we do not tend to assess what observations would be necessary to confirm such a statement. Taylor (1976, p.196) questions whether any observation is feasible. He concludes that the assessment of any event in connection with associated pleasures and pains requires 'hypotheses of the likely effects of possible actions, and some sort of imaginative comparison of the various effects thus envisaged'. To do this we use our memories of similar events in the past to make the comparison. While agreeing with some of his analysis I consider that his use of the word 'comparison' leaves open the possibility of an analogy with measurements since it indicates a conclusion of the type "this pleasure is bigger than that pleasure". However, the problem is

that it is not obvious how we should set about measuring pleasures and pains, present or future, or how we could gain the knowledge to enable us to do this. Both the choice of the measuring instrument and the mechanics of how to do the measurement are equally unknown.

If such a measurement were possible there is a further problem because it is necessary to know which future pleasures and pains must be measured. Socrates uses the word 'pleasure' to refer to a choice which maximizes pleasure and minimizes pain but he does not consider over what time span the future pleasures and pains should be considered. Only at the end of my life can I be sure how much pleasure and pain has been attached to a particular choice I made earlier. Other considerations may or may not give me a good life but could have some significance for my soul, for instance unselfishly choosing to give maximum pleasure to others. As Coby (1987, pp.159-160) discusses, a concern for my immortal soul would mean that a final account could not be given even on death. Throughout Socrates' account there is an implication that the present choice of action has clearly foreseeable consequences, which Socrates wants to use knowledge to measure. This is obviously only true if the whole of life, and possibly the whole of eternity, can be viewed. It is difficult to imagine any assessment which could take into account all the pleasures and pains that may occur up to my death and their possible effect on my immortal soul.

If the above problem is ignored a further difficulty arises if a quantitative result is required. There must be some unit by which we can measure the relative sizes of pleasures and pains and this leads to what Strauss (1999, pp.112-3) calls the rate of interest. Assuming I can give a value to my measured pleasures and pains then I would probably not think it worthwhile to forego a present pleasure of 100 units for a pleasure of 100 units in a year's time. It is uncertain whether I would give up the present pleasure for one worth 101 units in the future. Strauss assesses this in terms of considering what 'interest rate' I require in order to substitute a future pleasure for a present one. The situation becomes more complicated if I also consider pains. It is difficult to estimate how many units of future pleasure I would demand for 100 units of present pain. Strauss suggests that the rate of interest differs for different people and possibly for the same person at different stages of life, or on different days. We may even opt for a negative rate of interest, where we postpone a pleasure and hasten a pain because anticipating pleasure is pleasurable but anticipating pain is distressing. It appears that it is not possible to quantitatively measure the relative values of present and future pleasures and pains.

Although we may relate to the image, demonstrated by the distant object analogy, of different sized pleasures and pains, it appears that any estimation of the relative sizes is not easy to achieve. Socrates does not consider the practical problems of finding the sizes of either distant objects or pleasures and pains. Unfortunately, there are further problems associated with the analogy with regard to our actions if we could perform the necessary estimations, and I consider these next.

The choice of correct pleasures and pains

If we ignore the difficulty in estimating the relative sizes of pleasures and pains, there are problems concerned with the choice made after the measurements have been done. Firstly, Socrates assumes that, if I could somehow correctly determine the relative sizes of some pleasures and pains, I would act on the result. Klosko (1980, p.319) calls on psychological hedonism to support the assumption by Socrates. Under this influence, an agent would wish to maximize his pleasures and Klosko can see no conceivable circumstance when he would wish to go against the results of his measurements. I suggest a different scenario can be illustrated by using the analogy with lengths. I might decide that I want the biggest slice of cake on offer and measure the width of several pieces in order to make my choice. I could look at the biggest slice and decide that one of the others looks more appetizing and choose that instead. I have done the correct measurements but some other factor has caused me to reject the result. There could be several reasons for not acting on the result which knowledge has given me; I will consider two.

The first reason which might cause me to reject the result given by a correct measurement relates to the quality of the pleasures and pains. A difference in length is an absolute difference about which there can be no argument but with pleasures there could be different qualities which could affect the choice, and different people will not necessarily agree on the relative quality of different pleasures. Several commentators discuss the problem of assessing the quality rather than the quantity of a pleasure, with Richardson (1990, p.11) going so far as to state that if there are qualitative differences between pleasures then we may not be able to talk of maximizing them. If this is the case then, as Irwin (1977, p.112) notes, Socrates would lose 'the measuring science which settles disputes about good and evil'. Disagreement between people relating to which pleasure has the best quality is likely to be significant. For instance, I would rate three hours of opera as a high quality pleasure but there are many people who would disagree, and some may even designate the experience as painful. It would appear impossible to compare the qualities of pleasures and pains on any objective scale, which makes a choice based on measurement alone liable to error.

The second problem concerns how we view the distant pleasure or pain after a possible measurement of it. The distant object analogy can be used to demonstrate why I might not accept the measurement I have made. The object is then known to be larger than it appears but it is still seen as being smaller. Morris (2006, p.202-3) and Denyer (2008, p.192) both analyse this problem by considering the same optical illusion. The Müller-Lyer diagram comprises two lines of equal length, each with arrow heads at each end. One line has the arrows pointing inwards while the other line has the arrows pointing outwards and the lines appear unequal in length. The illusion persists even after a measurement shows that the lines are equal, meaning that we have contradictory views about the lines. If this is also true of pleasures and pains then the implication is that even if future ones could be measured, and their correct size known, they will continue to appear smaller than they really are, causing us to have what Morris calls 'contradictory preferences'. Socrates does not examine here what would make us abandon the illusion we continue to 'see' in favour of the measurement we know to be correct.

If we do accept the measurement, an apparently larger pleasure is not necessarily larger in reality. The example of a distant object fails to provide an analogy for what I designate as the 'multiplication problem' for pleasures. If the original object is replaced, both near and far, by one twice the size then, although the distant one will still appear smaller than it really is, it will appear twice as large as the original object which it replaced. Pleasures are not necessarily multiplied up in this way. To take a mundane example, choosing to eat a cream cake could give me great pleasure and any associated future pain could be minimal, maybe a slight discomfort in the stomach, for example. This means that I have net pleasure from my action. However, eating twenty cream cakes is unlikely to mean twenty times the overall pleasure. After the first few I suggest that all pleasure would have been replaced by the pain of being too full, with an associated feeling of nausea. Multiplying a pleasure does not necessarily mean enhanced pleasure and it may cause the resulting combination to flip over into pain. Interestingly, this is not true of multiplying a pain, where the result is increased pain. In addition to weakening the analogy with distant objects this highlights the fact that when choosing the greatest pleasure care must be taken to avoid extremes which may not turn out to be as pleasurable as expected. This is not considered by Socrates here.

This leads to my final problem with Socrates' argument in the *Protagoras*. Uncertainty about the future has the repercussion that an anticipated pleasure, or pain, may fail to materialise. The distance analogy fails here because although the distant object may be viewed inaccurately there is no question about its existence. However, someone who foregoes the

present pleasure of smoking because of the future pain of cancer may have a friend who continues to smoke and lives a long and healthy life. Future pleasures and pains have an associated probability which it is impossible to calculate accurately and Socrates does not consider this. There is another situation which may arise even if the future event does eventually occur. An example might be a holiday scheduled for a few months time. I may decide that the pleasure I will gain from it will be greater than the pain of having to save all my money now and forego several pleasures in the present. The holiday, when I eventually go on it, might not be as pleasurable as I anticipated because of a problem such as bad weather. Here the uncertainty is not in whether the future event actually occurs, but in whether it fulfils my expectations. In a similar scenario, my tastes may change so that when the anticipated event occurs I no longer find it pleasurable. Socrates does not consider a possible change in the size of the distant pleasure or pain, whether actual or because of a changed perception compared with the original view. His false assumptions are that when the apparent perception has been corrected by measurement it is then fixed, with no alteration to size and no uncertainty about the occurrence of the event.

The analogy gives a helpful image of an apparently diminished distant object, for comparison with the situation regarding pleasures and pains. However, my analysis has shown that there are problems with several aspects of the discussion in the *Protagoras*. Since I am considering the value of geometrical examples in enhancing our understanding of Plato's philosophy, these flaws in the application of the geometry appear to undermine the purpose of my work. I explore other dialogues to look for further geometrical references, which may add to the discussion on the measurement of pleasures and pains and address some of the problems.

Other references to the size of objects

The same distant object analogy, as that found in the *Protagoras*, is used in the *Republic*, at 602c, and the *Philebus,* at 41e-42a. In both of these dialogues it is also associated with the measurement of pleasures and pains. An examination of these passages will add to the account which Plato presents in the *Protagoras* and I analyse their content in later sections. Here, I consider some other references to the size of objects, to ascertain whether they have any relevance for my work. The *TLG* has been an invaluable aid in searching for such references.

There are many passages in the dialogues which compare the size of large and small objects. I show, by considering a few examples, that these are not relevant to the use of geometry, and they do not add anything to the *Protagoras* discussion. In the *Theaetetus,* when 'bigger'

and ‘smaller’ are mentioned by Socrates (155b), he is showing how a boy may grow from being smaller than a man to being bigger than the same man. No measurements are mentioned and geometry is not involved. In the *Statesman,* at 283d, the visitor thinks that the art of measurement relates to ‘length and brevity, and excess and deficiency in general’. He then divides the art of measurement into two parts. One is concerned with the relative greatness or smallness of things, and the other takes into account a mean value with which objects must be compared. An object can be large or small compared with the mean for that particular type of object. Although lengths must be measured, the manipulation of the values, to find a mean, would be based on arithmetic, rather than geometry. These, and other references involving the measurements of lengths directly with a measuring rod, or ruler, have no significance for my research.

There are other passages which mention objects at a distance but they are concerned with the fact that anything seen in the distance may not be seen clearly. A few examples demonstrate that there is no relevance for my discussion. In the *Parmenides*, Socrates compares objects seen from a distance with those seen nearby (165b-d), which could bring to mind the analogy used in the *Protagoras*. However, a closer examination of the text reveals that Socrates is not concerned with the false perception of the size of the distant object. The point he makes concerns the fact that the distant object is seen indistinctly: ‘must not such a thing appear one to a person dimly observing from far off; but to a person considering it keenly from up close, must not each one appear unlimited in multitude’. This is similar to an example found in *Laws* where ‘Looking at a thing from a distance’ (663b) is considered. However, it is said next that this ‘makes nearly everyone feel dizzy’ (663b), with no justification for this rather strange statement. In these examples there is the notion that distant objects are not seen clearly, which leads to errors in perception. It could be argued that distant pleasures and pains could similarly be seen indistinctly, but Plato does not make such a correlation, and geometry is not involved. Although this may be a worthwhile topic for further study, it lies outside the sphere of my investigation.

The *Sophist*, at 235e-236a, presents a situation which depends on the same geometry as that used to find the height of a distant object. The visitor talks about making a copy of something with length, breadth and depth the same as the original. If this is done for a large sculpture the final effect is wrong because ‘the upper parts would appear smaller than they should, and the lower parts would appear larger, because we see the upper parts from farther away and the lower parts from closer’ (235e-236a). Again, distance is causing a false perception. The visitor does not explain this but the example shows an awareness of the problem of

perspective. In painting the concept of perspective was not used until Alberti, in 1435 CE, but, as Anderson (2007, p.730) records: 'all the concepts they needed, and actually also all the geometry required for developing a mathematical theory of perspective, were available as an inheritance from ancient Greece'. Knorr (1991, p.208, note 2) records that some scholars suggest that Alberti used Euclid's constructions to establish the proportionality between magnitudes, and their representation in a picture. However, as Knorr points out, Euclid was not concerned with appearance in a picture but he was attempting to obtain, from angular measure, the actual dimensions of objects, using proportion and similar triangles. This is the geometry which is used to find the height of a distant object. However, in the *Sophist* there is no intention to measure the various parts of the statue, and the discussion leads to the possible skill of the sophist and the problem of speaking of '*those which are not*' (238b). I do not consider that a further study of this would enhance our understanding of the use of the distant object analogy or the associated discussions on pleasures and pains.

Other passages in Plato appear to be more relevant to the measurement of distant objects. In the *Republic*, Socrates mentions that sometimes 'sense perception seems to produce no sound result' (523b). Glaucon suggests: 'You're obviously referring to things appearing in the distance...' (523b) but Socrates denies that this is what he means. He is referring to situations where the same perception can give apparently opposite viewpoints, causing the soul to be puzzled (524a). He delays a discussion on the apparent diminution in size of distant objects until *Republic* 602c, which I examine later.

A passage in the *Euthyphro* is interesting because it contains a reference to measurement to distinguish the larger from the smaller (7c) and it is also noted that weighing would be used to determine the difference between 'the heavier and the lighter' (7c). I noted that weighing is mentioned in the *Protagoras* and the fact that it arises again, associated with length measurement, calls for further comment. In the *Euthyphro* passage Socrates is giving examples to show that 'subjects of difference' (7b) can often be resolved. However, some subjects, such as right, wrong, honour and dishonour cannot be decided so easily and similarly there is no agreement about what is pious and impious (7d-8a). When Socrates talks about measuring and weighing he is talking about objects which have a different size; he is not concerned with errors in perception when one object appears smaller than it really is because of its distance. Thus the *Euthyphro* passage has no relevance for my specific inquiry.

The above sample of references by Plato to large, small, near and far objects and associated passages is representative of many others but none of them use the application of geometry to find a length or add to the discussion about finding the relative sizes of pleasures and

pains. My search for other geometrical examples has revealed that there are no worthwhile ones for my purpose except where the same distant object analogy is repeated. Therefore, I restrict my analysis to the other texts which use the same analogy as that found in the *Protagoras.* The argument associated with this analogy, as used in the *Republic,* will now be examined and an assessment will be made about whether any of the problems found in the *Protagoras* are addressed in this account.

2.3 The *Republic*

In this section I explore the reference to the distant object analogy found in the *Republic,* at 602c. I assess the worth of the analogy for the discussions that are being held and combine the analysis in the *Republic* with that in the *Protagoras* as I look for any solutions to the problems found in the latter text. The *Republic* is a long work, exceeded in Plato's dialogues only by *Laws.* The text has been divided into ten books since antiquity, but it is not known whether Plato intended these divisions. Rutherford (1995, p.208), among many scholars, notes that the title *Republic* is a mistranslation from the Latin title of *res publica.* The Greek title was *Politeia* which he translates as 'state' or 'political constitution' although Schofield (2006, p.33) says that it has a core meaning of 'citizenship'. Blackburn (2006, p.7) declares that the *Republic* is commonly regarded as 'the culminating achievement of Plato as a philosopher and writer' and this perhaps provides a reason for its popularity. Craig (1996, p.291) notes that it 'must be the most written-about work in our philosophical tradition'. This creates a problem for me and I have had to be very selective in my use of works on the *Republic*. Many have no relevance for the argument I am exploring and they have not been included in the Bibliography. As with the *Protagoras* account I will give some background information for the dialogue before considering the argument supported by the distant object analogy.

The background to the dialogue

The conversation which takes place in the dialogue is recounted by Socrates to an unnamed person or group, as in the *Protagoras*, so again there are two audiences, the unknown recipient(s) of Socrates' reminiscences and the reader of Plato's text. Socrates describes how he went to the house of Cephalus, a retired manufacturer, who lives at Piraeus. Boyd (1962, p.1) notes that the company comprises characters drawn from across the Greek world. Plato's elder brothers, Glaucon and Adeimantus, are from Athens while Cephalus and his sons Polemarchus, Lysias and Euthydemus are from Syracuse, being resident aliens in Athens. Clitophon and Niceratus are given as being the sons of men of Athens and, in addition, there

are Thrasymachus from Chalcedon in Asia Minor and Charmantides of Paeania in Attica. There are some additional people who are with Socrates at the start but it is not clear whether they go with him to the house of Cephalus and it is unlikely that they could have any importance in Plato's eyes, being unnamed. It should be noted that none of those named here is named in the *Protagoras*. The lack of anyone being present at both discussions is important, as will be shown later. The other important fact to be drawn from the list of people present is that only one is known to be philosophically minded. Polemarchus of Thurii is said by Plato, in the *Phaedrus*, 257b, to have been converted to philosophy. He was executed by the Thirty in 404, which gives a connection with the setting of the dialogue, as I show next.

The dialogue takes place at the first festival of Bendis, where everyone is looking forward to a torch-race on horseback. This should allow the dramatic date to be known exactly but unfortunately the year of the first festival is unknown. Several scholars, including Nussbaum (2001, p.136), agree with a dramatic date of around 422. Various people mentioned in the dialogue, in addition to Polemarchus, were involved in the tyranny of the Thirty, either as participants or victims, and the resistance to the tyranny was based in Piraeus in 403 with a battle taking place in front of the temple of Bendis. Plato would be aware of the fate of his characters and such references would not go unnoticed by Plato's contemporaries.

With regard to the date of writing, Cornford (1946, p.xxv) considers it possible that the *Republic* was written some time in the two decades following the foundation of the Academy, which Scott (2009, p.271) gives as 387, although there is some uncertainty about the exact year. This is roughly supported by Blackburn (2006, p.22), and others, who suggest that the *Republic* was written around 375, when Plato was in his early fifties. However, several scholars believe that different parts were written at different times. Books 1 and 10 are sometimes taken to be separate from the others, although this is a controversial view. Mitchell and Lucas (2003, p.154) express the opinion, which many writers share, that the topics in Book 10 appear to be an appendix 'perhaps added in later editions as a rejoinder to criticisms'. An alternative view is given by Annas (1981, p.335) who suggests that Plato wanted to add to points which he felt had not been covered adequately in the other books of the *Republic*. I agree with this and I will show that the lines I analyse in Book 10 depend on earlier discussions, particularly those in Books 4 and 9.

The *Republic* covers Stephanus pages 327a to 621d. The distant object analogy is mentioned at 602c and so, as with the *Protagoras*, most of the dialogue comes before the geometrical example being investigated. The length of the *Republic* means that many subjects are

covered and there is some disagreement among scholars about its main purpose. Purshouse (2006, p.9) lists some of the topics which can be found, including ethics, psychology, education, epistemology, the nature of philosophy, the arts and the afterlife. He suggests that the core issue is justice and several writers agree with this, for instance Annas (1981, pp.10-12). In preparation for the work of the rest of this chapter I offer a summary of the sections which are relevant for the discussion on pleasures and pains associated with the distant object analogy.

Socrates ties in his discussion of pleasure with a description of the tripartite division of the soul and a corresponding division of characters in the city. This was originally examined in Book 4 but Socrates returns to the subject in Book 9. The proposal is that a soul has three parts, an appetitive part which desires bodily pleasures (580e), a spirited element which is honour loving or ambitious (581a) and a rational part which searches for truth, being intellectual and philosophical (581b). Any part of the soul can be dominant, so that there are three main classes of men, the avaricious, the ambitious and the philosophic. Each has his own corresponding form of pleasure (581c) but the philosopher will have experienced all the types of pleasure throughout his life and he is the only one who enjoys the pleasure of searching for the truth (582c). It follows that the philosopher's pleasure must be the best (583a). After a lengthy discussion Socrates concludes that people who are ignorant of truth and reality are confused about pleasure and pain (584e). The pleasures experienced by such people are mixed with pains, 'mere images and shadow-paintings of true pleasures' (586b). The discussion moves on to which life is most pleasant and a calculation is performed to show that a king, or philosopher-king, has the most pleasant life. This answers a query raised in Book 1 about whether an unjust man is better off than a just one, since injustice is often unseen and unpunished. The rest of Book 9 gives support to the view that a just life is better than an unjust one.

Book 10 starts with Socrates declaring that he wants poetry excluded from the city (595a). To explain why he takes this view Socrates considers the work of a painter. He cites the example of a bed. God makes the real idea of a bed and a carpenter manufactures a copy of this bed but the artist who paints the picture of a bed can only imitate what the others have produced. He is not a craftsman or a maker but, because his work is 'third from the natural one', only an imitator (597b-e). According to Socrates the works of poets, especially Homer, are also 'the third remove from that which is' (598e). It should be noted that some translators replace 'third remove from that which is' with such phrases as 'two steps away from reality' (Waterfield, 1994, p.349, for example). This appears to be more logical for the

examples which Socrates gives, although Plato uses a word referring to three. Socrates then asks on what part of human nature the poetry produces an effect and he declares that he needs to explain this using an analogy (602c).

The use of the analogy in the *Republic*

The analogy which Socrates chooses is the same as that found in the *Protagoras*:

> Something looked at from close at hand doesn't seem to be the same size as it does when it is looked at from a distance. ... And something looks crooked when seen in water and straight when seen out of it, while something else looks both concave and convex because our eyes are deceived by its colors [sic], and every other similar sort of confusion is clearly present in our soul ... And don't measuring, counting, and weighing give us most welcome assistance in these cases, so that we aren't ruled by something's looking bigger, smaller, more numerous, or heavier, but by calculation, measurement, or weighing... And calculating, measuring, and weighing are the work of the rational part of the soul... But when this part has measured and has indicated that some things are larger or smaller or the same size as others, the opposite appears to it at the same time. 602c-e

As in the *Protagoras,* Socrates uses several examples. In order to justify concentrating on the distance analogy, it is necessary to consider the other points which he makes. One example of an illusion which he gives is of a stick looking bent in water (602c). We look at such a stick and it appears bent even though we know that it is in reality straight. If we know the laws of refraction which produce the effect we still 'see' the stick as bent. However, this analogy does not have the advantage of demonstrating the differing sizes of the emotional pleasures which Socrates is about to discuss, since the apparent size of the stick is unaltered when it is placed in water. This is also true of the brief mention of colours which can deceive us (602d). Neither of these analogies was given in the *Protagoras* but Socrates continues by listing 'measuring, counting, and weighing' (602d). Weighing also appeared at *Protagoras* 356b, although I showed that it was not as useful an analogy as the distant object one. Here, 'weighing' is not discussed any further and Socrates returns to the size analogy when he says that measurement shows that 'some things are larger or smaller or the same size as others' (602e). This shows that the distant object analogy is the most important one which Socrates uses here, so that, as in my examination of the *Protagoras* account, I consider that I am justified in concentrating on the use of geometry within the discussion on the size of pleasures and pains.

Socrates refers to a distant object to demonstrate that it does not appear as it really is, but he is concerned here with problems which arise even if the true size is found. After we have done the measurements and obtained a value for the actual size, the appearance which

contradicts the result may persist (602e). Hence, we have two apparently conflicting opinions about the same object, at the same time. One comes from our perception and the other from the measurement. An earlier discussion, in Book 4 (436b ff), is summarized here: 'didn't we say that it is impossible for the same thing to believe opposites about the same thing at the same time?' (602e). Much has been written about this view of non-contrariety, for example by Stalley (1975) and Barney (1992), but the only relevance for the present discussion is the application to illusions.

We now have a paradox, in that we have conflicting opinions about the object and yet we cannot have conflicting opinions about anything. The solution to the paradox is related to the earlier theory about the division of the soul: since a single soul could not have two opinions about the same thing at the same time, the soul must have parts. The best part will accept the true information given by measurement and calculation, while the illusion persists in the lower parts of the soul (603a). Several writers, such as Waterfield (1994, pp.448-9) point out that Plato needs two divisions of the soul for this argument, rather than the three specified earlier. For my discussion it is only necessary to consider that there is a division into rational and non-rational and any further divisions are irrelevant.

Since the rational part of the soul can overcome the false impression of an optical illusion Socrates can now apply this result from his analogy to his discussion about pleasures and pains. He considers how a good man will behave if he uses the rational part of his soul to dispel illusions about the size of acceptable emotions. He concentrates on how the man would bear pain, such as the loss of a son, and decides that he will be 'measured in his response to pain' (603e). This is because he will use reason to enable him to bear his misfortune quietly and overcome other feelings which might cause him to dwell on his sufferings and behave like children who weep and wail after a fall (604c). Socrates says that instead of excessive grief we should attempt to replace 'lamentation with cure' (604d). Presumably he means that we should just "get on with life", or at least view our grief differently, since dwelling on the grief, with thoughts of past times, does not achieve anything. It is this view which can be corrupted by the dramatic arts.

The delusions we can have about the size of pleasures and pains can be created, according to Socrates, by seeing how characters in plays and poems overreact in situations. Socrates makes the penetrating observation that in poetry and the theatre a calm character is difficult to represent, and for popular success the characters must be 'excitable and multicolored [sic]' (605a). The emotions associated with their pleasures and pains are exaggerated so that they are removed from reality. There are many examples of this from the Greek tragedies

which would have been known to Plato. One brief quotation is sufficient to demonstrate Plato's misgivings. It is from Euripides' *Ion,* starting at line 1400, with Creusa filled with remorse when she thinks that she has found her abandoned son, Ion:

> CREUSA. You were my little baby then! I will leave the altar, though I die for it!
> ION. Take her, hold her! Apollo has driven her mad, she has left the altar. Bind her arms!
> CREUSA. Go on, kill me, kill me!

Such creations are a bad influence since people in the audience of a play, or listening to a poem, may accept the exaggerated emotions seen there and use them in their lives (606b). The concerns which Plato had about dramatic productions and written words continue up to this day, with debates about the value or necessity of censorship.

The argument so far has concentrated on pain, but Socrates extends his condemnation of poetry and theatre to humour. He declares that we may laugh at a comic situation which ought to disgust us (606c) and finally includes 'all the desires, pleasures, and pains that we say accompany all our actions' (606d). Hearing poetic representations of these emotions, three removes from reality, increases the passions whereas 'they ought to wither and be ruled, for that way we'll become better and happier rather than worse and more wretched' (606d). He returns to an earlier attack on Homer and suggests that reading such stories as appear in epic poetry can allow both pleasure and pain to be 'kings in your city instead of law or the thing that everyone has always believed to be best, namely, reason' (607a). Socrates concludes this section of the book with the thought that a man must choose between becoming good or bad, yet poetry can tempt us to neglect 'justice and the rest of virtue' (608b).

The discussion of pleasures and pains, and how they should be moderated, ends here. The next section of Book 10 is considered to be an enigma by some commentators who struggle to justify its existence and content. Here Socrates discusses the immortal soul and returns to an earlier discussion about whether a just man has the best life. How a man becomes just is tied in with his choice of pleasures and pains, both present and in the future, and so I suggest that this section has relevance for my discussion. In considering the greatness of a virtuous life Socrates questions whether anything can be termed great over a short span of time, and the whole of this life is short (608c). To assess the value of virtue, the whole of eternity is needed, which implies that the soul is immortal. The proof of the immortality of the soul has received much criticism, for example from Sheppard (2009, p.155) who says that there are several unsubstantiated assumptions. Fortunately, the validity or otherwise of the argument

is not relevant to my discussion. It is enough that Socrates considers that he has shown that 'the soul *is* immortal' (611b). A righteous man may suffer evil but everything will become good in the end for him, either in this life or the afterlife (612b).

The *Republic* ends with the myth of Er, giving details of the advantages which the soul of a just person will enjoy after death. Sheppard (2009, p.157) remarks that this section is usually ignored in modern commentaries. The events depicted may be strange to us now, but as part of my discussion they have some relevance. After the death of the body the soul is judged. Considerable detail is given but the important point is that each soul eventually chooses the nature of its next life. To make the best choice it needs to be able to 'reason out which life is better and which worse and to choose accordingly' (618d). To do this it needs the ability to choose between the correct pleasures and pains, as discussed above. A man must also go to Hades with knowledge about how to avoid extremes (619a). This supports the account of moderation in emotions and the necessity to cultivate the rational part of the soul which seeks knowledge. This ties the account of the myth with the section which contains the distant object analogy since the latter is used to claim that moderation is best. Measurement is needed to determine the correct size and the rational part of the soul is the one which will determine this.

The *Republic* argument combined with that in the *Protagoras*

I show first that the distant object analogy is not the best one to use for the arguments given in the *Republic.* This raises the question of why Plato would choose to use it. I show that the associated discussion allows some of the omissions in the *Protagoras* to be overcome. The fact that they are dealt with in the *Republic* lends support to my proposal that tracking the use of geometrical references in the dialogues allows discussions in different passages to be brought together. This adds to our understanding of the philosophy.

In the *Republic,* the diminished distant object is part of an illusion and it is compared to the illusion given by poetry. The rational soul is needed to dispel both false impressions. The reference to an illusion is helpful throughout Socrates' argument but I suggest that the analogy of calculating the size of distant objects is not the best choice in this case. Socrates is making the point that the characters in the theatre and in poetry are shown to have emotions which are larger-than-life, which can cause us to think, mistakenly, that exaggerated displays of emotion are acceptable. Since we should keep our emotions in check we must avoid excesses of pleasure and pain, whenever possible. Here there is no reference to future, or distant, pleasures and pains, and it is difficult to see how the analogy of distant objects which

are diminished in size can be applied in this case. Plato is making the point that theatrical events are portrayed as larger than they should be, not apparently diminished. A better analogy can be found elsewhere in the *Republic*. It is more apt because it describes a magnification effect. In his description of men imprisoned in a cave (514a ff) Plato describes how they are constrained to look forward, and can only see shadows on a wall made by firelight shining on artificial objects behind them. The shadows are an illusion and they would be larger than the objects they represent. This makes the shadow example more appropriate than the distant object analogy for the discussion of exaggerated characters and emotions in the theatre.

As discussed earlier, Plato uses other examples, such as weighing, but these are no more applicable to the discussion about exaggerated emotions. Since I am concentrating on the geometry which Plato uses, I confine my analysis to why he would use the distant object, which is not particularly appropriate, when there is an analogy which is more suitable for Socrates' purpose. I suggest that it is the repetition of the analogy which is significant. There are several omissions in the *Protagoras* account and it seems likely that Plato was aware of these and would want to explore the topic further. However, a new account would also have some apparent omissions if considered as a separate entity. One solution would be to combine the new with the old in some way and I suggest that the distant object analogy does this. It is distinctive enough to remind a reader of its previous use and a search of dialogues, already read, would reveal the earlier argument on pleasures and pains. The analogy, therefore, could enable a reader to connect together the two arguments, found in the *Protagoras* and *Republic.* There would be some support for this proposal if it can be shown that some of the omissions in the *Protagoras* account are considered in the *Republic,* resulting in an enhanced discussion on pleasures and pains.

I start with the problem which no one has solved up to the present day, relating to the type of knowledge needed to measure pleasures and pains. In the *Protagoras* this knowledge is needed to measure the relative sizes of pleasures and pains and to correct the false illusion of the size of distant pleasures and pains. In the *Republic*, the discussion is about the illusion of present pleasures and pains which can be exaggerated by the larger-than-life representations in poetry and the theatre. The rational part of the soul is necessary to overcome the illusion, but nothing is said about the knowledge which must be used. However, the discussion in the *Republic* does give some further information which can be related to the knowledge which is needed for both accounts. We know that we cannot find it in poetry because that is concerned with something that is a 'third remove from that which is' (598e). The division of

the soul into parts indicates that the knowledge must reside in the rational part of the soul because the irrational part 'cannot distinguish the large and the small' (605c). It is possible that only philosophers will be able to fully employ this knowledge, and in order to nurture the rational part they must receive the correct education. This is specified in Book 7 and includes training in literature, music, geometry, physical exercise and finally dialectic (521d ff). Although the type of knowledge needed is not defined, we do have more information about it.

A problem I noted in the *Protagoras* account was that, with regard to the distant objects, calculations must be done, but Socrates speaks only of measurement. White (1979, p.253) notes that when Socrates speaks of the reasoning part of the soul, in the *Republic*, one of his standard words is *logistikon*, derived from the verb *logizesthai*, to calculate. For my discussion, this opens up the possibility that in the *Republic* calculations are being considered in order to dispel illusions. This gives a method for finding the true size of a distant object while it is still far away since proportion can be used[9]. Unfortunately, this only clarifies the situation with regard to the analogy, of measuring distant objects, since it is difficult to imagine a calculation which could give as an answer the correct size of a pleasure or pain.

Another problem I discussed with regard to the *Protagoras* is what I termed the 'multiplication problem'. As part of my examination of this I suggested that Socrates should have considered moderation in pleasures, since multiplying pleasures does not necessarily give greater pleasure. In the *Republic*, his criticism of poetry relates to his belief that the excessive emotions seen in poetry and the theatre will be copied by the audience in real life and he disapproves of this. I noted earlier that multiplying pains does give greater pain and I do not think it is by chance that Socrates relies on a pain, that of grief, to demonstrate that moderation is best. However, he also mentions that care must be taken with pleasures.

The possibility that, knowing the choice I should make, I could still choose wrongly is not considered by Socrates in the *Protagoras*. With regard to this, I discussed that I might make a wrong choice if I based my decision on an illusion which I continued to see even after I had made a measurement which showed the illusion to be false. The Müller-Lyer diagram, described earlier, demonstrates this scenario. In the *Republic*, the persistent false illusion is discussed and the notion of the divided soul is used to solve the problem. Those, possibly only philosophers, who have the rational part of the soul as the strongest will correctly reject the illusion in favour of the correct size given by calculations. Lorenz (2006, p.147) notes that

[9] As shown in the Appendix.

the human soul, in the *Protagoras,* is very simple: there, if I have the correct view about what is best, I will act as I should. The use of the divided soul in the *Republic* has allowed an explanation of why this may not happen, if the non-rational part of the soul is strongest.

In the *Protagoras* Socrates assumes that the size of the distant pleasures and pains can be estimated and a choice made which maximizes the overall pleasure over pain. As I noted above, there is no mention of a time scale in his discussion. Socrates did not consider the problem of when the full consequences of any decision can be known. In the *Republic* he is clear about this. He proves, to his satisfaction, that the soul is immortal and in the myth of Er gives examples of the advantages enjoyed in the afterlife by a soul which has, in this life, made choices using the rational part of the soul. This means, in *Protagoras'* terms, that although it may not be obvious in this life that pleasures have been maximized, those who make the correct choices of pleasures and pains will gain their reward after death if not before.

Not all the problems noted in the *Protagoras* have been revisited in the *Republic.* The unit of measurement which could be employed remains a mystery as does the effect of the quality of pleasures and pains, and no mention has been made about the probability of future events. However, I suggest that the fact that some of the problems have been discussed indicates that Plato was building on his earlier work in the *Protagoras* when he wrote the relevant section of the *Republic.* I examine, in the next chapter, the third dialogue, the *Philebus*, where the distant object analogy appears to show that it addresses the remaining problems.

Summary of Chapter Two

In this chapter I examined two similar references to measuring the size of distant objects, and the associated discussions on pleasures and pains, given at *Protagoras* 356c and *Republic* 602c. I analysed the argument which the analogy supports in the *Protagoras*, and justified my decision to concentrate on the geometrical analogy, ignoring other methods of measuring which Socrates mentions. After noting that there are some problems, left untouched, I explored the account given in the *Republic.*

The analogy is not particularly apt for the work in the *Republic* and I suggested that the recurring use of the distant object analogy must have some significance. It would remind those reading the *Republic* about the incomplete argument in the *Protagoras* and allow a continuation of the discussion on pleasures and pains. I examined the *Republic* for any

reference to problems left from the *Protagoras* account and found several. The *Republic* gives a greater understanding of the knowledge needed to assess pleasures and pains: it resides in the rational part of the soul, nurtured by the type of education which Plato stipulates for philosophers. In the *Protagoras*, Socrates speaks of measurements to find the size of the distant object but he does not mention that, in addition, calculations would be needed[10]. Socrates' use of a word implying calculation, in the *Republic*, shows how the size of distant objects can be found and this could add to the understanding of how the true sizes of pleasures and pains can be estimated. However, a distant object continues to appear small, after its true size is found, and this was ignored in the *Protagoras* but is given some attention, in the *Republic*, by referring to the divided nature of the soul. I considered, in the *Protagoras*, the fact that multiplying pleasures does not necessarily result in a greater pleasure, a phenomenon which I termed the 'multiplication problem'. In the *Republic* some account is taken of this, in the sense that excessive emotions are criticized. Finally, the account of the afterlife, in the *Republic*, throws some light on the time scale needed to assess the overall effect of choices made.

I have demonstrated that our understanding of Plato's views on the relative sizes of pleasures and pains is increased by considering passages from both the *Protagoras* and the *Republic.* Since the discussions are connected by references to a distant object, which requires geometry to find its true size, I have some support for my assertion that Plato's geometry enhances his philosophy. This allows a tentative positive answer to my question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how? I say 'tentative' because some problems remain unsolved in the *Protagoras* account so that the possible enhancement could be queried. In addition, I claimed that the use of geometry would allow Plato to separate philosophers from the non-philosophers. Although there is some evidence for the claim that only philosophers have the knowledge required to find the sizes of objects and, by inference, of pleasures and pains, I have not yet explored the value of the analogy for each group of people. However, there is a third example of the same analogy, found in the *Philebus,* which is also associated with a discussion on pleasures. I consider this in the next chapter, in order to examine whether the remaining problems from the *Protagoras* account are covered in the *Philebus* and to analyse what can be gained from the analogy by both philosophers and non-philosophers.

[10] See Appendix.

Chapter Three

More on the distant object analogy and pleasures

3.1 The *Philebus* 60

The background to the dialogue 60

A summary of the discussions on pleasure 63

The use of the analogy 66

3.2 Analysis across the *Protagoras, Republic* and *Philebus* 69

A return to the unsolved problems from the *Protagoras* 69

The targets for the repetition of the analogy 72

Some alternative proposals 74

3.3 Insights given by the analogy and its geometry 76

The value of the analogy 76

Geometry and the measurement of pleasure 78

Philosophers and non-philosophers 80

Summary of Chapter Three 82

In this chapter I examine the final reference to finding the height of a distant object. This comes in the dialogue *Philebus,* and I demonstrate that the arguments there complete those discussed in the last chapter. I start with a description of the background to the *Philebus*, including location, people present and estimated dates of the writing and the action. The *Philebus* contains many of Plato's views about pleasure and I examine these next, taking into account the text and comments by other writers. This provides a basis for the assessment of the appropriateness of the analogy, in the context of the *Philebus* account. I argue that, although there are problems with the reference to the analogy, it enhances the discussions in the *Protagoras* and the *Republic,* covered in my previous chapter. This demonstrates the value of the analogy as a connection between the discussions.

This concludes my investigation into the distant object analogy and I finish with a reflection on its importance as a geometrical example in these dialogues. I draw together points which

have been made throughout this, and the previous chapter, in order to show the value of taking note of Plato's use of geometry in a study of his philosophy. This enables me to provide an answer to my question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how?

3.1 The *Philebus*

It is difficult to state Plato's main argument in the *Philebus* since, as with many of Plato's dialogues, different scholars concentrate on different subjects within the text. Bartlett (2008, p.142) calls it a 'notoriously difficult and labyrinthine dialogue' and suggests that the question which unites the *Philebus* is the best way of life for a human being. He qualifies this by explaining that he means whether we should seek pleasure or knowledge. Gadamer (1991b, p.102) takes some words from near the beginning of the dialogue, where Socrates mentions 'what is good for all creatures' (11b) and takes the topic of the *Philebus* to be the concept of the good. Harte (2004, p.111) considers that the important issue in the dialogue is whether pleasures, like beliefs, can be false. She points out that Protarchus, as a defender of hedonism, objects to Socrates' suggestion of the possibility of false pleasures. The problem of the role of hedonism in the *Philebus* has led to many conflicting views and Carone (2000, p.258) lists some of the authors, such as Frede (1993) and Gosling (1975), who have propounded opinions with which she disagrees, demonstrating that there is no agreement amongst scholars on this topic. Fortunately, however, I am not primarily concerned with such debates. My task is to show that the *Philebus* discussion which is centred on the distant object analogy can be associated with the arguments about pleasures and pains covered in Chapter Two.

The background to the dialogue

Unusually for Plato, very little detail is given about the dramatic setting of the *Philebus.* Taylor (1963, p.408) assumes that the discussion is located in Athens but this is not specified in the dialogue. There is also insufficient information to allow the date on which the action takes place to be established. Taylor refers to 58a, where Gorgias is mentioned, and deduces that the dramatic date of the dialogue must be after the first visit of Gorgias to Athens. There was a visit in 427 but this does not allow an accurate dating of the dialogue. There were similar uncertainties in the dramatic dates of the *Protagoras* and the *Republic,* with some consensus amongst scholars for the 430s for the *Protagoras* and around 421 for the *Republic.* This could place the discussion in the *Philebus* between those found in the other two dialogues being explored. I consider that this does not affect my analysis of the three

dialogues. If the setting of the *Philebus* comes before the *Republic* this means that the reparation of some of the omissions in the *Protagoras* account are merely tackled in a different order, by the Socrates of the dialogues.

With regard to the date when it was written, some interpreters think that the *Philebus* was one of the last dialogues which Plato wrote, placing it immediately before *Laws*, but others, including Waterfield (1980, p.271-2), argue for an earlier dating. Waterfield notes that in the *Philebus* Socrates is the main speaker whereas in other late dialogues he plays only a small part or is absent, as in the *Laws*. He suggests that if the *Philebus* had been written earlier it would fall within a group of dialogues in which Socrates is the main spokesman and he places the *Philebus* immediately after the *Republic* (p.276). This would explain some similar points which are made in the *Philebus* and the *Republic.* Frede (1985, p.153, note 4) agrees with Waterfield about connections between the *Republic* and the *Philebus* and also mentions similarities with the *Symposium,* although she qualifies her suggestion with the thought that Plato could have 'shelved' problems without losing track of them. If the *Philebus* is one of the last written by Plato it would have been written in the late 350s. Hackforth (1972b, p.4) argues for a date between 360 and 354 because of the lack of any political references. He suggests that such political detachment would fit with Plato's dislike of Dion's call to arms, after Plato's last visit to Syracuse which, according to Scott (2009, p.276), was in 361. Taylor (1963, p.408) suggests a date 'years after 367' which is the year when Aristotle arrived at the Academy, although he gives no support for his assertion. It appears that there is only agreement that the *Philebus* comes after both the *Protagoras,* an early dialogue, and the *Republic.* The order which I am using in my analysis thus follows the probable order in which Plato wrote the dialogues, but it should be noted that my work is not dependent on a correct chronology.

Only Protarchus and Philebus participate in the debate with Socrates. Protarchus is given as the son of Callias (19b) and a pupil of Gorgias (58a) and Waterfield (1982, p.10) suggests that he is the same person mentioned by Aristotle at *Physics* 197b. There is uncertainty about the identity of Philebus and he takes little part in the discussions. It is, therefore, puzzling why the dialogue is named after him. Klein (1972, p.158) notes that of 2369 lines in the dialogue Philebus speaks only 23 and although the title, *Philebus,* has been handed down to us from antiquity it is never mentioned by this name in writings by Plato's contemporaries. Neither Protarchus nor Philebus are named as being present in the other discussions connected with the analogy arguments, in the *Protagoras* and the *Republic.* There are several references to a group of spectators, for example at 16a, when Protarchus says: 'don't you see what a crowd

we are and that we are all young?' after which Socrates refers to them as 'my boys'. They take no part in the discussion and their youth would indicate that they are not philosophers.

Several scholars note a connection between the content of the dialogue and some known arguments of Eudoxus of Cnidus. Aristotle (*Nicomachean Ethics,* 1172b) mentions that Eudoxus said that 'pleasure is the good' and he suggests that Plato used similar arguments to those of Eudoxus, which the translator (Ostwald, 1962, p.275), cites as being in the *Philebus*. Hackforth (1972b, p.6) reports that some writers have suggested that Philebus is a fictitious name for a character who is meant to be Eudoxus. He cites Taylor (1963, p.410) as an example but reading Taylor's work shows that he only draws a comparison between the views of Eudoxus and Philebus. Hackforth (1972b, pp.82-3) also mentions Speusippus whose argument about pleasure and pain, good and evil, is mentioned by Aristotle (*Nicomachean Ethics,* 1153b). Hackforth interprets Speusippus' argument as promoting the middle or neutral life and this idea is found in the *Philebus*. However, as Hackforth admits, it is not known when Speusippus announced his views. It should be noted that a portrayal of the historical Socrates arguing about the views of Eudoxus or Speusippus would be an anachronism since Eudoxus is believed to have been born between 408 and 390 and Speusippus was born around 410. Crombie (1962, p.252) gives a different view when he suggests that the dialogue is obscure because Plato was writing for people in the Academy where the various topics found in the text had been discussed.

The *Philebus* runs from 11a to 67b with the reference to distant objects at 41e. There is, therefore, a little over half the dialogue before the geometrical reference under consideration. The dialogue starts with a reminder of a discussion which has just occurred and Socrates summarizes what has been decided. Philebus believes that pleasure is all that is needed for the good life but Socrates favours knowledge, understanding and remembering with the associated right opinions and true calculations (11b). Socrates attempts to analyse what pleasure is and notes that, unlike Protarchus, he thinks that not all pleasures are good (13a-b). There is then a prolonged discussion about the one and the many with the eventual conclusion that the best life has a mixture of both knowledge and pleasure (22a). Socrates wants to examine which of these is most important in making a life good. These early discussions lay the foundation for the arguments which run through the rest of the dialogue.

A summary of the discussions on pleasure

The *Philebus* contains a thorough analysis of different types of pleasure in addition to the attempt to decide whether pleasure or knowledge gives the best life. I give the main points covered, before and after the use of the distant object analogy, concentrating on those with a particular relevance for the discussions I have covered in the *Protagoras* and the *Republic.*

Socrates eventually decides that he can only examine pleasure if he considers it with pain since they can arise together (31b). He lists several examples to illustrate this point, one being the claim that hunger is a form of disintegration and pain, with eating being the corresponding refilling and a pleasure (31e). There is also the anticipation of the soul of the two kinds of experience, with hope before pleasure being comforting and the expectation of pain being frightening (32b-c). Socrates proposes a third condition where there is neither pleasure nor pain (32e) and suggests that a person who chooses a life of reason could live in this neutral state (33a). He then returns to an examination of the pleasure which belongs to the soul and suggests that it depends on memory (33c). After an analysis of this he realises that sometimes a man can undergo pleasure and pain simultaneously, when he is in need of replenishment, with associated pain, but has a pleasurable hope of replenishment, through his memory (36b). The association of pleasures and pains has been a theme throughout the three dialogues which contain the distant object analogy.

There is then a discussion about true and false pleasures. Socrates thinks that some pleasures and pains can be false, but Protarchus denies this, although he accepts that there are false judgments (36c). Socrates points out that when someone makes a judgment he does so whether the judgment is right or wrong and he suggests that, in a similar way, someone who is taking pleasure cannot be deprived of this taking of pleasure. It is difficult to understand, therefore, why a judgment can be true or false but a pleasure cannot (37a-b). Socrates points out that a pleasure can arise with a false judgment but Protarchus, although agreeing with this, believes that it is only the judgment which is false, not the pleasure (38a).

Socrates proceeds by saying that memory and perception lead to judgment and gives the example of looking at something from a distance, although here he means that the view may not be clear. The viewer may not be certain whether he is looking at a statue or a man in the distance (38d). Whatever he decides, Socrates imagines the judgment written in the soul, as in a book (39a). If what is written is true he has formed a true judgment but if what is written is false he has formed a false judgment. In addition there might be a 'painter' who provides illustrations in the soul (39b) and again these pictures may be true or false. This applies to

experiences in the past and present but Socrates concentrates on the future. He relates the pictures to 'anticipatory' pleasures and pains (39d) and uses the example of a man who has a mental picture of himself gaining much gold in the future, with associated pleasure (40a). Both good and evil men can have such pictures but because the good are loved by the gods the pictures they have will turn out to be true. The wicked will have false pictures in their minds so that they are enjoying false pleasures (40b-c). A picture of a future pleasure is true if it portrays a pleasure that actually occurs. Thus, a mental picture of my being pleased with the future receipt of a sack of gold is true if, and only if, I receive a sack of gold *and* it brings me pleasure. This reference to the uncertainty of future pleasures is relevant to my work on the analogy since it is not found in either the *Protagoras* or the *Republic*. It will be related to these dialogues in the next section.

Socrates reminds Protarchus that anyone who judges anything is always judging, whether the judgment is true or false (40c-d). He wants to view pleasures and pains in a similar way so that anyone who has a pleasure always has it even if it is a false pleasure (40d). The only way to distinguish between good and bad judgments is by their truth or falsity and he claims that the same can be applied to pleasures, which can only be bad if they are false (40e). Protarchus disagrees with this since he insists that some pleasures can be bad, because there is some badness involved with them (41a). Socrates shelves the case of bad pleasures for later and proceeds with the discussion of false pleasures. He reminds Protarchus that they had decided earlier that when we have desires the body and the soul have separate experiences (41c). There can, therefore, be pleasures and pains existing side by side (41d). Socrates wants to make a decision about which is greater or smaller, pain compared with pleasure, pain compared with pain or pleasure compared with pleasure (41e). This leads to the geometrical analogy, which I analyse in the next section.

Apart from discussing the magnitude of pleasures and pains Socrates also examines ways in which we can experience them; in a life of pleasure, a life of pain and a neutral life (43d). He agrees with Protarchus that a life free from pain is not the same as a life of pleasure although some people may think this (43d-e). Such people believe that they are experiencing pleasure because they are not in pain, resulting in a false judgment about pleasure (44a), and Socrates refers to some unnamed enemies of Philebus who believe this (44b-c). In order to dismiss this view Socrates realises that he must determine the nature of pleasure and the discussion continues with an analysis of how pleasures can be both mixed and unmixed with pains. As an example of a mixture of pleasure and pain Socrates mentions the pleasurable relief felt from painful itching when it is rubbed (46a). Such experiences can have an even amount of

pleasure and pain but some may have more of one than the other (46d). There is a long discussion which includes mixtures of pleasure and pain found in the soul, such as when watching a tragedy which can provoke a mixture of laughter and weeping (48a).

Further examples are given and eventually Socrates decides he must now examine unmixed pleasures (50e), those with no associated pain. These can also be called pure or true pleasures, with examples such as pure colours, shapes and sounds although the pleasure of a shape is not related to an object but to the pure shape of a geometrical figure (51b-c). Plato is not concerned here with who enjoys which pleasures. However, it is likely that 'most people', as referred to by Socrates in a derogatory way in several dialogues (for example, *Republic* 351c), would be unlikely to agree that a geometric shape would give one of the best pleasures in life. I suspect that Plato is here talking about the philosopher who, having had several years of training in geometry, would appreciate the pure pleasure of such a figure. Socrates then adds the pleasure of learning as a pure pleasure because there is no pain associated with a hunger for learning (51e-52a) and no pain associated with a lapse of knowledge (52b). He acknowledges that the pleasure of learning belongs 'not to the masses, but only to a very few' (52b). As Hackforth (1972b, p.100) suggests, this implies that the term 'knowledge' does not apply to everyday learning but only to such learning as is associated with philosophers and mathematicians.

Throughout his discussions, Socrates refers to mixed, false, pure and true pleasures, but there is considerable confusion about these terms. Dybikowski (1970, p.244-5) notes that several writers find no distinction between mixed and false pleasures in the *Philebus.* However, he disagrees because the list of false pleasures at 36c-44b all include a mistaken belief, for example someone in pain may believe a neutral state to be pleasurable. When Socrates speaks of pure and mixed pleasures the criteria for the classification of a pleasure into one of these categories lies in whether there is associated pain or not. Thus a pure pleasure has no associated pain, although Socrates qualifies this requirement to include 'imperceptible and painless lacks' (51b). The confusion noted in several writers could be because, as Dybikowski (1970, p.246) says, Plato designates pure pleasures as true, and being true can be contrasted with being false. However, in this context 'true' is used in the sense of not being associated with pain and 'false' as being associated with error.

Frede (1992, p.452) notes a problem with the whole concept of a true pleasure since 'pleasure' is described as filling a painful lack. Socrates is aware of the apparent discrepancy when he states that true pleasure can be associated with 'imperceptible and painless lacks' (51b). His list of pure pleasures shows that there are few which fulfil this requirement and

some of these can be criticized. One such example is the inclusion of learning as a pure pleasure. The notion of the absence of any pain associated with the pleasure of learning has produced much debate. Warren (2014, pp.23 ff) refers to several passages which indicate that Plato was aware that there are pains linked with the process of learning. These include the notion that reflecting on knowledge which is forgotten can cause me distress if I realise that it is now needed (p.25). A different view is taken by Ionescu (2008, p.453) who suggests that Socrates believes that awareness of ignorance is necessary for learning and this would be a perception of a painful lack. In support of learning as a pure pleasure, Taylor (1956, p.80) makes a valid point when he observes that mixed pleasures involve oscillations between a want and its removal, as with a man who, when he is hungry, enjoys a meal and in order to enjoy another one must become hungry again. To enjoy knowledge it is not necessary to forget repeatedly what has been learnt and so learning, obviously a pleasure in Socrates' eyes, must be a pure pleasure. The conflicting opinions of modern writers indicate that the definitions of different types of pleasure are somewhat ambiguous.

The use of the analogy

In the *Philebus* Socrates again uses the analogy of a distant object and, unlike the texts examined in Chapter Two, he does not use other analogies. He says:

> ... does it happen only to eyesight that seeing objects from afar or close by distorts the truth and causes false judgments? Or does not the same thing happen also in the case of pleasure and pain? ... if you take that portion of them by which they appear greater or smaller than they really are, and cut it off from each of them as a mere appearance and without real being, you will neither admit that this appearance is right nor dare to say anything connected with this portion of pleasure or pain is right and true. 41e-42c

Socrates says that he had thought earlier that 'it was true and false *judgments* which affected the respective pleasures and pains with their own condition' (42a). Now the pleasures themselves are seen as true or false 'because they are alternately looked at from close up or far away' (42b). Here the method of measuring the objects is one of comparison. A difference in size can be determined by separating off the excess from one of the objects and this was a common method in geometry to compare the size of two shapes. Socrates declares that we must 'take that portion of them by which they appear greater or smaller than they really are, and cut it off from each of them as a mere appearance' (42b-c). This will show that the appearance of the distant object is wrong. The use of the same analogy, of the false apparent size of distant objects, is again associated with an argument about pleasures

and pains. The reader of the *Philebus* could now be reminded of the arguments already explored, when the same analogy was used in the *Protagoras* and the *Republic.*

As in the *Protagoras* and the *Republic* Socrates wants to show that pleasures can be viewed incorrectly and his analogy of the distant object serves to demonstrate that things are not always as they seem. Perhaps the most obvious problem with his analogy in the *Philebus* arises from his declared method of comparing the sizes of the greater and the smaller. The notion of comparing two different lengths by cutting off from the larger the amount by which it exceeds the smaller is a valid technique in diagrams, but it is difficult to see how this can be applied to either distant objects or pleasures and pains. I can only imagine it being used if there is a near object which is known to be the same size as the distant one. Then the amount by which the distant one is apparently diminished could be found. However, this would mean that the size of the distant object is already known and the purpose of such a calculation would be to calculate, using proportion, the distance away. This is not what Socrates is attempting to find. The method would similarly fail if applied to near and distant pleasures and pains. There is a further problem with the use of the analogy here because the falsity of anticipatory pleasures is not mainly concerned with their distance from us, but arises because the pleasure may not materialise or be as expected.

Another problem arises from the assumption, implicit in the use of the distant object analogy, that future pleasures appear deceptively small. Some writers criticize this view. Gosling (1975, p.214) suggests that there is the possibility that some future pleasures appear 'larger than life' because of present distress which affects judgment. Later Gosling (p.219) acknowledges that distance tends to make things look smaller but thinks that the situation with regard to pleasure is more complicated. Similarly Taylor (1963, p.423) notes that we expect a coming pleasure to be greater than it turns out to be by contrast with present pain, and Gadamer (1991b, p.174) says that the anticipated pleasure seems more enjoyable in contrast with present pains. Parry (2010, p.224-5) also observes that if pleasure in the future is contrasted with present pain then the pleasure appears greater. Reidy (1998, p.349) notes that if the future pleasure is only perceived alongside a present pain then 'correctly perceiving the relevant pleasures and pains in themselves becomes difficult, if not impossible'. In terms of the geometrical analogy this is equivalent to us not knowing the size of the object when it is near as well as when it is far away. Benardete (1993, p.188) contrasts the 'spatial index', which tells us that a distant object is larger than what we apparently see, and the 'time index' which should tell us to diminish the pleasure. These writers all suggest that the future, or distant, pleasure is perceived as larger, not smaller, than it really is.

Other writers have different views. Ionescu (2008, p.442) takes a more neutral position when she mentions, in discussing how the intensity of a future pleasure can be wrongly estimated, that we may 'underestimate or overestimate what we anticipate as future pleasure'. However, this does not correspond to the object analogy where the size of the distant object is always underestimated. A few, such as Hackforth (1972b, p.78), have views which support the analogy because they believe that we underrate a future pleasure or pain so that when it becomes a present feeling it seems to have increased. The lack of agreement amongst modern writers reflects ambiguity in the *Philebus*, which implies that the distant object analogy can be criticized in this context.

Most of the authors cited considered the assessment of a future pleasure when in present pain. There are other examples which Plato could have used to take account of the current situation in which the assessment of the size of the pleasures and pains is made. I can demonstrate my point by referring to examples which he uses elsewhere. At *Theaetetus* 152b he discusses measuring temperature. As far as Plato was concerned temperature could not be measured quantitatively. Cohen and Drabkin (1948, p.255) record that the Greeks knew that things expanded on heating, but there is no evidence that expansion was used to create a scale of temperature. Although temperature measurement would not be a useful analogy for measuring pleasures and pains when Plato wants to suggest that quantitative measurements of these are possible, the situation here concerns qualitative values. In the *Theaetetus* it is said that what one man considers to be a cold wind, another will not. Such an example used in the *Philebus* would allow for the point that our present condition can affect our view. Just as the man who is hot might think that something which is warm is cold, so a person who is at present in pain could see a neutral state as pleasure or misjudge the magnitude of a pleasure. The distance analogy does not include the possibility of different assessments because of different conditions. It could be argued that this analogy was in Plato's mind when he wrote the *Philebus,* since he mentions hotter and colder quantities at 24d.

Another appropriate analogy which Plato uses is found in the *Republic,* where he explains the situation of a straight stick which looks bent when placed in water (602c). This is also a false perception and so has the same use as the distant object as an analogy to demonstrate a false view of a pleasure. I criticized this in the context of the *Republic* passage, but this analogy has an advantage, for the *Philebus* discussion, over the distant object one, since a truly bent stick would also look bent in water. This gives some uncertainty to the outcome when the stick is removed from the water, which corresponds to the uncertainty about future

pleasures. They may not materialise as expected and so might be either larger or smaller than they appear.

Neither of my suggestions gives a clear correspondence with the situation where an anticipated pleasure never happens, but neither does Plato's distant object analogy and, with regard to misjudging a future pleasure, my examples offer some advantages. Since better examples exist, found in other passages in the dialogues, Plato must have had a reason for using the same example as he used in the *Protagoras* and *Republic,* where he develops similar arguments. I have been critical of Plato's use of the analogy in the *Philebus* but I examine next its value within a complete discussion of the measurement of pleasures and pains.

3.2 Analysis across the *Protagoras, Republic* and *Philebus*

Although some of the problems left unresolved in the *Protagoras* argument are discussed in the *Republic* passage, there are some which are ignored in both texts. I show that there is an acknowledgement of these in the *Philebus*, with an attempt to solve them. This reinforces the importance of taking note of Plato's geometrical references, in this case in order to obtain a complete assessment of the situation regarding the size of pleasures and pains. The repetition of the analogy could have a further significance, as a reminder of previous discussions, and I explore this suggestion and some difficulties which arise from it. I leave a discussion on the possible value for separating philosophers and non-philosophers to the next section.

A return to the unsolved problems from the *Protagoras*

There are several problems from the *Protagoras* which are not addressed in the *Republic* and I start with the uncertainty about future pleasures and pains. The uncertainty can arise from two different scenarios. Firstly, the anticipated pleasure, or pain, may not materialise and secondly, if it does occur, it may turn out not to be as pleasurable, or painful, as expected. In the *Philebus* Socrates mentions anticipatory pleasures and pains when he uses the example of the man who imagines that he will have a pile of gold and sees an inner picture of the pleasures this will enable him to have in the future (40a). However, the image does not always portray what will actually happen and so these pleasures can be counted as false (40c). This statement occurs a few lines before the reference to the distant object analogy. The discussion moves to a consideration of how to judge the size of pleasures and pains which can be viewed close up or far away (42b), thus connecting it to the arguments in the *Protagoras* and the *Republic.*

There are two ways in which the image of the pleasures to come with the gain of gold could be false. The man may never achieve the amount of gold which he imagines will bring pleasure so that the pleasure he envisages never occurs. Alternatively, he may become very rich but find that money does not bring the pleasure he falsely anticipated. Both aspects of the problem, not discussed in the *Protagoras*, have now been considered. Socrates even offers a solution of sorts. A good man will not see false images of the future. This can be interpreted to mean that if the good man thinks hopefully of gaining gold he will actually gain the gold at some time in the future and he will enjoy the pleasures which he imagined that the gold would bring. It is only the bad man who will have a false picture of the pleasures which the gold will bring (40b). Either he will not obtain it or the enjoyment he anticipates is false.

Although the *Philebus* addresses the problem of the uncertainty of future pleasures there are difficulties with Socrates' account. When Socrates talks of a painter producing images in our soul he claims that he paints the truth for those who are good and he paints false pictures for those who are bad (40a-c). Harte (2004, p.125), amongst others, notes that if the falsity arises because the gold is never achieved then there is an implication that good people will acquire the gold, which means that good people can predict the future. Socrates' statements can only be acceptable if it is the pleasure in having the gold which can be false and a good person will not anticipate taking pleasure in becoming rich. The pleasure for bad people is false because taking pleasure in becoming rich could be considered to be a bad source of pleasure or, alternatively, wealth does not always bring pleasure. A further complication is noted by some writers, for example Delcomminette (2003, p.229) who suggests that if there is an anticipatory pleasure then this exists whether or not the pleasurable anticipated event actually materialises. For example, I may gain much pleasure from reading guide books about a place I intend to visit. I may later suffer some pain and disappointment if my trip is cancelled but nothing can remove the pleasure which I felt as I made my plans. There are many debates about the treatment of future pleasures in the *Philebus*, for example by Gosling (1959, p.52) and Kenny (1960, pp.51-2), in a reply to Gosling's article. For my purposes the important point is that Plato shows an awareness of the unpredictability of future pleasures and attempts to offer a solution: that good people will not anticipate falsely.

Another problem from the *Protagoras* which is not discussed in the *Republic* is the one concerning the effect of the quality of the pleasure. The estimation of size which is promoted in the distant object analogy will only determine a greater pleasure but not make any allowance for the fact that some pleasures may be intrinsically better than others, even if

smaller. In the *Philebus* a hierarchy of pleasures can be deduced from various comments by Socrates throughout the dialogue. At the bottom is freedom from pain. Although some people would classify a life free from pain as pleasant, Socrates disagrees (43d). The next levels of the hierarchy of pleasures are formed by what Socrates calls mixed pleasures. These are mixed with pain and there are several different levels here since there can be different ratios of pleasure to pain (46d). Finally there are the pure pleasures. Socrates does not distinguish different levels of purity but Ionescu (2008, pp.448-9) suggests that he never claims that pure, unmixed, pleasures are always true and never false. This leaves open the possibility that some pure pleasures are less true than others and could even be false. Ionescu gives an example to demonstrate this. She considers the pleasure of smell, which is considered to be a pure pleasure. The pleasure of smelling a rose could be falsely exaggerated if I compare it with a smell I just had from something like a daffodil which does not have a strong smell. This would mean that there are levels of different quality within the pure pleasure category. If all these criteria are applied to any pleasures being contemplated it should be possible to identify which are intrinsically better, although the best comprise a very limited number of pleasures.

The remaining problem, which is not discussed anywhere in the *Protagoras* or the *Republic,* is the one concerning the unit of measurement of pleasures and pains. Harvey (2009, p.18, note 44) suggests that it is modern thinkers who expect to be able to attach 'a precise quantity to any possible state'. It is, therefore, possible that the uncertainty about a numerical value, when measuring pleasures and pains, was not a concern for Plato. However, there are some comments earlier in the *Philebus* which could be taken to be a recognition of the problem. The skill necessary for quantitative measurements may not be available to human beings. Socrates mentions that all artistic discovery is a gift from Prometheus, and it allows men to determine the one and many, and the unlimited, and then look for 'the exact number of every plurality that lies between the unlimited and the one' (16d). However, although the gods gave us such abilities Socrates claims that even the 'clever ones' of his time cannot achieve the correct results (16e). This is not specifically connected to the measurement of pleasure and pain, later in the dialogue, and Socrates does not arrive at a solution nor does he propose any unit of measurement. However, I see this as some recognition of the problem at least, and this is more than is found in the *Protagoras* or the *Republic*.

In the *Philebus* several solutions have been proposed for problems regarding the size of pleasures and pains, associated with the analogy argument, which were not discussed in

either the *Protagoras* or the *Republic*. This means that looking at the three dialogues together has produced a more complete discussion than concentrating on any one of them. I explore next the significance of an analogy which is repeated in this way.

The targets for the repetition of the analogy

It is worth noting that in the three dialogues examined, Plato does not waste time repeating points which he appears to believe he has already covered satisfactorily. Each dialogue tackles questions which were left unanswered in the other two, in a complimentary way. It is, therefore, difficult to accept that for Plato the three discussions were not connected. If, as I argue, the connections were made intentionally, then the next consideration must be the intended recipients for the reminders.

I first consider the situation from the point of view of the Socrates of the dialogues. If he wished to continue a discussion on pleasures and pains it is feasible that he would want his companions to remember any earlier conclusions. The various dramatic dates given show that the events recorded took place some years apart. The arguments given in the *Protagoras* probably took place in the 430s, with some uncertainty about the actual date, and the estimated date for the events recorded in the *Republic* is around 422. It is possible that the setting for the *Philebus* is between these two dates, although there is less certainty about the Philebus dating. However, it appears likely that the events depicted in the three dialogues were separated by several years. This offers some support for the use of a reminder by Socrates since his associates may not remember, initially, the previous discussions. However, a closer examination of the background to the dialogues reveals that this is not the case.

The notion of a participant recollecting an earlier argument, with or without the use of the analogy, is impossible because the list of the people present at each discussion is different. Rowe (2006, p.21, note 9) generalises this point in his argument against using references to other dialogues in an account of any one text. He says: 'Any explicit reference would mean keeping at least some of the same *dramatis personae*; that Plato rarely does this between any two dialogues is perhaps one mark of how little interested he is in cross-references'. I showed in my descriptions of the background to the *Protagoras, Republic* and *Philebus* that there is no one, other than Socrates, present in more than one of these dialogues but I strongly disagree with Rowe's statement. Rowe has omitted to consider, in addition to Socrates, one participant in all the dialogues, the person who reads Plato's texts. In Plato's day the dialogues might have been aimed at listeners, as the texts were read aloud to an

audience, but the same argument would apply and for ease of reference I will refer to readers at this point in my examination. I return to this problem in my final chapter, when I analyse the reader/listener debate more closely.

Taking into account the reader moves the discussion from the point of view of Socrates, as a participant in the written dialogues, to that of Plato, as the author of the dialogues. If Plato wanted someone who was reading, for example, the *Republic,* to remember a previous discussion in the *Protagoras,* there is more than one way in which he could do this. He could have a character in the *Republic* repeat the whole argument from the *Protagoras,* but this would mean including many pages which the reader had already met, with a corresponding loss of interest. Alternatively, a final conclusion could be reiterated but, unfortunately for this proposal, the way in which Plato presents Socrates' arguments means that there is no simple, concise, conclusion which can be given to summarize a previous discussion. An alternative method would be to remind a reader of the previous argument without repeating it all word for word. I propose that this is the value of the repetition of the distinctive distant object analogy which had already been used in the *Protagoras.* Thus, the reader of the *Republic*, at the point where the distant object analogy appears, may be reminded that the same analogy appeared elsewhere. A search would then reveal that a similar discussion was being held, on the measurement of pleasures and pains, as was pursued in the other text. The same reasoning can be applied to the *Philebus* so that the reader can look for the other two dialogues where the same analogy appeared. I suggest, therefore, that the repeated geometrical examples are aimed at the readers of Plato's texts. I explore the significance of this, with regard to Plato's views on writing, in my final chapter, when I critically analyse theories about cross-dialogue connections.

Such a proposal could explain why Plato used the same analogy in the three dialogues, even though it is not particularly apt. However, my proposal might be criticized because it implies that Plato *intended* to remind his readers of another text and any theory about what Plato meant, or intended, can only be conjecture. I suggest that it is acceptable to make a proposal about the *possibility* that Plato used the repetition of his geometrical examples intentionally, if I can show it to be feasible. I have shown that the discussions are enhanced when the three texts are taken together, which offers some support for my proposal, but I now consider some other possibilities.

Some alternative proposals

I start by looking at the purpose of the analogy. I have suggested that the distant object analogy serves to enhance our understanding of Plato's philosophy in several ways and one of these is the connection that it achieves between several dialogues. However, there is a more mundane reason for Plato to repeat the analogy. It could be simply a good way of demonstrating the point which he wished to make, regarding the choice of pleasures and pains. I have shown that this suggestion fails because the analogy is not the best example to use. In the *Protagoras* it provides a suitable comparison with distant pleasures and pains but there are many problems with the account, as I explored above. In both the *Republic* and the *Philebus* I showed that the analogy is not particularly relevant for the aspect of pleasures and pains being discussed, and I indicated that there are better examples for the situations being explored there. It could be argued that the use in the *Republic* can be explained if all Plato wanted to do was give some examples of optical illusions, in order to enhance his argument that poetry has illusionary aspects which should be overturned. I claim that this does not weaken my proposal since there was no necessity to choose the distant object analogy over other more appropriate examples. The fact that Plato made this choice suggests that he had in mind the *Protagoras* discussion. The use in the *Philebus* appears to be even less appropriate, giving further support to the notion that there must be some other reason for its use. My claim is that the most obvious one is to remind his audience of the other passages where the same analogy appears.

Against this suggestion, there are some features of Plato's work which could indicate that he had no intention of connecting the dialogues together. For instance, it might appear feasible that if Plato wanted to continue some earlier thoughts he would set each of the dialogues in the same location. The fact that he did not do so could be seen as a possible criticism of any connection between them. However, a comparison of their locations shows how apt each one is for the part of the argument which takes place there. The *Protagoras* is known to be set in a particular house in Athens, that of Callias, who is entertaining several sophists. The discussion is on finding the relative sizes of pleasures and pains, which needs knowledge to be applied, and the sophists claim to teach knowledge. The part of the argument concerned with responding to emotions with moderation, in the *Republic,* is conducted when Socrates has gone to the temple of Bendis in Piraeus. As stated above, this is where a battle took place against the Thirty Tyrants, who were overthrown partly because they did not rule with moderation. Scott (2009, pp.13-14) notes that although their rule was initially 'fairly mild and temperate', it soon became an attack on democracy with 'an extremely exclusive members-

only club'. Plato's contemporaries could connect the call for moderation in the dialogue with the known behaviour of some of those present in the discussions. The setting of the *Philebus* is not specified but some of the views expressed in the text have been connected to people known to have attended the Academy in Athens and so it is possible that the location is intended to be Athens. The content of the *Philebus* is such that there is some support for the view that it was intended to be read by members of the Academy. I suggest that such connections could have been made by Plato's contemporaries and he might have thought that this was more important than setting the dialogues in the same place.

I have implied that Plato chose his settings carefully, and there is some support for this in both ancient and modern works. Sheppard (2009, p.20) relates an anecdote which demonstrates this. Dionysius of Halicarnassus states that Plato repeatedly revised his writings and on his death a writing tablet was found containing different versions of the opening sentence of the *Republic*, 'I went down to the Piraeus yesterday...' (327a). Although this story is probably apocryphal, it demonstrates that Plato was renowned for taking care with the settings of his dialogues and they can, therefore, be assumed to have some relevance. Sheppard (2009, p.21) confirms this view by analysing the historical context of the *Republic,* including the setting and the people present. With regard to my work, the setting is important within the context of each dialogue and a common setting is not essential for the three dialogues. The distant object analogy alone is sufficient to bring to mind the earlier arguments.

It could also be proposed that if the repetitions have any importance they would be easier to understand if some of the participants appear in more than one of the dialogues. Socrates could then be seen to be reminding those present of the previous discussions at which they were also present. Unfortunately, in order to do this Plato would have had to forego choosing the most appropriate interlocutors for the particular topics covered in each dialogue. As indicated above, the people are associated with both the locations and the discussions. In the *Protagoras* the presence of the sophists is appropriate for a discussion on applying knowledge to determine the relative sizes of pleasures and pains. Plato is often dismissive of the sophist's claim to have or teach knowledge. The *Sophist* lists many criticisms of them, for example the visitor notes that they appear wise to their students 'without actually being wise ... the sophist has now appeared as having a kind of belief-knowledge about everything, but not truth' (233c). In the *Republic*, the people with Socrates were involved in one way or another with the turbulent events which Plato knew would come later in the reign of the Thirty Tyrants. Less is known of Protarchus and Philebus, the only

participants in the discussion with Socrates in the *Philebus*, but Plato's contemporaries would have been able to relate to them and presumably make a connection with the subject matter of the dialogue. The choice of different people for the three dialogues has reasons which outweigh the advantage of using the same people throughout.

My approach throughout Chapters Two and Three appears to depend on theories concerning the chronology of the dialogues, and these change as new systems of analysis are found. My claim is that the chronology is not necessarily relevant. For instance, if it was somehow found to be the case that the *Republic* was written before the *Protagoras* my proposal would not suffer. In this new situation the *Protagoras* lines would be a repetition of the *Republic* text and Plato would have dealt with the problem of exaggerating present pleasures and pains before moving on to pleasures and pains in general and specifically those in the future. Similar comments could be made about the placing of the *Philebus* with regard to the *Protagoras* and the *Republic.* My assertion is that the arguments are enhanced by considering all three texts, not that they necessarily progress from one text to the next.

Adding the account in the *Philebus* to those in the *Protagoras* and *Republic* has allowed the omissions in the latter texts to be addressed. Plato has now presented a full discussion on finding the relative values of pleasures and pains in order that a correct choice can be made. This ends my research on an aspect of practical geometry. Before proceeding to my next geometrical example I bring together several strands of my discussion, from this chapter and the previous one, to give a complete picture of my exploration of Plato's geometry so far.

3.3 Insights given by the analogy and its geometry

In this section I consider all aspects of my work so far in order to assess the value of using Plato's geometrical references in a study of his philosophy. I concentrate first on some different aspects of the use of the analogy of the distant object, in order to demonstrate its value. I look for enhancements to Plato's philosophy which the references to the geometry may give. These include some observations on the topic of *akrasia*. Finally, I explore how the geometry associated with the analogy could be used to separate philosophers from non-philosophers so that each group gains different insights from the discussions.

The value of the analogy

I have examined the distant object analogy in terms of the geometry needed to find the true size of the object. Although I have criticized its appropriateness there would be little point in using it if it had no value in itself. At a basic level the analogy serves to give a picture of how

things can appear other than they really are. In particular, an object in the distance is seen as apparently diminished and a comparison is made with distant pleasures and pains, whose magnitudes can also be perceived incorrectly. Although, as I highlighted earlier, there are some discrepancies about the sizes of such pleasures and pains, they are generally taken to appear to be deceptively small, allowing a worthwhile comparison with the distant object. Since there is a way to find the height of the object, in calculations about which Plato would have had knowledge, the implication is that the sizes of pleasures and pains can be found in a similar way. Once the relative sizes are known, the correct choice can be made between different ones. In spite of its faults, the analogy gives an image which can be easily understood, and serves to demonstrate the fallibility of our assessment of the size of pleasures and pains.

The reference to a distant object provides a further insight. Plato makes the point several times that such objects may only be seen indistinctly. Using the comparison with distant, or future, pleasures and pains means that we are aware that as well as appearing to be diminished they may also appear to be indistinct. Depending on the nature of the lack of clarity this could demonstrate a problem when we try to assess the distant pleasures and pains, for example it may be that they are not seen in their entirety. Although Plato does not explore the indistinct aspect of a distant object in the context of the analogy, it adds to the value of the discussion about the correct choice of pleasures and pains.

The analogy serves a further purpose since tracking it across several dialogues reveals a continuation of the discussion about the relative sizes of pleasures and pains, and the choices which should be made. I showed that the debate in the *Protagoras* is incomplete and the one in the *Republic* covers some of the missing aspects, but not all. The same can be said of the *Philebus,* so that all three dialogues are needed. Any one of the accounts can be criticized as being inadequate, but connected together they offer an enhanced picture. Without the analogy, and merely observing, in the three dialogues, that the relative magnitudes of pleasures and pains must be found in order to make a wise choice, would not provide a distinctive means of connecting the three discussions together. As an advocate of the notion that Plato wrote nothing without reason I consider that one possible purpose for the analogy is to make the link more obvious. Finding connections between dialogues is a subject which causes some controversy in modern times, as I explain in my assessment of interpretation theories in the final chapter. However, as Gill (2006, p.137) relates, starting with Aristotle and continuing until the 'end of antiquity', there was a view that several of Plato's doctrines can be found in a connected system across some or all of the dialogues. I argue that merely

observing that the relative magnitudes of pleasures and pains must be found in order to make a wise choice, would not provide a reliable means of connecting the three discussions together. The distant object analogy is very distinctive, and when it is repeated, earlier encounters are easily brought to mind.

In this section I have demonstrated the value of the analogy since if it is ignored some important aspects of Plato's work are missed. It serves to highlight the connection between the three discussions, in the *Protagoras*, the *Republic* and the *Philebus*, and an examination of these has provided a greater insight into Plato's views on the relative sizes of pleasures and pains. Although I have referred to the geometrical aspects of the analogy, I have not yet summarized the worth of the geometry itself, and I consider this now.

Geometry and the measurement of pleasure

The geometrical calculations needed to find the height of a distant object were recent discoveries in fourth century Athens. Plato would realise that some years earlier people would think that the determination of the height of an inaccessible object was an impossible task and only approximate sizes could be given. In his time the practical methods to enable this to be done, by calculation involving measureable lengths, were known, but no one knew how to measure, quantitatively, the size of pleasures and pains. When Plato puts into Socrates' mouth talk of the necessity of being able to do this, it is possible that Plato was thinking that sometime in the future the way to make such measurements would be found, as the way to find the size of a distant object had been found recently.

However, there is a problem about which I suggest Plato was aware. I have assumed throughout my analysis that the assessment of the size is accurate for objects and, if so, a similar process might be possible for pleasures and pains. With regard to finding the height of a distant object it is necessary to find the height of a near object and some lengths along the ground[11]. When we measure a length in modern times we expect to obtain the same value as other people who use a similar ruler, although scientists who require precise values assess error limits. Before Plato's time, Solon had attempted to standardise measurements, but there was no certainty that a value given by one measuring rod would be the same as that given by another. It was not until the eighteenth century CE that a standard metre rod was established in Paris, by which all rulers could be standardised[12]. While a measuring tape

[11] See Appendix for details.

[12] In 1960 the standard was changed to a fixed number of wavelengths of the light emitted by Krypton-86 and the latest standard is the distance a ray of light travels in a vacuum in a fixed time.

bought in a local shop would not be checked against the standard metre, we expect that the accuracy is sufficient for agreement by people using different tapes and measuring the same length, for example the length of a shelf. I suggest that this is not the case in the ancient world.

At a time when the likelihood of making some errors would be the norm, Plato would be aware that the calculations needed to find the height of a distant object could produce different answers, if slightly different values were obtained for the various lengths used. The calculations given in the Appendix show that the height of the distant object is found by manipulating lengths which must be measured, with a measuring rod, or ruler. Although the results should be accurate enough to demonstrate the fact that the distant object is larger than it appears, there could be problems if two, or more, distant objects are being compared, using different rulers. Depending on the accuracy of the rulers, and the difference in size of the objects, it is possible that the result could be wrong and the smaller object declared to be the larger. This has a significance for the corresponding measurement of pleasures and pains, as I show below.

When Socrates uses the analogy he suggests that the size of pleasures and pains can be found in a similar way to the height of a distant object. The problem of inaccurate measurements means that there could be differences in the value obtained for this height by different people. The implication is that several values could be obtained for the sizes of the pleasures and pains under investigation. This is particularly important in the situations which Socrates is exploring since it is the relative sizes about which he is concerned. Dissimilar results for two measurements could mean that the order of size of two pleasures, for example, would be reversed.

This has some implications for the problem of *akrasia*, or weakness of will. If I make a choice based on measurements which I have made of two pleasures, the result depends not only on my possible desire to choose wisely, but also on the accuracy of my measurements. This supports Socrates' belief, given in the *Protagoras*. He declares that it is 'absurd' to say that a man 'knowing the bad to be bad, nevertheless does that very thing, when he is able not to do it' (355b). An obvious flaw with this statement is that people do apparently choose wrongly, without being forced to do so. The use of the geometrical analogy allows a way out of this impasse. I consider below the situation with people who do not have the expertise they need in order to find the sizes of the pleasures and pains they are about to choose. Here, I assess the problem faced by people who have the correct knowledge but make inaccurate measurements when they attempt to find the relative magnitudes of, for example, several

pleasures. This allows two people facing the same choices, and with the same determination to 'do the right thing', to end up choosing differently, not necessarily for the best. This is an aspect of Plato's views on *akrasia* which could benefit from more attention but, unfortunately, there is insufficient space to investigate it further here.

Philosophers and non-philosophers

The understanding of geometry necessary to fully appreciate the analogy separates philosophers from non-philosophers, as I mentioned in Chapter One, and it is philosophers who have the greatest need to make the correct choice of pleasures and pains. There are several passages in the dialogues which demonstrate this. One example comes in the *Republic* (485d), where Socrates declares that someone who desires knowledge is concerned with the pleasures of the soul and would abandon bodily pleasures, if he is a true philosopher. Such statements can be connected to the distant object analogy since, in order to choose the correct pleasures, a philosopher would have to assess the relative values of the pleasures of the body and those of the soul. The purpose of the analogy is to demonstrate that magnitudes can be calculated for inaccessible objects. However, only people who are trained in geometry would be able to do this for objects, with the implication that only they can find the correct size of pleasures and pains. I examine how this divides philosophers from non-philosophers.

Everyone is concerned with making the correct choice between several pleasures and pains and the discussions associated with the analogy have some value for non-philosophers as well as philosophers. In addition, everyone is aware that distant objects appear diminished, even though the calculations need geometrical expertise. This means that we can all appreciate that distant pleasures and pains appear deceptively small. It is the notion of relative sizes which implies that the sizes of different pleasures and pains must be known accurately. With this analogy, therefore, Plato is able to separate the non-philosophers and the philosophers. The former can have some awareness of the problems with future pleasures and pains, and the philosophers can understand the full implications of the choices which must be made. The use of geometry hints that only philosophers have the expertise which allows a correct choice of pleasures and pains.

If the analogy is not present, a non-philosopher could think that the measurement of pleasures and pains is possible by anyone and that everyone could then make the correct choice. It may be that the use of measurement, proportion and similar triangles would not enable the relative sizes of pleasures and pains to be found, but the point to keep in mind is

that Plato, through Socrates, is making the connection. The inference is that those who could not find the height of a distant object would not be able to find the magnitudes of pleasures and pains. Without the analogy, and its dependence on geometry, everyone, including non-philosophers, can choose pleasures and pains correctly and achieve the best life. With the analogy, philosophers know that the geometry which they have learnt offers an insight into the estimation of pleasures and pains. The necessity to use geometry to assess the distant object, with the implication that a similar knowledge is necessary for finding the sizes of pleasures and pains, means that Plato can exclude non-philosophers from a full understanding of how to correctly choose pleasures and pains. I return to this point when I study the second geometrical reference, the word *diagramma*.

There are several references in the dialogues to a mixture of pleasure and knowledge, relating this to the best life. One instance is in the *Philebus*, when Socrates admits that: 'We declared the life that combines pleasure and knowledge the winner' (27d) before spending much of the dialogue investigating whether this is the case. The geometry of the analogy demonstrates a slightly different way to look at this. A person without any knowledge cannot achieve the best pleasures, except by chance, since the knowledge to estimate their correct sizes will be absent. This implies that a life of pleasure alone cannot be a good life since there is no guarantee that the correct pleasures will be chosen. Since only philosophers have both the knowledge and desire for the correct choice of pleasures this means that only philosophers can have the best life. This is supported by various statements by Socrates, such as the analysis in the *Republic* when the conclusion is that the philosopher has the 'most pleasant life' (583a).

This has an implication beyond the people of fourth century Athens. I claimed earlier that the repetitions of the analogy are aimed at the readers of the dialogues, rather than Socrates' companions within each dialogue. My assessment of the understanding of non-philosophers and philosophers shows that not all readers can gain equally from the analogy. Only those who understand geometry and are philosophically minded will benefit fully from the reference to the analogy. However, these are presumably the people that Plato aimed to reach with his work so that there is no reduction in the value of the analogy.

Summary of Chapter Three

The aim of this chapter was to continue my examination of the distant object analogy. A passage in the *Philebus* contains the same analogy as the *Protagoras* and the *Republic*, associated with a further examination of pleasures and pain. The *Philebus* considers the pleasures necessary for a good life and I gave a summary of the discussion on pleasures. When I examined how the analogy is used in the *Philebus* I showed that some problems which had not been considered earlier were discussed in this dialogue. The possibility of false pleasures is examined, particularly those in the future. A solution is provided to the uncertainty of these, since good people will not anticipate falsely. The quality of pleasures is assessed and a hierarchy of pleasures is given. There is also some reference to the difficulty of knowing how to find a numerical value, which could be applied to the measurement of pleasures and pains. The accounts in the three dialogues examined together, therefore, give a more complete discussion of pleasures and pains than any one of them taken in isolation.

Each debate is associated with the distinctive analogy of the measurements needed to overcome the false perception of size of a distant object, providing some support for my suggestion that the analogy is used to bring to mind previous arguments. None of the participants in the three dialogues appear in more than one, which led me to propose that the reminder is aimed at the reader of Plato's texts. Next I assessed an alternative explanation regarding the use of the analogy, and some possible problems with my proposal that Plato intended to present the repetitions in order to enhance his argument. I suggested that taking the analogy as a means of joining together the discussions on relative pleasures and pains is feasible. This implies that the geometrical reference has an importance for enhancing our understanding of Plato's philosophy.

I ended by summarising my findings in order to give details of how taking note of the geometry can clarify several aspects of the discussion. I explained how the problem of accuracy has repercussions for the notion of *akrasia* since the wrong choice could be made inadvertently. In addition to reminding a reader of the previous discussions, the use of the analogy suggests that only philosophers can hope to assess correctly the relative sizes of pleasures and pains.

I return to my original question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how? I have shown that my work offers answers to both parts of the question since I have given several examples demonstrating how Plato's philosophy is enhanced by the use of the distant object analogy

and the associated geometry. However, little can be surmised from only one example of geometry in the dialogues, and it could be thought that only practical geometry has any worth. To explore further, and obtain more corroboration for a positive response to my question, I consider an example chosen from theoretical geometry as I examine Plato's use of the word *diagramma*.

Chapter Four

Diagramma and the acquisition of knowledge

4.1 Plato's theory of knowledge 86

The Forms 86

A hierarchy of knowledge 88

The significance of *diagramma* in the search for knowledge 91

4.2 The *Lesser Hippias* and lies 93

The background to the dialogue 94

The use of *diagramma* in the *Lesser Hippias* 96

The problem of lies 98

4.3 The *Cratylus* and errors 100

The background to the dialogue 100

The use of *diagramma* in the *Cratylus* 103

Diagramma and errors 105

4.4 The *Theaetetus* and false theories 107

The background to the dialogue 107

The use of *diagramma* in the *Theaetetus* 111

Philosophical knowledge and false theories 112

Summary of Chapter Four 114

In this chapter I explore Plato's use of the word *diagramma*, adopting a similar approach to that used with the distant object analogy, in order to give further support to my earlier work. *Diagramma* is employed in seven dialogues. It appears at *Lesser Hippias* 367d and e, *Cratylus* 436b, *Theaetetus* 169a, *Euthydemus* 290c, *Phaedo* 73b, *Republic* 529e, and *Epinomis* 991e. Apart from the possibly spurious *Epinomis*, all the references are in passages connected with discussions where the acquisition of knowledge is investigated. The rarity of the word means that it stands out from the surrounding prose and it can be used to follow the debate about knowledge across the dialogues, resulting in a more complete analysis than is found in any one text. The word is often translated as 'diagram' but, as I explained in Chapter One, it has an added meaning involving the use of a diagram to demonstrate constructions or prove

theorems. I claim that its specialist nature implies that the knowledge under discussion is expert knowledge, such as philosophers would require. Thus, the use of geometry again separates philosophers from non-philosophers. I use the transliterated Greek, *diagramma,* whenever I refer to it, ignoring the changes of ending which the Greek language applies.

I argue that Plato maintains that some knowledge can only be acquired by philosophers, and I relate this to the use of *diagramma* by considering, in this chapter, three of the texts where the word is found. They are all associated with warnings about how knowledge cannot be learnt from other people. In the *Lesser Hippias* the possibility of lies told by experts is considered and in the *Cratylus* there is mention of errors which can be made by others. The *Theaetetus* contains a discussion about false theories, which involves a critique of the views of other philosophers. The other references to *diagramma*, which explain how knowledge can be reliably acquired by a philosopher, are explored in the next chapter, where I also analyse the use of *diagramma* in the *Epinomis*.

There is uncertainty about the order in which the dialogues under discussion were written but there is general agreement that the *Lesser Hippias* is an early dialogue and the *Theaetetus* is a late one. There is more uncertainty about the location of the *Cratylus* within the chronology of the dialogues but the consensus appears to be to place it somewhere between the other dialogues being considered here. I examine them, therefore, in the order: *Lesser Hippias, Cratylus* and *Theaetetus*, although it should be noted that the correct chronology is not crucial to my argument. As in previous chapters, some description of the background to each dialogue is required for the analysis that follows. This includes information about the setting of the dialogues, possible composition dates and an account of the characters involved in the discussions. After giving a summary of the main arguments found in each dialogue, I assess how *diagramma* is used in the texts.

Throughout the dialogues Plato refers to several different types of knowledge. In order to associate *diagramma* with knowledge which only philosophers can acquire, it is necessary to show that Plato envisaged different levels of knowledge, available to different people. In the space available here it would be extremely difficult to give a full account of Plato's theory of knowledge but in the first section of this chapter I give a brief survey, based on passages from several dialogues.

4.1 Plato's theory of knowledge

Any attempt to describe Plato's views about knowledge is hindered by the fact that, although the word 'knowledge' appears many times in the dialogues, an examination of the associated texts does not reveal a definitive account of what knowledge is. One example can be taken from the *Theaetetus* where, in response to the question 'What do you think knowledge is?' (146c), there is an extensive discussion which ends with no agreement being reached. Some scholars concentrate on whether Plato distinguished 'knowledge that', 'knowledge how' and 'knowledge by acquaintance'; others base their analysis on the objects of knowledge. I take a different approach to Plato's theory of knowledge by considering the people who have access to each type of knowledge. In the hierarchy which emerges from a study of the dialogues, only philosophers can access the highest levels of knowledge although some knowledge can be gained by other people.

The Forms

Writers before Plato had considered which individuals could achieve the highest levels of knowledge, and several authors separated humans from the gods. One example is that of Xenophanes, who 'flourished' in the fifth century BCE. Bryan (2012, p.48) notes, in an examination of Xenophanes' fragment B35, that he had a 'commitment to the cognitive disparity between gods and men' and she suggests that this excludes the possibility of divine revelation (p.52). However, there is evidence that Plato did not agree with this. As Bryan later notes, Plato believed that we can have 'a share of the divine and thus, perhaps, a share in the epistemic potential of god' (p.187). This must provide the highest knowledge which man can envisage and, for Plato, this is knowledge of entities which he calls the 'Forms'. I suggest that, for him, this knowledge is restricted to philosophers.

There is some uncertainty about what Plato means by the entities he designates as 'Forms'. To describe them he uses both *ideai*, or Ideas, and *eidē*, normally translated as the Forms. Some writers use both terms but most choose to use only one, irrespective of the Greek word found in the particular text being discussed. 'Idea' appears most often in older works but many recent authors use 'Forms' in order to lose the association with thoughts of the mind since, as Armstrong (1959, p.37) explains, the Forms are entities which exist independently of minds. I will generally use 'Form' as I explain the main aspects of Plato's theory. Moline (1981, pp.79 ff) points out that the question 'What is a Form?' is never asked in the dialogues although Socrates frequently asks questions of the type 'What is X?', where X stands for a variety of things, such as truth or justice. In addition, the people in discussion with Socrates

always appear to understand what he means when he mentions the word Form. It should be noted that although Socrates assumes the existence of the 'Forms' in Plato's dialogues it is generally agreed, as Irwin (2000, p.145) records, that the Theory of the Forms is a Platonic theory which was not necessarily accepted by the historical Socrates. Moline (1981, p.105) notes that Plato never argues about the existence of the Forms, only about the characteristics which they have.

I do not intend to enter into the controversy concerning exactly what Plato intended the Forms to be, but some understanding of the concept is necessary before connecting them with the use of the word *diagramma.* The *Parmenides* gives something close to a definition when it is said that 'these forms are like patterns set in nature, and other things resemble them and are likenesses; and this partaking of the forms is, for the other things, simply being modelled on them' (132d). Armstrong (1959, pp.37-9) observes that in various references Plato indicates that anything has a Form, so there is a Form of Bed, a Form of Justice etc. The highest Form is the Form of the Good.

Other authors give descriptions of the Forms taken from various references found in Plato's work. One such author is Rowe (2003, p.105) and I paraphrase here the most important points which he gives. The Forms are the things which a philosopher aims to grasp when attempting to understand anything important, such as goodness, beauty, and justice. Each Form is what explains or even causes the particular things at the phenomenal level that share its name. However, the Forms exist independently of particular phenomenal things and also of minds, human or divine. They are eternal and unchanging and, therefore, they are unlike phenomenal objects which change, coming into and passing out of existence. Thus the Forms are a stable set of objects for knowledge.

Letter 7 contains several references which can be taken as referring to the Forms. When Plato lists what is necessary for knowledge of every real thing, the nature of a Form is implied: 'first, the name; second, the definition; third, the image; knowledge comes fourth, and in the fifth place we must put the object itself, the knowable and truly real being.' (342a-b). He gives an example to clarify what he means. When considering a circle, we first have the name and then a definition, which Plato gives as 'the figure whose extremities are everywhere equally distant from its center [sic]' (342b). Thirdly, there is a drawing of the circle and then, fourthly, there is knowledge, based on reason and right opinion which are found in our minds. These are distinct from 'the circle itself' (342c), in the fifth place. Circles found in life are not perfect representations of circles and names are not fixed, so a circle

could be known as a 'straight line', if we wished to call it such (343b). Kenny (2004, p.50) points out that it is the 'circle itself' which is the Form of Circle.

Plato then states how the Forms can be known. Sense perception cannot reveal this fifth element and neither can quickness of learning nor a good memory (344a). All the elements, names, definitions, visual and other perceptions, coupled with questions and answers in conversation with a teacher, must be added to reason and knowledge 'at the very extremity of human effort' to illuminate the nature of any object (344b). This is not something which is available to everyone. There are several points in the description of the Forms which suggest that only philosophers can achieve knowledge of them since only they can acquire the highest knowledge, of things themselves. In the *Republic* Plato is very specific about this. Socrates asks: 'As for those who in each case embrace the thing itself, we must call them philosophers, not lovers of opinion?' The reply given is an emphatic 'Most definitely' (480a). Philosophers are uniquely educated in order to achieve such knowledge, since expertise in 'questions and answers', or dialectic, is what philosophers are taught after they have mastered geometry (*Republic*, 536d). The question that this raises is what knowledge can be achieved by non-philosophers.

A hierarchy of knowledge

My concern is with whether non-philosophers can know the Forms. One author who believes that they can is Martinez (2011). He considers knowledge which a non-philosopher is said to have, and suggests that since 'all knowledge is knowledge of Forms ... we must accept that the user of products of crafts, since she has knowledge, must have cognitive access to the Forms' (p.330). Since knowledge of the Forms is a basis for ruling a city, Martinez divides knowledge into two separate spheres, that of moral knowledge and all other knowledge (p.332). Only philosophers can attain the moral knowledge, or knowledge of the Form of the Good, needed to rule others. I question the necessity to insist that non-philosophers have knowledge of some of the Forms. In his arguments, Martinez examines practical knowledge but does not mention mathematical knowledge. I suggest that taking the knowledge of geometry into consideration allows a return to the usual division between non-philosophers and philosophers, namely that only philosophers can access the Forms. This is because, for Plato, the knowledge of geometers comes between the knowledge of the majority of people, including craftsmen, and that of philosophers, but geometers do not know the Forms. This implies that the craftsmen discussed by Martinez cannot have access to knowledge of the Forms. I justify this claim by a brief survey of some passages where Plato explains what is, in effect, a hierarchy of knowledge.

We all need to know about things in the world in order to lead our normal lives, for example how to dress ourselves, but Plato is not concerned with such mundane knowledge. The first people he singles out as needing knowledge are craftsmen. He often mentions knowledge of a skill, or technical knowledge, such as in the *Republic* where knowledge of building houses is said to differ from other kinds of knowledge (438c). He notes, at *Euthydemus* 289a, that there is no value in knowing how to make things, for example medicines, unless it is also known how to use what is made. However, in the *Cratylus*, he imagines a carpenter making a shuttle, but it is the weaver who uses it who will know whether the shuttle is a good one, or not (390a-b). The knowledge of the craftsmen, therefore, can be knowledge of how to make something or an understanding of how the object can best be used. The other types of knowledge which Plato discusses are mathematical, specifically geometrical, and knowledge of the Forms.

The knowledge of geometers is different from that of craftsmen because the latter use their knowledge to make or use manufactured items, whereas geometers do not make their diagrams, 'they simply discover those which already exist' (*Euthydemus*, 290c). In the *Philebus* Plato contrasts the art of calculating and measuring, used by craftsmen, with the geometry and calculations used by philosophers (56e). It is suggested that those 'that are animated by the spirit of the true philosophers are infinitely superior' (57d). Geometrical knowledge is superior to other knowledge which has been discussed because the objects of geometrical knowledge are not the shapes that we see in the world but they are the pure shapes of theoretical geometry. Thus, there are examples of circular objects which can be seen in the world but these can never be perfect circles. The geometer is only concerned with perfect mental circles which can be investigated using theorems. This hints at knowledge of the Forms but as Aristotle points out, there is an alternative point of view. He notes that mathematical objects are eternal and immutable, like the Forms, but they differ from the Forms 'in that there are many similar objects of mathematics, whereas each Form is itself unique' (*Metaphysics*, 987b 14-18). This can be demonstrated by the fact that there are many perfect circles but only one Form of Circle. This suggests that the knowledge of geometers is not knowledge of the Forms.

It is in the *Republic* that Plato is more specific about a hierarchy of knowledge. He first considers opinion, which he places between ignorance and knowledge (478c-d) and he claims that the 'majority of people' study, for example, beautiful things but not the beautiful itself (479d). Plato says that they have opinion but no knowledge (479e). This is contrasted with the knowledge that belongs to philosophers who are lovers of wisdom and 'embrace the

thing itself' (*Republic,* 480a). Plato also suggests that the knowledge of philosophers is associated with understanding and those without understanding are like 'blind people who happen to travel the right road' (506c). In this hierarchy of knowledge, geometrical knowledge falls between the two extremes. Plato suggests that the mental state of geometers is 'thought but not understanding, thought being intermediate between opinion and understanding' (511d). Thus the understanding acquired by philosophers gives the highest knowledge, with 'thought but not understanding', as exhibited by geometers, coming between the knowledge of philosophers and the opinion of ordinary people.

I will refer to the highest knowledge possible as philosophical knowledge. By this I do not mean philosophical knowledge as the phrase is used in modern times, often connected with scepticism when, as Gascoigne (2002, p.8) explains, 'the possibility of philosophical knowledge itself is called into question'. Gascoigne continues (p.9) to speak of philosophical knowledge (*pk*) in relation to *pk*-scepticism and he starts his investigation into several different types of knowledge with an assessment of perceptual knowledge. My use of the phrase 'philosophical knowledge' is to denote knowledge which Plato thinks only philosophers can attain. Although Plato talks of perceptual knowledge, amongst other types of knowledge, my designation of philosophical knowledge is tied in to his doctrine of the Forms.

In order to be a philosopher, Plato claims that intense training in geometry, amongst other subjects, is needed. This means that all philosophers can be classed as geometers. However, as I showed earlier, not all who train to be geometers will automatically progress to be philosophers. This means that some people who have mastered geometry are left as non-philosophers. They can have knowledge of, for example, perfect circles but not of the Form of Circle. If only philosophers can know the Forms a question remains about the nature of the knowledge of craftsmen, highlighted by Martinez (2011, p.330). Sedley (2003, pp.165 ff) suggests that there could be two levels of Forms, so that there is a 'difference in levels of understanding' (p.167) between philosophers and non-philosophers. In this case, it would still be philosophical knowledge which Plato is exploring when he uses *diagramma* but the phrase 'philosophical knowledge' would relate to value Forms 'construed as transcendent entities' (p.167) rather than knowledge of all the Forms. When Plato gives advice about how to attain knowledge he does not always specify whether he is aiming his comments at philosophers, or non-philosophers, or both. Some of the statements, such as warnings about liars, could be useful to everyone. However, I claim that the use of the word *diagramma*

allows Plato to indicate to philosophers that the topics being discussed apply to philosophical knowledge as well as, or instead of, the knowledge which ordinary people require.

The significance of *diagramma* in the search for knowledge

A single word in a text might not have any great significance in most cases but *diagramma* is so rarely found in extant ancient Greek texts that I suggest it calls for some investigation. The word is used eight times in Plato's works and one of these occurs in the *Epinomis* which, as Cooper (1997, contents, p.vi) reports, is generally agreed by scholars not to have been written by Plato. A brief examination of this text will be left until after a full analysis of the use of the word in dialogues which are accepted as non-spurious.

I claimed earlier that *diagramma* means something more than 'diagram' or 'geometrical diagram'. Plato's use of it, in association with advice about acquiring knowledge, offers further support for this, since he is dismissive of diagrams used by geometers. One reference which illustrates this is in the *Republic*, where Socrates says:

> ... although they use visible figures and make claims about them, their thought isn't directed to them but to those other things that they are like. They make their claims for the sake of the square itself and the diagonal itself, not the diagonal they draw, and similarly with the others. These figures that they make and draw, ... they now in turn use as images, in seeking to see those others themselves that one cannot see except by means of thought' 510d-e

Graphousin is the word translated as 'figures' and these cannot be compared with the 'things themselves'. However, a different situation exists in the passages where the term *diagramma* is used, since it incorporates a connection with theorems or constructions. Unlike the 'figures that they make and draw' described above, a geometrical theorem is not a human invention. Although it must be discovered it states an eternal, unchanging fact, one example being the theorem accredited to Pythagoras, which expresses the fixed relationship between the sides of any right-angled triangle. 'Eternal' and 'unchanging' are words which can be used to describe the Forms so that a reference to geometrical theorems suggests that the associated discussions are on how to obtain knowledge of the Forms.

This means that the sections being considered are not aimed at everyone and the knowledge under discussion is not that which ordinary people might be able to acquire. A modern example demonstrates how writers can target a specific type of person. If I start to read an article entitled, say, 'Computers in the twenty-first century' it is possible that I will find it to be an interesting account of how computers can enhance our lives, written in prose aimed at the non-specialist reader. Alternatively, I could find technical terms which I do not

understand and I would suspect that the article is aimed at professionals in the field of computing. They may gain much from the author's account whereas I would have to accept that the article was not written for my benefit. I argue that this is the situation with Plato's use of *diagramma*.

In support of the claim that *diagramma* is used when Plato is targeting people who would understand fully the geometrical significance of the term, I will consider the *Meno*. The word *diagramma* does not appear in this dialogue, although there are several pages devoted to descriptions of geometrical diagrams, for example at 82c ff. Socrates is attempting to demonstrate *anamnesis* to Meno and he uses a slave boy who knows no geometry to demonstrate that all knowledge is present in everyone and can be retrieved. Socrates attempts to help the boy to retrieve some geometrical knowledge concerning what we would call square roots, which in ancient Greece were designated as diagonals of a square. When talking about the geometrical diagrams Plato uses several different words, such as *schēma*, translated as shape (74d-e). Other words appear as Socrates mentions lines and spaces. Several writers who compare the *Meno* to the *Phaedo* in terms of references to *anamnesis* do not note the discrepancy with regard to the use of the word *diagramma*. I believe that the absence of a word which is apparently so apt for the text of the *Meno* demands some explanation and I offer here two reasons why Plato might have omitted to use the term *diagramma.* I show that the one which appears to be most likely supports my suggestion that *diagramma* is a specialized geometrical term, aimed at philosophers who are experts in geometry.

The most obvious explanation for the absence of an appropriate word in Plato's texts is that the word was new, either to the Greeks in general or to Plato. The *TLG* lists very few occurrences of *diagramma* in writings either before or during Plato's lifetime. However, consideration of the chronology of Plato's works indicates that the *Meno* was written after at least one other dialogue which contains the word *diagramma.* Several authors offer support for this view. Irwin (2008, p.79) places the *Meno* after the *Euthydemus* since it reflects some themes found there and in some other earlier dialogues. Kenny (2004, pp.40-41) surveys several theories of chronology and states that there appears to be general agreement that the *Meno* was written after the *Lesser Hippias* and possibly after the *Euthydemus.* The order of Plato's texts is a contentious issue and uncertainty about the chronology prevents a clear decision about Plato's knowledge of the word *diagramma* when he wrote the *Meno.* However, Griswold (2002), who is sceptical about our ability to produce an accurate chronology admits (p.133) that 'even the severest skeptic [sic] must agree that we still know a

little about chronology'. I suggest that it is more likely that Plato knew of the word than that he did not, even if the writing of the *Meno* is earlier than the *Euthydemus.* This would mean that he made a conscious decision not to use it in his account of *anamnesis* in the *Meno*.

If the word was known to Plato it might appear to be an obvious one to include in the *Meno*. It refers to geometrical diagrams being used to demonstrate geometrical facts and in the *Meno* Socrates is doing exactly that. However, I have suggested that *diagramma* is a word which only specialist geometers would fully understand and if this is the case it would not be understood by any non-specialist. The point Socrates is attempting to make in the relevant sections of the *Meno* is that a slave boy, with no knowledge of geometry, can 'remember' geometrical knowledge he had before birth. The argument would be undermined if the boy initially understood an advanced geometrical term used within the discussion. This gives a good reason for its omission in the *Meno* and offers some support to my suggestion that it is a specialized term which is aimed at those who have an advanced knowledge of geometry. This includes philosophers, as has been shown. Only philosophers can hope to achieve the ultimate knowledge of the Forms, which leads to my suggestion that this is the knowledge under discussion in the passages containing *diagramma.* In the rest of this chapter I examine three of the references to the word *diagramma*. They give warnings about different ways in which knowledge gained from other people may be false, and using *diagramma* to connect the discussions leads to a more complete account than is found in any one dialogue.

4.2 The *Lesser Hippias* and lies

Some authors think that the *Lesser Hippias* may not have been written by Plato. There are several reasons for this such as the one given by Kahn (1996, p.118), that the dialogue can be taken to represent Socrates 'defending false theories with fallacious arguments', an unlikely topic for Plato to choose. However, it is cited by Aristotle who says 'the proof in the *Hippias* that the same man is false and true is misleading' (*Metaphysics* 1025a6). This ensures that it is accepted as genuine in spite of its strange arguments. Aristotle does not state that it is written by Plato, but it is generally agreed that he would have stated the author if it had not been Plato. With regard to the unusual choice of topic, Kahn (1996, p.118) suggests that Plato may have been using artistic freedom to make a philosophical point. I show that the use of *diagramma* in the *Lesser Hippias* is connected with advice which warns those who wish to acquire knowledge to be wary of liars. I give some background to the dialogue before I analyse the context in which *diagramma* appears. I then assess Plato's use of *diagramma* in a

discussion about lies. The dialogue is short, from lines 363a to 376c, with the word *diagramma* occurring at 367d.

The background to the dialogue

Socrates is in conversation with Hippias, a sophist, who declares that he teaches others for a fee (364d), but he is also a mathematician and astronomer (367d) in addition to being a specialist in many arts and crafts (368b-e). References to him as Hippias of Elis appear in antiquity with regard to his geometry. Heath (1921, pp.225-6) refers to various ancient texts to obtain support for a belief that Hippias' quadratrix[13] was used in attempts at both the trisection of an angle and the squaring of a circle. Both of these geometrical problems were at the forefront of the minds of geometers in Plato's time, as explained in Chapter One. There is also a third character, Eudicus, son of Apemantus, who initiates Socrates' entry into the discussion but then takes little part in it. Dusanic (2008, p.43) says that a man called Eudicus is known to have existed but some modern scholars believe Plato's character of this name is fictitious. One line in the text hints at an audience for the discussion between Socrates and Hippias since there is reference to 'these people here' (369c) who will decide who is giving the best argument.

The dramatic setting of the dialogue must have been in a time of peace, because Hippias has come to Athens from his home in Elis, which was a member of the Spartan league and therefore an enemy of Athens. The peace of Nicias was from 421. Lampert (2002, pp.233-4) combines this date with the references to the Olympic Games (for example at 364a) which, he suggests, are consistent with 'intense Olympic fever throughout Greece'. He proposes a dramatic date of Spring 420 so that the dialogue takes place a few months before the Olympic Games, in August and September of 420. However, Dusanic (2008, p.47) mentions a diplomatic conference in Athens in 385 and says that it is likely that Hippias would have been sent as a representative. Hoerber (1962, p.122), who is often cited by later authors, concludes that the evidence is so slight that it is better to leave the date undetermined. He notes that Hippias is also in the *Protagoras,* but that date is not known with certainty, and there is no evidence that the two conversations occurred on the same visit of Hippias to Athens.

There is less uncertainty about where the *Lesser Hippias* should be placed within the chronology of Plato's dialogues. Surveys based on style led to nineteenth century analysts

[13] The quadratrix was the first curve to be plotted point by point rather than with a compass and straight edge.

placing it in the early group of dialogues. Modern scholars agree with this result, for example Waterfield (1987b, p.270) suggests it was written in 395-390, saying that the dialogue appears to be somewhat naive. Analysis based on content also supports the early positioning of the dialogue since there is a lack of references to the Forms, and these are often absent in the early dialogues. However, although the Forms are not mentioned in the *Lesser Hippias,* I suggest that the references to knowledge which I examine could be precursors to more serious thoughts about the content of philosophical knowledge. This would give the *Lesser Hippias* a greater importance for Plato's theory of knowledge than it has hitherto enjoyed.

The dialogue starts with Socrates asking Hippias to give his views on the characters of Homer's Achilles and Odysseus. An observation is made that the truthful man and the liar could be different kinds of people (365c) and there is then an analysis of what is needed to be a liar. There is a suggestion that in order to lie, a liar must be powerful, intelligent, knowledgeable and wise (366a). Socrates uses Hippias, and his talent for calculating, as an example. Since he is so good at calculating he could also give a false answer to a problem with most consistency (366e), since if an ignorant person wished to lie about arithmetic he could, by accident, tell the truth (367a). Socrates concludes that the same person can both lie and be truthful in the field of arithmetic and calculations (367c-d). He wants to check whether the same is true in other areas. He first considers the topic of geometry and it is here that he refers to *diagramma*, which I examine after summarizing the rest of the arguments.

Socrates then discusses many other topics in which Hippias boasts that he is an expert, for example astronomy and the craft of engraving rings. In all of these the person who is most knowledgeable could be the best liar, although he could also be truthful if he so wished (368a-369b). The result is then applied to Achilles and Odysseus, but Hippias is unhappy with this and suggests that when Achilles lied he did so involuntarily whereas the lies of Odysseus were 'voluntary and on purpose' (370e). Socrates attempts to show that a voluntary liar will be a better liar than an involuntary one, just as the wise person will be a better person than an ignorant one (371e). He admits that he is confused by the argument (372d) and attempts to make sense of it all by considering a runner. A good runner runs quickly and a bad runner runs slowly but if both run slowly he suggests that the one who runs slowly voluntarily is the better runner (373d). Thus 'the one who accomplishes bad things involuntarily is more worthless than one who does them voluntarily' (373e). Socrates applies this conclusion to other activities, such as wrestling and singing (374a-c). He also suggests that it is preferable to limp voluntarily than to limp involuntarily although having a limp means 'having worthless

and awkward feet' (374d). After considering using the senses and such techniques as using a bow (374d-375c), he finally mentions justice and concludes that the better soul, in a good man, will be the one who does injustice voluntarily whereas it is the bad man who will do injustice involuntarily (376a-b). This shocking result is mediated somewhat by Socrates declaring it is only to be understood in regard to the good man who voluntarily does what is shameful and unjust 'if there is such a person' (376b).

The use of *diagramma* in the *Lesser Hippias*

A reference to *diagramma* occurs twice within a few lines, but I will take this as one use of the word. Socrates says:

> Isn't it the same way in geometry? Doesn't the same person have the most power to lie and to tell the truth about geometrical diagrams [*diagrammatōn*], namely, the geometer? [Hippias agrees]. Is anyone else good at these things, or the geometer? [Hippias replies 'No one else']. The good and wise geometer, then, is the most powerful in both respects, isn't he? And if anyone could be a liar about diagrams [*diagrammata*], it would be this person, the good geometer? For he has the power to lie, but the bad one is powerless; and one who does not have the power to lie cannot become a liar, as you agreed. 367d-e

Not all translators use the same terms for *diagrammatōn* and *diagrammata*, for example Jowett (1924a, pp.444-5) uses the word 'diagrams' both times, omitting the first 'geometrical'. Fowler (1970b, p.445) uses 'geometry' in the first instance and 'diagrams' in the second. This demonstrates the problem which translators face with a word used as rarely as *diagramma*.

Socrates wishes to use a geometrical example to show that a good geometer is better able to lie than a poor one, in the same way that a good arithmetician can best lie about calculations. The use of *diagramma* to illustrate his point is not a good choice because it is difficult to think of a lie used in a diagram which would withstand observation. If I say that all the angles in an isosceles triangle are equal I am lying, but a non-mathematician may accept my statement as true. However, if I use a diagram to produce a proof or some construction to illustrate this contention, the lie would become obvious immediately. I suggest that Plato could easily have found a better example to demonstrate that a good geometer is able to lie about geometry. For example, he could have used the same example that I have just given, limited to a false statement about triangles without reference to *diagramma*. Alternatively, he could have used a word which means a 'diagram', without the association of a theorem, since non-geometers can be misled, for instance by being given an equilateral triangle when such a specialized triangle is not intended. It is only when such a diagram is *used*, for a construction

or a proof of a theorem, that the mistake becomes noticeable. My proposal is that one reason why Plato refers to *diagramma* is so that he can target philosophers since *diagramma* is a specialized term.

There is support for the view that Plato is concerned specifically with philosophers when we look at other terms which he incorporates into his argument in the *Lesser Hippias*. Hoerber (1962, p.126n2) points out that Socrates uses *epistēme* when he speaks of arithmetic, geometry and astronomy and *techne* for the crafts which Hippias has. I find some significance in this distinction because the theoretical subjects are those which Plato elsewhere has on his curriculum for philosophers (*Republic,* 525b ff). These subjects are individually emphasised, whereas the practical subjects are considered with little detail. This could mean that Plato is thinking mainly of the knowledge which philosophers seek rather than the general knowledge needed daily by non-philosophers. There is further support for this view at the end of the *Lesser Hippias*. Socrates states that the 'more powerful and wiser soul was seen to be better' (375e), although he proceeds with the questionable conclusion that this soul will do injustice voluntarily. Plato often expresses a belief that the philosopher has the wisest soul. One example comes in the *Phaedrus*, where it is said that:

> ... only a philosopher's mind grows wings, since its memory always keeps it as close as possible to those realities by being close to which the gods are divine. A man who uses reminders of these things correctly is always at the highest, most perfect level of initiation, and he is the only one who is perfect as perfect can be. He stands outside human concerns and draws close to the divine. 249c

The type of knowledge being investigated in the *Lesser Hippias* is not specified, but the reference to the wiser soul implies that Plato has in mind the knowledge which philosophers wish to acquire. Plato is warning philosophers who are seeking knowledge to be wary about lies, and the best liars are those who are experts.

If the reference to *diagramma*, with its rather obtuse meaning, is aimed at philosophers and the knowledge under discussion is philosophical knowledge there are several inferences which can be made. The Forms are not specifically mentioned in the *Lesser Hippias* but I claim that the presence of *diagramma* suggests that Plato has some higher knowledge in mind, which will eventually be the basis for his theory of the Forms. Philosophers who wish to acquire such philosophical knowledge are warned, in this dialogue, to guard against lies being told by alleged experts, which can include professional teachers, such as the sophists. However, it is not the case that the Socrates of the dialogues is addressing philosophers who are present. Hippias, as a mathematician, would understand the full meaning of the term *diagramma* but he is a well-known sophist. Plato's views on sophists, in several dialogues,

are such that he cannot be treating Hippias as a philosopher, which leaves the readers of the dialogue as targets for Plato's advice. This is the same claim which I made with regard to my work on the distant object analogy and I return to the point later, to give a more complete analysis.

The problem of lies

One problem with Plato's account of lies in the *Lesser Hippias* is that his use of the term liar can be criticized. Aristotle explains why this is the case in the *Metaphysics*: 'for when he [Plato] says that the man who limps willingly is better that he who does so unwillingly, he means by limping *pretending* to limp' (1025a10). Rather than calling a man who is able to lie a liar, Plato should have specified a man who chooses to tell lies. However, choosing to tell a deliberate lie implies, in addition to an intention, a knowledge of the truth, otherwise there can be no certainty that a lie has been told. This explains Plato's insistence that an expert is better able to lie than anyone else, although he ends by wondering whether such a person exists.

Much of the *Lesser Hippias* is concerned with the notion that experts make the best liars but we do not usually expect an expert to tell us lies. Aristotle provides a reason for this to happen when he considers liars in the *Nicomachean Ethics*:

> It is not the capacity that makes a boaster, but the moral choice. His characteristic and the kind of person he is mark him as boastful. Similarly, one man is a liar because he enjoys lying as such, and another because he desires reputation or profit. 1127b14

Unfortunately, several examples can be given which show that experts, perhaps tempted by fame and fortune as Aristotle suggests, have lied. I give one case to demonstrate this.

Heinrich Schliemann is famous for excavating the site of Troy, in the nineteenth century CE. He reported fantastic finds, including gold jewellery, which he photographed his wife wearing. This gave him a fame which persisted for over a century. While he is still admired for the excavations which he undertook, questions have been raised about the finds which he claimed to have made. Traill (1995) is one of several authors who detail various deceits that were practised by Schliemann in his autobiographical writings, including references to his excavations and the so-called treasure of Priam. Traill concludes that 'we need to be sceptical at all times, but especially when it comes to the most dramatic finds' (p.304). He gives many details, including the appearance in photographs, taken before the discoveries, of some of the gold later claimed to be found at Troy. The dating of many of the finds was under suspicion almost from the first reports of them. Allen (1999, p.3) begins her

investigation into Troy with: 'the story of the discovery of Troy is but partly told by those who have penetrated the deceptions with which Schliemann surrounded himself in his accounts of the event'. There is no doubt that Schliemann achieved fame and possibly fortune but there is evidence that he gained these by lying about some of his achievements. The possibility of experts lying appears to be an unchanging characteristic of scholarly life and it is easy to imagine Plato being suspicious about the intentions of the sophists of his day. Any outlandish lie quoted by them could bring them fame, as they might appear to be more knowledgeable than others in Athens, and fortune, in that their exalted status would attract more students.

A problem I find with the account of expert liars is Plato's use of Hippias. Hippias of Ellis was renowned in the ancient world as a mathematician and we might expect Plato to admire him, with no hint that he might tell lies. However, Socrates hints that what he says, about experts being the best liars, applies to Hippias since he chooses examples which are subjects which he [Hippias] claims to know. Separating philosophical knowledge from other knowledge offers a possible explanation for Plato's use of Hippias. An expert in a specialized subject can teach within that field. If we consider a craftsman such as a carpenter then, with his specialized knowledge of carpentry he can teach an apprentice carpenter. Similarly, a geometer can explain to a student of geometry how to derive various geometrical facts. While Plato may accept Hippias as a teacher of geometry, perhaps the problem is that Hippias did not progress beyond geometrical knowledge to become a philosopher.

Plato could expect someone who teaches how to acquire knowledge in general and specifically the highest level of knowledge possible, that of the Forms, to have a superior education, such as that of a philosopher. This brings us back to the problem of the liar. In the *Lesser Hippias*, Plato is apparently saying that sophists may deliberately lie and because of their expertise they will be able to lie convincingly. If they do not have philosophical knowledge then it is not this in which they are experts. The implication in Plato's argument is that an expert in philosophical knowledge would be the best person to lie about this specialized knowledge but it does not appear feasible that Plato would accuse philosophers of deliberately lying about their knowledge. Hence the statement 'if there is such a person' (376b). However, it is also the case that a lie may not be deliberate, and I show that Plato considers this situation in the *Cratylus,* when he again refers to *diagramma*.

4.3 The *Cratylus* and errors

I examine, in this section, a further reference to *diagramma* which is also associated with a warning about the false presentation of knowledge, although here there is no assumption of deliberate falsification, implicit in the word 'lie'. The *Cratylus* is about names, their origin and nature and the dialogue extends from lines 383a to 440e. However, the discussion can be shown to have wider implications. When Plato uses *diagramma* in the *Cratylus,* at line 436d, he is again warning about the false path towards knowledge which can result from listening to other people, since they can be in error. Noting the repeated use of *diagramma* allows the discussion in the *Lesser Hippias* to be continued, and I argue that there is further support for the notion that the word *diagramma* is a specialized word aimed at philosophers. I again start with some background details.

The background to the dialogue

The characters of the *Cratylus*, in addition to Socrates, are Hermogenes, son of Hipponicus, and Cratylus. Hermogenes is mentioned several times in Xenophon (for example at *Memorabilia* 4.8.4 ff) as a close associate of Socrates and he is named as being present at Socrates' death (*Phaedo*, 59b). It is impossible to draw a conclusion about whether he was, or was not, a philosopher but it appears unlikely. If he had any connection with philosophy, Plato would surely have mentioned it. With regard to Cratylus, Aristotle notes that the young Plato, before he followed Socrates, 'became acquainted with Cratylus and the Heraclitean doctrines – that the whole sensible world is always in a state of flux' (*Metaphysics* 987a32). The intention of this passage is unclear, but Sedley (2003, p.2-3) suggests that Cratylus was the first major intellectual influence on Plato before Socrates. Cratylus believed that names are correct by nature and Sedley suggests that this may be why Plato changed his name from Aristocles to Plato (p.31) which, according to Diogenes Laertius (3.4), implies 'the breadth of his style'. However, Riginos (1976, p.38) notes that 'Plato' was a common name in fourth century Athens and his father could have given it to him. Several modern writers, such as Soltes (2007, p.156), repeat the story from antiquity of Cratylus eventually becoming so disillusioned with language that he abandoned it and resorted to using only gestures.

The dramatic date of the dialogue cannot be accurately determined but several scholars have attempted to reach a solution. Allan (1954, p.273) dates it to 399 because Socrates mentions the views of Euthyphro (for example, at 400a) and Allan assumes that these are taken from when Socrates met Euthyphro, as recorded in Plato's *Euthyphro.* This occurred just before Socrates' trial and so can be accurately dated to 399. This date is not accepted by other

writers, for example Sedley (2003, p.3, note 5) puts the dramatic date before Plato was influenced by Cratylus, which would place it at least ten years before Socrates' death. Ademollo (2011, p.14) claims that when Socrates tells Cratylus that he is 'still young' (440d) he implies that Cratylus is at least twenty years younger than himself. Cratylus would then have been born around 450, or later, indicating an earlier dramatic date for the dialogue. None of these assumptions can be verified and I conclude that the dramatic date must be left as uncertain.

The time when the *Cratylus* was composed is similarly uncertain. Barney (2001, p.3, note 4) notes that the common view is that it is a 'late-early' dialogue which was written before the *Phaedo*, which is generally accepted as coming before the *Republic* (see, for example, Rowe, 2003, p.103). Support for this comes from an assessment of the development of the arguments about the Forms. However, Barney thinks that there is a common theme in the *Cratylus, Theaetetus* and *Sophist* and since the two latter dialogues are accepted as coming after the *Republic,* this would indicate a post *Republic* date for the *Cratylus.* Baxter (1992, p.3) lists further opinions which exist, including the notion that the *Cratylus* was written after the *Phaedo* but before the *Republic.* Luce (1964, pp.136 ff) summarizes many of the views that have been held about where the *Cratylus* should be placed in a chronological list of Plato's dialogues. He admits that there is no firm evidence for a date of writing but finally (p.153) quotes a German article by Wilamowitz who suggests that the *Cratylus* was written early in the existence of the Academy as a warning to students that etymology does not reveal the 'ultimate constitution of things'. This would give a date shortly after 387. Sedley (2003, p.16) offers a solution to these conflicting views by suggesting that 'the *Cratylus* is a possibly unique hybrid, a product of more than one phase in Plato's thought'.

The *Cratylus* is concerned with names, but there has been some argument among commentators about whether Plato intended words in general, including verbs and adjectives as well as nouns, to be included in the discussions. The arguments can be fully explored if the names of objects alone are considered and this is the approach which I will take. Hermogenes starts by criticising Cratylus' view that everything has a correct name which 'belongs to it by nature' (383a). Hermogenes believes that names are given to objects by 'rules and usage' (384d). On his conventionalist theory, individuals or communities were responsible for originally naming things, which appears to be a reasonable assumption. Unfortunately, Hermogenes then takes an extreme position in which, since any name can be a correct name, any individual can use any name for the same object. Thus, what the public call a 'man', Socrates can call a 'horse' and still be correct (385a). Socrates wants to

investigate whether the same would apply to things, as well as names. He reminds Hermogenes of Protagoras who says that 'man is "the measure of all things," and that things are to me as they appear to me, and are to you as they appear to you' (385e). Hermogenes declares that he does not believe Protagoras (386a). Socrates examines the doctrine which, he says, would imply that it is impossible for some people to be wise and some foolish. Since this is obviously untrue he draws the conclusion that Protagoras is wrong and everything is not necessarily as it appears to me and it is, therefore, possible to speak falsely (387c). If we can speak falsely then there can be true and false names, since these are a part of speech.

After a discussion about craftsmen Socrates suggests that naming is a craft and so not everyone will be skilled enough to award a name to an object. A specialized namemaker is needed as a rule-setter (389a). Socrates calls such a person a *nomothetēs*, usually translated as law-giver, but the translator, Reeve (1997, p.107, note 6), notes that in this dialogue it is someone who establishes the rules of usage of names. Therefore, in his translation of the *Cratylus,* he uses the term rule-setter, although other translators use the more obvious name-giver.

There follows an etymological investigation into names. This long section of the dialogue has caused some dissent among commentators about its purpose. Keller (2000, p.297), among others, thinks that Plato wants to criticize an intellectual topic that was popular in his time. He believes that the sophists were particularly interested in etymologies and Socrates mentions these teachers, and the high prices they charge. Prodicus is said to run a course on the correctness of names for fifty drachma (384b) and other sophists teach the subject for 'a great deal of money' (391c). Socrates eventually believes that he has thoroughly analysed the correctness of names and he and Hermogenes call on Cratylus to enter the discussion. Socrates suggests that a name shows the nature of the thing named and so names can give instruction (428e). Cratylus agrees and affirms his belief in the role of the rule-setters but he disagrees that the art of naming is like other arts. A painting can be a good or bad representation of its subject but a name must always have been correctly assigned (430d). Socrates demonstrates that this is not always the case. Just as he could show a man a picture which is supposedly his portrait but may not be, so he can call him a name which may not be correct (430e).

Socrates tries to persuade Cratylus that convention must play a part in naming and finally asks him what he thinks the function of a name is, and what good it accomplishes. I suggest that Socrates expects Cratylus to realise that for conversation to be possible some convention about the names of objects is essential. Cratylus, however, replies that knowing a name gives

knowledge of the object (435d). He goes further and insists that 'the best and only way' to know something is to know its name (436a). Socrates exposes a flaw in Cratylus' theory by considering the position of the original name-givers. If, as Cratylus believes, the names awarded are the natural names objects should have, then the names given will correspond to the understanding of the nature of things which the name-givers had (436b). If this was incorrect then the names will be incorrect and so we cannot rely on knowledge obtained from names. Cratylus disagrees. He offers proof that the name-givers did not err based on the fact that names are 'entirely consistent with one another' (436c). Socrates, however, thinks that consistency can follow from an initial error, and it is here that he refers to *diagramma* to support his argument.

After the reference to *diagramma* Socrates returns to the problem of the original name-givers. If they had to know the things to which they gave names, in order to give the correct names, but knowledge of objects comes from knowledge of their names, there is an obvious problem: the name gives knowledge, but knowledge is needed in order to give the name. However, the paradox can be resolved. Although for us the best way to knowledge is through the names which already exist, the name-givers may have had access to prior knowledge. Cratylus lays a foundation for this when he suggests that the name-givers could have been divine (438c), with access to the true nature of things, which mortals do not have. Socrates disagrees and the discussion moves to a consideration of the problem of knowing things which change. The doctrine of Heraclitus is mentioned, but Socrates is unsure about its validity (440c). Cratylus supports Heraclitus (440d) and the dialogue ends with Socrates sending him on his way with no conclusions being reached.

The use of *diagramma* in the *Cratylus*

Socrates says:

> Geometrical constructions [*diagrammatōn*] often have a small unnoticed error at the beginning with which all the rest is perfectly consistent. That's why every man must think a lot about the first principles of any thing and investigate them thoroughly to see whether or not it's correct to assume them. For if they have been adequately examined, the subsequent steps will plainly follow from them. 436d

Other translators offer slightly different words for *diagrammatōn*: for example Jowett (1953, p.101) and Fowler (1970a, p.179) both give 'geometrical diagrams'.

Socrates is criticising Cratylus when he refers to *diagramma* but Barney (1997, pp.149-50) gives an analysis of Hermogenes' contribution to the argument which makes his views also applicable to the *diagramma* comments. Barney suggests that Hermogenes distinguishes

between two actions. One is the 'setting down' of a name, which is the basis for the convention that it is the name for a particular object, and the second is the practice of 'calling' that object by the given name. I interpret this as *using* the name, just as the term *diagramma* incorporates the notion of *using* a diagram, rather than just drawing it. Barney continues by suggesting that different criteria for correctness apply in the two instances he has quoted. In the *giving* of a name any name can be the correct name, but the *use* is only correct if the given name is used. He notes that most interpreters of the dialogue do not distinguish between these two cases. I show how this can also be applied to *diagramma.*

Any figure of lines and curves could be designated as correct and it is only when some proposition is made, and an attempt made to prove it, that certain correct conditions must be met. An example demonstrates this. If I wanted to draw a triangle I might draw one which was equilateral, with all sides of equal length. This diagram would be correct as a representation of a triangle. If I then wanted to obtain some propositions about right-angled triangles I could use my diagram to obtain many consistent results but not all of these would be correct for right-angled triangles. If someone looked at my work they could be misled if they did not notice the initial error, the construction of the wrong type of triangle for my proofs. I used a similar example to criticize the use of *diagramma* in the *Lesser Hippias* but there I was considering a demonstration that all the angles in an isosceles triangle are equal, which can easily be seen to be wrong. Here, I am assessing the use of *diagramma* by considering the example of a triangle being the basis of proofs about various geometrical facts. Thus, when a diagram is *used* it is necessary that it shows the correct geometrical figure for the particular conclusions which are being obtained.

The application of the term *diagramma* is relevant for several points in the discussion, including the best way to learn about things. Whatever the nature of the name-givers, Socrates concludes that 'it must be possible to learn about the things that are, independently of names' (438e). He goes further and suggests that a better way to learn about things is through the things themselves, rather than through their names (439b). I see *diagramma* as demonstrating this point. The name of a trapezium, for example, gives no information about its area[14]. A geometrical diagram can give an image of a trapezium to show what it looks like and if the diagram is used to produce proofs then knowledge of the properties of a trapezium can be obtained, giving a formula for its area.

[14] A trapezium area is half the sum of the parallel sides multiplied by the perpendicular distance between them.

Although, as I have shown, *diagramma* can be seen to be relevant for much of the discussion in the *Cratylus* it is a strange example to find within a dialogue that is concerned with the question of whether errors can be found in names. The use of *diagramma* in the lines in which it appears looks a little forced since an error in a diagram would often be immediately obvious, which is the point I made in my work on the *Lesser Hippias*. Initial errors in a mathematical proof, without diagrams, would be more difficult to detect. A recent example can be given to demonstrate this: the proof of Fermat's Last Theorem was published near the end of the last century, but several errors were found in what was originally taken to be a valid proof. These were unintentional and could be corrected to give the proof which is now accepted. Plato's observation about unrealised errors obviously applies here. It is interesting that Aristotle devotes much time, in *Prior Analytics II,* to describing ways in which conclusions can be drawn from premises and he warns that 'we can draw a true inference from false premises' (64b7). He notes that someone who does not notice the error might also fall into the same error in his own mind. Since this is the point made by Plato in the *Cratylus* it is tempting to imagine that Aristotle is writing about what he had been taught by Plato.

***Diagramma* and errors**

Before the reference to *diagramma* there is no description of different types of knowledge which may have an initial error, the only examples given being names. After the use of *diagramma* there are hints that Socrates' arguments can be extended to other knowledge, which some scholars, including Silverman (2001, p.32), suggest could be knowledge of the Forms. Socrates' view is that the best way to know something is through the thing itself (439b). This phrase brings to mind other references to the Forms and incorporates the notion that there can only be knowledge of things which are unchanging. Otherwise, as soon as something is known about an object it will have changed and the knowledge will be obsolete. Socrates mentions knowing 'the beautiful itself' (439d) and 'the good' (440b) and it is these references which have led some commentators to examine the *Cratylus* as a dialogue which explores Plato's notion of the Forms. The knowledge being acquired would then be knowledge of interest to philosophers, and this supports my view that *diagramma* is used as a specialized word, targeting philosophers.

Since the *Cratylus* is obviously concerned with names, several writers take the Form being examined to be the Form of Name. Wolf (1996, p.31) examines some implications of this when he asserts that unlike the views of both Hermogenes and Cratylus, Socrates' account of names makes dialectic possible. Wolf suggests that Socrates sees a world where there are regularities and names are imperfect attempts to explain the nature of things. The Form of

an entity presents an ideal name which the name-giver attempts to match but imperfections occur which are dependent on the skill of the name-giver. If the names were perfect representations of the objects to which they refer then there would be no necessity to search for knowledge as it would be there in the names. This echoes an earlier account by Weingartner (1970, p.11) who points out that Cratylus' view is disastrous for dialectic. A question such as 'What is justice?' contains the answer, if the word 'justice' reveals the nature of justice. However, on Wolf's analysis of Socrates' views, the names given must always be imperfect copies of the Form of the name.

I suggest that this could also be applied to *diagramma* since, for example, no drawing of an equilateral triangle can possibly be a true drawing of three sides of exactly the same length. The errors that are present mean that a diagram cannot reveal everything about such a triangle. The angles of an equilateral triangle are equal but this fact could not be accurately determined from the diagram. Constructions and reasoning are needed to produce a theorem which states that the angles are all equal. This demonstrates the importance of using *diagramma*, with its connection to theorems, rather than a word for a diagram such as *schēma*.

There is a connection with the use of *diagramma* in the *Lesser Hippias.* In that text we are told that experts are best able to lie, although Plato is obviously unsure whether any experts who deliberately lie actually exist. I suggested that it is philosophers who are not likely to lie, yet errors can be found in scholarly works, in ancient times as well as modern. I see the statement in the *Cratylus* as an extension to the discussion instigated in the *Lesser Hippias*, with the word *diagramma* providing the connection between the two texts. With the use of this distinctive and unusual word Plato can connect two warnings, to be wary of lies and also of false premises. Socrates' warning in the *Cratylus* contains no hint that the falsity may be deliberate; thus Plato is exploring the possibility of philosophers being misled by accidental errors, in addition to deliberate ones.

There is some argument about the place of the *Cratylus* in the chronology of Plato's dialogues and it is possible that it was written before the *Lesser Hippias.* If this is the case my argument about the reference to *diagramma* is still valid. The use of a poor example is explained in both if it is a specialized geometrical term and, independently of which of the two dialogues was written first, there is a problem for philosophers running through both. There are no known philosophers present in the discussion with Socrates, in the *Cratylus*, so it appears that Plato is again addressing readers of the dialogues. However, with so much uncertainty about the knowledge which can be acquired from others, a question arises about how philosophers

can be helped to gain knowledge. Obvious candidates for the role of purveyors of philosophical knowledge are other philosophers and in the next section I show that Plato considers these more fully in the *Theaetetus.*

4.4 The *Theaetetus* and false theories

Although people may lie or be in error there is the possibility, conversely, that some people may be trustworthy and accurate. Perhaps Plato could have given some advice about how to distinguish such people from the rest of the populace. A budding philosopher might then search out such men and adopt the knowledge which they have found and are willing to share. There is no doubt that Plato felt that philosophers should attempt to pass on whatever higher knowledge they have gained, as the simile of the cave demonstrates (*Republic* 514a ff). There, the people who escape from the cave and become enlightened force themselves to return to the cave in order to share their knowledge with the unfortunate people who are still imprisoned. However, there is the possibility that enlightened men with good intentions may fail to pass on correct theories about knowledge and I suggest that this is the situation which Plato examines in the *Theaetetus*.

The word *diagramma* is again associated with advice about the acquisition of knowledge but Plato connects it to an assessment of Protagoras' statement 'man is the measure', Heraclitus' theory of flux, and Theaetetus' definition of knowledge. I propose that this is to analyse the way that unintentional errors can be made by experts. Plato shows that previous philosophers may have made such errors, which means that they should not be used as sources of knowledge. Before considering the use of *diagramma* I will give a brief overview of the background to the *Theaetetus* and discuss the arguments presented in the dialogue.

The background to the dialogue

A reference in the dialogue (at 142b) reveals that it takes place in the year when Theaetetus died from wounds received in a battle at Corinth. This means that it is set in one of the two years during Plato's lifetime when Athens was involved in such a battle, either 394 or 369. Chappell (2004, p.30) says that 'it is now almost universally accepted' that 394 is too early. This is because Theaetetus would only have been in his early twenties in 394 and so could not have completed the work which gave him fame as a mathematician.

A slave reads out to Euclides and Terpsion, both of Megara, an account written by Euclides of a discussion led by Socrates that took place some years earlier, in 399. This date is known because it is mentioned that the meeting took place 'not long before his [Socrates'] death'

(142c). The setting for the reading is Megara which Chappell (2004, p.25) notes was famous, in the fourth century BCE, for studies in logic. Euclides admits that he was not present at the discussion, but he was told about it by Socrates, and, after writing his account, he checked it with Socrates for accuracy (143a). There could be a problem with this statement since Socrates had been condemned to death. However, we know that there was a slight delay in carrying out the execution so the account could have been checked during that time.

There are several characters present at Socrates' discussion. Theodorus is a distinguished mathematician from Cyrene, and Taylor (1963, p.322) notes that he appears in a list of Pythagoreans given by Iamblichus. Also present is Theaetetus, who is declared to be a gifted pupil of Theodorus (144a-b). In antiquity he was credited with completing work on the five regular solids usually ascribed to Plato. Both of these men would be able to understand the meaning of *diagramma*, but both are primarily mathematicians and, in Chapter One, I used the words of Theodorus to demonstrate that geometers were not necessarily interested in philosophy. There are some references to other people being present, for example at 144c, but only one is named, a young namesake of Socrates (147d).

The situation of Euclides calls for some comment. He founded the Megarian school of philosophy (Nails, 2002, p.145) so here we have a philosopher who is part of a dialogue within a dialogue. He wrote the account of Socrates' conversation but because a slave reads out the account Euclides becomes part of the audience. I suggested, when examining the *Lesser Hippias* and the *Cratylus*, that Plato was aiming his advice at philosophers who were the audience for his dialogues. In the *Theaetetus* we have a renowned philosopher taking on the role of a member of the audience. Once again, no one who is in discussion with Socrates in the main part of the dialogue is a philosopher but we now have two different philosophical audiences. One audience, which comprises Euclides, listens to the account, while the readers of the *Theaetetus* receive the entire dialogue, as it is written. It should be noted that within the dialogue Protagoras is conjured up from the dead so that, in one way, there is a philosopher present in the discussion with Socrates, but he could hardly be said to be an intended recipient of advice about how to acquire knowledge.

The format of the dialogue is similar to Plato's early dialogues. Socrates is the main speaker and he declares several times that he knows nothing, as when he declares that he is 'barren of wisdom' (150c). However, Waterfield (1987c, p.132) refers to stylistic evidence and the thoroughness of the arguments to support a general agreement amongst present day scholars that the *Theaetetus* is a late dialogue. He suggests a date around 360 and thinks that

it shows that Plato is having doubts about some of his earlier ideas. He explains the prominence of Socrates by suggesting that the dialogue is 'a kind of homage to Socrates'.

The dialogue covers lines 142a to 210d and *diagramma* occurs at 169a. Most of the dialogue is concerned with attempting to answer the question posed by Socrates 'What do you think knowledge is?' (146c). Theaetetus gives some examples of knowledge which Socrates will not accept as an answer to his question, and Socrates then applies what he calls his midwifery skills to bring forth other ideas. Theaetetus suggests that 'knowledge is simply perception' (151e), and the ensuing discussion contains the reference to *diagramma.* When Socrates eventually shows that this definition of knowledge cannot be correct (186e) Theaetetus offers two more suggestions: first that knowledge is true judgment (187b) and then later he gives a definition of knowledge as 'true judgment with an account' (201d). There are many books and articles which analyse the validity of the arguments relating to these three definitions but since my purpose here is to investigate the use of *diagramma* I will concentrate on the first, where the word occurs.

The argument can be summarized briefly. Socrates claims that if knowledge is perception then the views of both Protagoras and Heraclitus must be correct. He proceeds to show that both views are wrong and, therefore, the definition of knowledge which Theaetetus has given must also be wrong. It is interesting that the *Cratylus*, where *diagramma* is also used, has some criticism of both Protagoras (386c) and Heraclitus (440c). Scholars have different opinions about what Plato is trying to show in the *Theaetetus* with regard to these men but I suggest that he is applying his warnings about errors to the theories of earlier philosophers. A fuller account demonstrates this.

Socrates starts by apparently supporting Theaetetus, using Protagoras to do so. When Theaetetus offers the definition that knowledge is perception Socrates suggests that this is equivalent to Protagoras' statement that 'man is the measure of all things: of the things which are, that they are, and of the things which are not, that they are not' (152a). The problem with Theaetetus' definition of knowledge as perception is that what we know must be true, or it cannot count as knowledge. Sense perception is only always true if we accept Protagoras' theory that whatever seems to a particular person to be the case is true for that person, stated as 'man is the measure'. To demonstrate what Protagoras meant Socrates uses the example of a wind which to one person may appear cold and to another as not cold (152b). He also mentions that the same thing can appear small or large in different situations (152d). When Theaetetus agrees with him Socrates connects 'it appears' with 'he perceives

it' and concludes that things are, for an individual, as he perceives them, and so perception is knowledge.

Socrates then mentions false perceptions, such as those we have when we are dreaming (157e). Since there can be false perceptions knowledge cannot be perception, since knowledge cannot be false. However, Socrates again changes his views and appears to agree with Theaetetus' statement that knowledge is perception when he says 'my perception is true for me – because it is always a perception of that being which is peculiarly mine; and I am judge, as Protagoras said, of things that are, that they are, for me; and of things that are not, that they are not.' (160c). He considers that this shows that various views are equivalent to that of Protagoras, for example Heraclitus and 'all things flow like streams' (160d). This appears to be a reference to the well-known statement of Heraclitus given by Plutarch which Kahn (1979, p.53) gives as part of fragment L1: 'one cannot step twice into the same river'. This quotation was famous before Plato's time and Plato quotes it more fully in the *Cratylus,* when Socrates relates that Heraclitus wrote 'you cannot step into the same river twice' (402a). It is known that Cratylus took it further, saying 'it cannot be done even once' (reported by Aristotle, *Metaphysics,* 1010a14) since a person is constantly changing. The connection, in the *Theaetetus*, between Protagoras' 'man is the measure' and Heraclitus' view of a continuously changing world, is not made clear, and different writers focus on different aspects of the arguments. Waterfield (1987c, pp.150-1) claims that the cause of the obscurity is because Plato wants to establish a connection between Protagoras and Heraclitus, and he ties it to Theaetetus' suggestion that knowledge is perception. Chappell (2006, p.121) sees the relation between Heraclitus' doctrine of flux and Protagoras' man-measure relativism as being close because they are both about how to deal with contradictions. The important aspect for my thesis is that Plato is assessing the theories of both philosophers.

Socrates then again starts to find fault with the theory of Protagoras, which by implication will also throw doubt on Heraclitus' views, since Socrates has connected the views of both Protagoras and Heraclitus with the statement of Theaetetus. He concludes 'we have got to say that perception is one thing and knowledge another' (164b). He then feels that he may have been a little rash and he re-examines the situation, imagining that Protagoras is present to justify his opinion. He also brings Theodorus into the discussion. This is where he mentions *diagramma.* Throughout this section of the dialogue Socrates apparently switches several times between support for and criticism of Protagoras and Heraclitus. There is

copious literature examining the validity or otherwise of Plato's arguments. The main point for my work is that *diagramma* is again being used within a discussion about knowledge.

After a long discussion with Theodorus in which Theaetetus takes no part, Socrates declares they are set free from Theodorus' friend Protagoras, and 'we are not going to grant that knowledge is perception' (183c). He calls on Theaetetus to attempt another definition for knowledge and the dialogue continues with the other definitions being scrutinised until Socrates finally rejects them all saying 'knowledge is neither perception nor true judgment, nor an account added to true judgement' (210b).

The use of *diagramma* in the *Theaetetus*

Socrates says:

> Come with me at any rate until we see whether in questions of geometrical proofs [*diagrammatōn*] it is really you who should be the measure or whether all men are as sufficient to themselves as you are in astronomy and all the other sciences in which you have made your name. 169a

Other authors, such as Jowett (1924b p.226) and Fowler (1967, p.103), use 'diagrams' for their translations of *diagrammatōn.*

The use of *diagramma* in the context of this dialogue appears to be appropriate for Plato's criticism of the earlier philosophers. Protagoras is usually quoted as saying that 'man is the measure' without adding that the statement should be qualified. This can be done by specifying that what appears to one man is true for that man. This is not the same as claiming that it is an absolute truth for all men. There has been much discussion about this point in ancient and modern literature, and Plato sometimes includes the qualifier and sometimes omits it. For example, at 152b Socrates suggests that Protagoras means that the wind 'is cold for the one who feels cold, and for the other, not cold', carefully specifying that neither view is correct for everyone. However, when he imagines Protagoras defending his own theory he sometimes omits to qualify the statement. One such example is at 167b when he says that 'what one is immediately experiencing is always true', omitting to say 'always true for that person'. The use of *diagramma* can be applied to either version of Protagoras' theory. Proofs from geometrical diagrams give indisputable theorems which are not dependent on any point of view and so they demonstrate that there are universal truths, recognizable by any one man or all men. What is obtained from geometrical diagrams must be accepted, even if it overthrows a theory which is commonly accepted as true. An example from ancient Greece supports this statement. The Pythagoreans thought that lengths must

be commensurable but geometrical diagrams presented them with the problem of incommensurable sides and diagonals of squares. By Plato's time numbers which we now designate as square roots were well researched. Thus, the facts obtainable by using a geometrical diagram transcend an individual's limited point of view. Since the truths exhibited by geometrical reasoning in diagrams are unchangeable, the use of *diagramma* also reinforces Plato's criticism of Heraclitus' changing world.

Plato's criticism of Protagoras and Heraclitus has produced many different points of view among modern philosophers and Chappell (2004, pp.48-9) summarizes two of these. One interpretation is that Plato does not want to discredit completely the views of Protagoras and Heraclitus. He is seen as limiting their theories to one area, the perceptible or sensible world. For Plato, the sensible world is not everything since he also has the world of the Forms, although he does not discuss these in the *Theaetetus*. The discussion in the dialogue then shows that Protagoras and Heraclitus only see parts of a larger truth. The second interpretation has Plato refuting the theories of Protagoras and Heraclitus completely, saying that they are not true even in the sensible world. For my analysis of the use of *diagramma* it is irrelevant which of these interpretations is best since in both interpretations Protagoras and Heraclitus fail to take account of what Plato sees as the world of the Forms. My claim is that Plato's use of the word *diagramma* indicates that he is discussing knowledge in which only philosophers are interested, namely of the Forms. The empirical world, where people can see the world from different viewpoints and the entities which can be known are in a Heraclitean flux, is the world about which a non-philosopher can learn: but this is not what interests Plato in this context.

Waterfield (1987c, p.137) questions why Socrates is having the discussion with two mathematicians and suggests that either the discussion really took place and Plato is simply recording it or he chose such *dramatic personae* because mathematics is an exact science and knowledge involves certainty and truth. If the latter is the case then, from my point of view, Plato chose his characters well. It is in the *Theaetetus* that he demonstrates that a mathematician who understands geometry would not be interested in the philosophical knowledge revealed. This leaves only philosophers to gain from the discussions associated with the word.

Philosophical knowledge and false theories

Philosophers searching for knowledge are being told by Plato that the writings of renowned philosophers may offer false theories. Unfortunately, there is a problem with Plato's criticism

of other philosophers. If the writings of some philosophers cannot be relied upon because their theories may be false, then Plato can similarly be accused of promoting false theories, however unintentionally. One such false theory could be that which declares that other philosophers cannot be relied upon, which leads to a paradox. If we assume that Plato is correct, philosophers may be wrong but, as a philosopher, he may also be wrong, which could undermine his statement that others can be wrong. If they are correct, Plato's original statement is incorrect, but the initial assumption was that it was correct. In *Letter 7* Plato offers a suggestion which could possibly solve this paradox. He states that any written work cannot convey philosophical doctrines and he is critical of anyone who tries to write about any subject which he, Plato, has discussed because 'this knowledge is not something that can be put into words' (341c). As Morrow (1962, p.67) highlights, Plato did not write down full details of philosophical knowledge, of the Forms, saying that discussion is necessary in order to understand them. However, the letter continues, rather immodestly, to declare that 'if these matters are to be expounded at all in books or lectures, they would best come from me' (341d). If this is accepted it allows for others to be wrong while Plato is correct. He can thus state, believing there is no paradox, that other philosophers can be mistaken in their theories.

If the term *diagramma* singles out philosophical knowledge specifically, this claim has implications for the analysis of the *Theaetetus* with regard to the Forms. These are not specifically mentioned in the dialogue, but various scholars have attempted to show that they are mentioned indirectly. One example is at 175b when Socrates talks of the philosopher who 'in his turn draws someone to a higher level'. This brings to mind the philosophers who, in the simile of the cave, return to help to raise the ones left behind to greater knowledge (*Republic*, 520c). Cornford (1964, p.99) suggests that Plato's point in criticising Heraclitus is that if all things are changing then language can have no fixed meaning and Cratylus, who influenced Plato, did reach this conclusion. Plato's purpose then becomes to persuade us that there can be no true knowledge unless there are some things which are exempt from the Heraclitean flux. Cornford concludes that 'Plato is determined to make us feel the need of his Forms without mentioning them'. Burnyeat (1990, p.38) notes that 'X itself', when used in the dialogues for various X, often implies a reference to a Form, and such phrases are used in the *Theaetetus*. However, he suggests that this is insufficient to imply a theory of Forms. Cobb-Stevens (1989, p.247) expresses the views of many writers when he says that 'the *Theaetetus* is perhaps one of the most puzzling dialogues in the Platonic corpus' because the dialogue aims to define knowledge but does not discuss the Forms.

I find it interesting that none of the above commentators consider the word *diagramma*. My suggestion is that the use of *diagramma* has a significance which the writers who ignore its existence fail to see. It allows a connection to be made with the other passages, in the *Lesser Hippias* and *Cratylus*, where it appears. Thus, it allows an exploration, across several dialogues, of discussions about acquiring knowledge, and the use of such a specialized term implies that specialist knowledge is under discussion. This implies that the discussion is aimed at philosophers and it is philosophical knowledge, or knowledge of the Forms, which is intended. This adds support to those scholars who believe that Plato is indirectly referring to the Forms in the *Theaetetus*. It also means that, in his criticism of the theories of two renowned philosophers, Plato is warning philosophers who are searching for knowledge to be wary of what other philosophers say. Man cannot be the measure of everything if the existence of universal knowledge, such as that of the Forms, is to be possible, so Protagoras must have been mistaken. Also, to allow for knowledge to exist not everything can be changing, meaning that Heraclitus was wrong. The discussion adds to the warnings found in the *Lesser Hippias* and *Cratylus* about experts who may deliberately lie and people who may inadvertently present false premises. The implication is that someone who wants to acquire philosophical knowledge should not turn to other people for help.

Summary of Chapter Four

I started by indicating that the highest knowledge available is knowledge of the Forms, which philosophers can attain, but I suggested that non-philosophers could not acquire it. I looked at one author who suggests that this may not be the case but noted that, with his scenario, there is still a limit on the knowledge available to non-philosophers, since they cannot know the Form of the Good. I demonstrated that *diagramma*, as a specialized geometrical term, could be used to target those who understood geometry. Since geometers who have not progressed to philosophy would not be interested in acquiring philosophical knowledge, the word can be seen as being aimed at philosophers. I referred to the *Meno*, where *diagramma* does not appear although there are many references to geometrical diagrams, to support my claim. This means that Plato's advice about the acquisition of knowledge is aimed at philosophers who are searching for philosophical knowledge, or knowledge of the Forms.

I then considered the use of *diagramma* in three dialogues and showed that the advice, in each passage associated with the word, complemented that in the other dialogues. In all three, Plato foresees some problems which may occur when there is an attempt to acquire knowledge from other people. The *Lesser Hippias* claims that experts can be the best liars, so

that the words of teachers, such as the sophists, should be treated cautiously. However, it is unlikely that philosophers would lie deliberately. Plato solves this problem in the *Cratylus* where he discusses errors in premises, which must not be accepted without close examination. The *Theaetetus* demonstrates that even renowned philosophers such as Protagoras and Heraclitus can present false theories unintentionally. The passages associated with the word *diagramma* all warn philosophers to be careful when other people attempt to present them with knowledge.

I suggested that the word *diagramma* is not always the best one for Plato to use as he warns about how not to gain knowledge. This reinforces the importance of *diagramma* for reasons which are not immediately obvious in each of the three texts examined, namely connecting together discussions in the three dialogues and separating philosophers from non-philosophers. The overall conclusion to be gained from the *Lesser Hippias*, *Cratylus*, and *Theaetetus* is that philosophers must be wary of acquiring knowledge from other people. We are left with the question of how a philosopher can obtain the knowledge he craves and in the next chapter I show that Plato discusses this in the other texts where he uses the word *diagramma*.

Chapter Five

Further implications of *diagramma*

5.1 The *Euthydemus* and discovery 117

The background to the *Euthydemus* 117

The use of *diagramma* in the *Euthydemus* 120

Diagramma and discovery 122

5.2 The *Phaedo* and recollection 124

The background to the *Phaedo* 124

The use of *diagramma* in the *Phaedo* 127

Diagramma and recollection 128

5.3 The *Republic* and reason 131

The arguments in Book 7 of the *Republic* 131

The use of *diagramma* in the *Republic* 132

Diagramma and reason 134

5.4 The value of *diagramma* in Plato's dialogues 136

The *Epinomis* and its authenticity 136

More on the meaning of *diagramma* 140

Insights given by *diagramma* 141

Summary of Chapter Five 143

In this chapter I track *diagramma* across three more dialogues, the *Euthydemus*, the *Phaedo,* and the *Republic*, and show that it is connected with further discussions about how to acquire knowledge. I consider each text in the order stated, since that is the generally accepted chronological order of writing. In each case I give a brief account of the content of the dialogue before assessing the role of *diagramma*. In all three dialogues Plato offers suggestions for how a philosopher can acquire philosophical knowledge through his own efforts. This is necessary since, in the three dialogues explored in the last chapter, Plato dismisses knowledge offered by other people, since it may be false. Here the message is more positive. In the *Euthydemus* Plato advocates personal discovery, although he is reticent about how this may be accomplished. However, in the *Phaedo* he offers a suggestion when

he mentions *anamnesis*, or recollection of knowledge known before birth. Finally, in the *Republic* he stresses the need to use reason to look beyond the physical world. All six occurrences of *diagramma,* in non-spurious dialogues, have then been analysed, and I summarize the advice from all of them to show that Plato provides a philosopher with the means of attaining philosophical knowledge.

In the final section I examine the occurrence of *diagramma* in the *Epinomis*, now generally considered to be written by someone other than Plato. I show how the use of *diagramma* in this specific dialogue supports this view and allows a suggestion for the identity of the author. Finally, I assess the value of the use of *diagramma* and its significance for our understanding of Plato's philosophy. I end with a summary of the insights which I have uncovered.

5.1 The *Euthydemus* and discovery

I examine Plato's use of *diagramma* in the *Euthydemus* to continue my demonstration of the value of the geometry in the dialogues. As I follow the repetition of the geometrical reference in a further text I show that the discussion about how to acquire knowledge is being continued. In the *Euthydemus* Plato is again critical of the sophists but Socrates also shows how philosophical knowledge can be gained. This is an aspect of the dialogue which does not generally figure in commentaries on the text. After a preliminary survey of the background to the dialogue I look briefly at the various arguments within it before relating the use of the word *diagramma* to the acquisition of knowledge. I find support for my claim that it could be specifically aimed at philosophers.

The background to the *Euthydemus*

The *Euthydemus* runs from lines 271a to 307c, with *diagramma* at 290c. There are few commentaries on the *Euthydemus*; Chance (1992, p.1) notes that the dialogue has been neglected more than any other dialogue of importance. He gives several reasons why this might have happened, including the fact that Aristotle later examined fallacious arguments, which Chance says is seen as a main topic of the *Euthydemus* (pp.9-10). Most scholars, ancient and modern, have concentrated on Aristotle's analysis rather than Plato's account. Sprague (2000, p.4) says that it was rare to find any specific commentary on the dialogue until the 1970s.

The *Euthydemus* is usually placed among the early dialogues, although this is not accepted by all scholars, and Kahn (2000b, p.92) points out that there are several common themes with later dialogues. He picks out, as one example, the reference to the mathematicians turning

over their discoveries to the dialecticians, and he refers to the *Republic*, Book 7, where a study of mathematics is proposed as a preparation for dialectic. He decides that this is a special case of intertextuality where 'the reference to a second text is not only justified but actually required for a satisfactory understanding of the first.' Although he does not mention the use of *diagramma* this supports my assertion that these dialogues, and the others where *diagramma* appears, give a better philosophical insight when taken together, than when read in isolation.

Kahn (2000b, p.94) also notes that the doctrine of the Forms is often used to position dialogues within a chronology. He declares that the *Euthydemus* has, 'in its own way', references to the Forms, and he concludes that it is approximately contemporary with the *Phaedo* or *Cratylus*. I have already pointed out that *diagramma* in any dialogue, if seen as a specialized geometrical term, can indicate that philosophical knowledge of the Forms is being discussed. Ausland (2000, p.22) mentions various proposals for the date of writing the dialogue but says that in the nineteenth century scholars had the *Euthydemus* 'all over the map', and several conjectures have resurfaced in more recent times. He relates the preoccupation with dates to the period when spurious dialogues were being identified and notes that the authenticity of the *Euthydemus* was questioned in the nineteenth century, but it has been accepted as genuine ever since. The dramatic date of the dialogue is easier to determine. As Sprague (1965, p.vii) records, Socrates is said to be elderly, for example, the phrase 'practically senile' is used (295c), and Alcibaides is said to be alive (275a). He died in 404 so Sprague suggests around 405 as a dramatic date.

The *Euthydemus* is a frame dialogue with Socrates recounting to Crito an encounter he had the previous day with two sophists, the brothers Euthydemus and Dionysodorus. Aristotle is dismissive of the intellectual powers of Euthydemus when he ascribes false reasoning to him: 'such was the argument of Euthydemus, to prove, for example, that a man knows that there is a trireme in the Piraeus, because he knows the existence of two things, the Piraeus and the trireme' (*Rhetoric,* 1401a27*).* Socrates notes that both Euthydemus and Dionysodorus had been taught fighting but they are now 'skilled in fighting in arguments and in refuting whatever may be said, no matter whether it is true or false' (*Euthydemus*, 272a-b). This brings to mind Aristophanes' *Clouds* where Strepsiades tells his son that 'these people train you, if you give them money, to win any argument whether it's right or wrong' (lines 95-97). It is likely that such criticisms of the sophists would have been familiar to Plato so it appears that neither Euthydemus nor Dionysodorus could be described as a philosopher. Clinias and

Ctesippus are also present. Not much is known about these characters, or Crito, but I suggest that Plato would have made it clear if they had any connections with philosophy.

Different commentators choose different topics to take as the main discussion in the *Euthydemus.* Some see the dialogue as being about the way to happiness (for example, Dimas, 2002) while others concentrate on the comparison being made between Socrates and the two sophists (for example, Weiss, 2000). Sprague (1965, pp.x-xiii) lists nine main points which are covered in the dialogue. Rutherford (1995, p.112) summarises the reason for the different points of view when he declares that the *Euthydemus* is unusual for Plato because there is no formal subject under discussion. He declares that the dialogue is 'more about method than about a particular philosophic issue'. All of these writers show a lack of concern with the appearance of the word *diagramma.* I give a brief account of the main points to be found in the dialogue and then give my interpretation of the important conclusions which can be drawn when the use of *diagramma* is taken into account.

Euthydemus claims that he and his brother can teach virtue (273d) and Socrates challenges them to demonstrate their teaching skills. They do this by asking Clinias a series of questions, and each time they cause him to contradict himself. Socrates thinks that some of the terminology which the brothers use is ambiguous, and he asks them to demonstrate the value of wisdom and virtue (278d). This leads back to the question of whether wisdom is teachable. After further debate Socrates questions what type of knowledge should be acquired. *Diagramma* is mentioned in the ensuing discussion, and its use will be examined in the next section.

The conversation continues with Socrates recalling his earlier frustration; he wanted someone to 'make plain what this knowledge can be which we ought to have if we are going to spend the remainder of our lives in the right way' (293a). The dialogue continues with some fallacious arguments about knowledge, put forward by Euthydemus, including the thought that people either know everything or know nothing (294a). Since Socrates knows something, the brothers try to show that he has known everything since before he was born. The arguments continue to exploit linguistic ambiguities in supposed logical statements but, as they continue, Crito reports the comments of a passer-by who declares that the discussions are 'chattering and making a worthless fuss about matters of no consequence' (304e). Crito decides that he will not be able to persuade his son to study philosophy, but Socrates points out that even if people give a 'laughable performance' (307b) in a pursuit that is no reason to avoid pursuing it. He tells Crito that, with regard to philosophy, he should 'take heart, pursue it, practice it, both you and yours' (307c).

In the *Euthydemus* Plato is again criticising the sophists by showing that they do not provide a path to true knowledge. He defines the type of argument which the brothers use as 'eristic' (272b). Waterfield (1987a, p.299) gives the literal meaning of this as 'designed for victory or contentious'. Socrates derides the arguments used by the sophist brothers, and several commentators, such as Chance (1992, pp.18-20), suggest that Plato's purpose in the *Euthydemus* was to distinguish between eristic and Socratic dialectic. These had much in common and Guthrie (1975, p.275) notes that some contemporaries of Plato, for example Isocrates, declared that Plato and Socrates were as eristic as any sophist. Among others, Guthrie (p.283) suggests that the unnamed critic who appears in the crowd (304d-e) is supposed to be Isocrates, but at the date at which the dialogue appears to take place he would have been a young boy. Plato, therefore, avoids naming him but contemporary readers would have been able to draw parallels between the criticisms stated in the dialogue and the known views of Isocrates.

The use of *diagramma* in the *Euthydemus*

The discussion which contains the term *diagramma* starts at around 288d, where Socrates relates that his investigation is about the love of wisdom, or philosophy, which is an acquisition of knowledge. However, gaining knowledge is not sufficient since it is also necessary to be able to *use* the knowledge. Socrates refers to an earlier discussion (280c-e) and now gives a more specific example, that if we knew where all the gold on earth is buried it would be of no value to us unless we also knew how to use the gold (289a). Such things as money-making or making medicine are equally useless without knowing how to use the thing we have made (289a). Making a lyre, or a flute, is an art which is distinct from the art of playing the instrument (289c) and speech-writers may be unable to use the speech they have written (289d).

Socrates then reports a statement which he claims was made by Clinias:

> Whenever the hunters catch what they are pursuing they are incapable of using it, but they and the fishermen hand over their prey to the cooks. And again, geometers and astronomers and calculators (who are hunters too, in a way, for none of these make their diagrams [*diagrammata*]; they simply discover those which already exist), since they themselves have no idea of how to use their prey but only how to hunt it, hand over the task of using their discoveries to the dialecticians' 290c

Lamb (1967, p.445) translates the lines slightly differently, giving 'the geometers, astronomers, and calculators .... are not in each case diagram-makers, but discover the realities of things'. It is interesting that 290c comes in an interlude when Socrates is relating his conversation with Clinias, who is not a known sophist, whereas much of the dialogue

takes place between Socrates and the sophists, Euthydemus and Dionysodorus. Crito doubts that Clinias is clever enough for such a profound statement and hints that 'some superior being' was present and said the words (291a), presumably meaning Socrates.

The reference to 'discover those which already exist' is puzzling if Plato intends a reference to a geometrical figure alone but it is understandable if theorems are implied. This demonstrates the inadequacy of using 'diagram' as a translation for *diagramma.* If Plato intended to refer to a diagram, without the added implication of a theorem which *diagramma* gives, he could have used other terms, as he does in texts such as the *Meno*, where Socrates says *schēma* (74d). This would be readily understood by non-philosophers and could act as an analogy for ordinary knowledge. The use of *diagramma* is associated with advice that knowledge can be found by discovery, and used by the application of dialectic. The reference to dialectic implies that the knowledge under discussion is philosophical knowledge of the Forms. This offers further support for my assertion that the word *diagramma* is connected with advice aimed at philosophers.

With the exception of the *diagramma* example the type of knowledge discovered and used is fairly mundane, for example how to catch and cook fish. The use of *diagramma* is also set apart by a consideration of the people involved. With regard to the other types of knowledge it is implied throughout the text that the discoverers are a separate group of people from the users. Thus, the possibility that a huntsman could be a cook, or a cook a huntsman, is not considered. The division between geometers and dialecticians is more complicated. Although the connection between them is not explored in the *Euthydemus* there are other texts which provide evidence for Plato's views, and I referred to some of these in earlier sections. According to Plato, philosophers can only study dialectic after successfully completing all the earlier stages of learning, which include calculation, geometry, solid geometry and astronomy (*Republic*, 526a ff). However, not everyone who studies geometry progresses to philosophy. Since all dialecticians are also geometers but not all geometers are dialecticians there are two distinct possible situations. The people who are only geometers must hand over their discoveries to the dialecticians, but the philosophers, as experts in geometry, can make their own discoveries from the geometrical diagrams and then use the discoveries in their role as dialecticians. This is equivalent to the situation where a hunter could also be a cook.

The fact that a geometer and a dialectician can be one person, who is both a discoverer and a user, removes the problem of the expert who lies or is mistaken. The geometer who passes on a discovery could give false knowledge for the dialecticians to use, either intentionally or

unintentionally, but combining the role of geometer and dialectician means that the discovery comes from within, and is not dependent on the veracity of others. The use of a geometrical diagram can be seen as an analogy for philosophical knowledge and an important point is that the knowledge must be discovered, not invented. After the examination of how *diagramma* is used in the *Lesser Hippias, Cratylus* and *Theaetetus* to warn about false knowledge we now have a passage which offers some advice about how philosophical knowledge can be achieved. After its discovery, dialectic must be employed in order to use the knowledge which has been gained.

There are many topics which writers ascribe to the *Euthydemus.* I have shown that it also contains advice about the way that philosophers can successfully learn knowledge. Other writers have hinted at this theme, but without giving the detail which comes from tracking the use of *diagramma.* Kahn (2000b, pp.88-9) makes a general point about the whole dialogue when he says 'the *Euthydemus* is, among other things, an educational tract'. Roochnik (1991, p.213) picks out the philosophers when he suggests that Socrates' arguments fail to show that everyone should philosophize. However, he claims that they are successful enough to urge some people to pursue philosophy and they teach them how to do it.

***Diagramma* and discovery**

The quoted passage (290c) raises the question of why dialecticians are needed to use geometrical discoveries. Hawtrey (1978) addresses this question and his work supports my suggestion that although Plato refers to geometrical diagrams, the knowledge he is discussing is something more than geometrical knowledge. Hawtrey first assesses the possible meaning of 'dialectician' and decides that if it refers to someone 'who knows both how to ask questions and to answer them' then it is 'obscure what he will do with geometrical figures' (p.15). He decides that Plato is thinking of dialectic as the highest level of education, as proposed in the *Republic* (p.16). Other writers, such as Nehamas (1990, p.10), assert that the term dialectic implies an involvement with the Forms. This would mean that philosophical knowledge is the goal at which Plato is aiming and presumably he is not restricting all such knowledge to that based on facts demonstrated in geometrical diagrams. *Diagramma* thus provides an apt analogy, since both theorems and the Forms are unchanging and eternal.

A few writers on the *Euthydemus* give a brief account of the use of *diagramma*. In his analysis of the dialogue Hawtrey (1978) refers to some of the other occurrences of the word *diagramma* in Plato. He cites *Phaedo,* 73b, and *Republic* 529e (p.17, note 4), *Lesser Hippias*

367d and f [sic] and *Epinomis* 991e (p.17, note 5), but he does not mention the *Cratylus* or the *Theaetetus* and he does not attempt to make any connections between the different passages. He states that it 'is clear' that the obvious translation of *diagramma* can be 'diagram' but admits that it can also mean 'geometrical *figure*' although he dismisses the notion that it is connected with conclusions drawn from the figures (p.14). In his assessment of the section in the *Euthydemus* around 290c, Wedberg (1955, p.93) notes that *diagramma* had several meanings in Greek mathematical terminology. It could be a diagram as we know it, or a mathematical proposition or a mathematical proof, but he thinks that in the *Euthydemus* the first interpretation is the best. I disagree with this point, however, since such a figure is drawn, or made. It is the propositions or proofs of theorems, with the associated conclusions, that are discovered, not made, and this notion leads to a comparison with philosophical knowledge, whose unchanging nature means that it also must be discovered and not 'made', or invented. The importance of this point can be demonstrated by considering a non-geometrical example from modern science.

We talk of Einstein's equation, $E = mc^2$, but this connection between energy (E), mass (m) and the velocity of light (c) has existed since the beginning of the universe. Einstein did not invent it, he discovered it. Similarly, the facts shown on a geometrical diagram have always been true. Scientists reasoned out the implications of the energy-mass equation and Plato asserts that philosophers can reason out, using dialectic, the implications of geometrical discoveries and, by extrapolation, discoveries in all types of knowledge. Thus, we do not speak of someone inventing a new geometrical fact but rather of someone discovering universal truths which existed before the geometers drew the diagrams to reveal them.

I have claimed that *diagramma* is used to select philosophers who both understand the term and are eager to acquire philosophical knowledge of the Forms. From the account given in the *Euthydemus* It appears that the only way to truly know the highest truths is to discover them for oneself. Unfortunately, how the discovery can be made is not explored in this dialogue, although hints are given. Socrates states several times that he found his knowledge from within his soul, for example at 295e. In addition, it is necessary to concentrate solely on philosophy. He criticises those who attempt to combine philosophy with another subject, politics for example, saying that although both disciplines are good 'while partaking of both, they are inferior to both' (306c). It is with the next occurrence of *diagramma*, in the *Phaedo*, that more detail is given about how to achieve the discovery of knowledge.

5.2 The *Phaedo* and recollection

The text associated with the use of *diagramma* in the *Pheado* continues the positive advice about how to acquire knowledge. The dialogue runs from 57a to 118a with *diagramma* at 73b. I show that taking note of the geometrical example provides further enhancement to our understanding of Plato's message. I suggest that it offers support for my claim that this is again aimed at philosophers, who can be reminded of the rest of the advice by the repetition of the rare word. The *Phaedo* contains proofs for the immortality of the soul and featuring in these proofs is the notion of *anamnesis*, the recollection of things known before birth and forgotten. There is copious literature on the validity of the proofs, but my concern here is not with their validity, but with the use of *diagramma*, which appears in the second proof of immortality. It is again difficult to find any published analysis of *diagramma* but I show that there is a significance in its presence which is missed by those who ignore the word. I start with some background to the dialogue and give a summary of the arguments to be found there.

The background to the *Phaedo*

There is some uncertainty about when the *Phaedo* was written. Bluck (2000, p.2) suggests it appeared shortly before or after 386. He later (p.144) claims that 'no one would be likely to deny' that the *Phaedo* was after the early Socratic dialogues and before the last although it is not clear whether it precedes the *Symposium* or not. By looking at other dialogues written at about the same time Bluck considers Plato's workload as he founded the Academy and concludes that the earliest date for the *Phaedo* would be 392. Grube (1977, p.1) gives an analysis of the date of writing which is in approximate agreement with this. He suggests that the *Phaedo* was written between the *Meno* and the *Republic* and dates it to about 390 or shortly afterwards. Taylor (1963, p.174) also places the *Phaedo* before the *Republic.* This is because there is a possible reference to the *Phaedo* at *Republic* 611b when Plato refers to other arguments for immortality, although Taylor acknowledges (p.174, note 5) that this could be a reference to the arguments used by the Orphics or Pythagoreans. He dates the text to around 387. Hackforth (1972a, p.7) also gives this date, based on the fact that there is no mention of politics in the *Phaedo* and when Plato returned from Sicily he was repelled by politics. Scott (2009, p.270) gives the date of the visit as 388. Hackforth insists, like most scholars, that the *Phaedo* was written before the *Republic.*

The slight uncertainty among scholars on the exact date of writing the *Phaedo* gives way to certainty about the date of the action. It is a reported dialogue but the events recorded took

place on Socrates' last day and we know that this was in 399. Phaedo, who was with Socrates at his death, tells Echecrates, from Phlius, about the events and discussions of Socrates' last hours. It is known that Plato was not there but it is generally accepted that the account given is fairly accurate as many of the people present would still be alive when the *Phaedo* was published. However, Phaedo's act of recounting the story, to Echecrates, is not necessarily historically correct. The setting is Phlius, where Phaedo is on his way back to Athens and Taylor (1963, p.175) notes a significance in this. The location of the conversation could be the meeting house in Phlius, which was used by the local Pythagoreans.

With regard to the characters found in the dialogue, Sedley (1995, p.6*)* claims that Plato's choice of speakers always had some significance. In later dialogues the choice was largely symbolic, 'representing his carefully thought-out apportionment of philosophical debts'. In earlier dialogues the speakers were known proponents of views which were against those of Socrates, or they were opinionated and confused individuals whom Socrates could target. Sedley claims that the *Phaedo* is a transition between these two methods of character selection. We now know that Phaedo became an independent philosopher, setting up his own school at Elis. Sedley (1995, pp.8-9) notes that he wrote the dialogue *Zopyrus*, in which a physiognomist, who claimed to tell people's characters from their appearance, declared that Socrates would, therefore, be stupid and a womaniser. Since this is obviously not the case Phaedo may have been stressing that Socrates' intellect had overcome the proposed laws of body-soul interaction. Plato could, therefore, have chosen Phaedo as the narrator of the dialogue since it is about the 'philosophical soul's liberation from the body, even in this life' (Sedley, 1995, p.9). However, Hackforth (1972a, p.13) suggests that Phaedo was chosen because he was the one who initially told Plato about Socrates' death. Echecrates was a suitable choice to question Phaedo about the events because as a Pythagorean he would be interested in the immortality of the soul and the theory of the Forms. Also, Echecrates was a non-Athenian and so would be less likely to know the details of Socrates' death than an Athenian.

Several scholars remark on the relevance of the choice of Simmias and Cebes as interlocutors in the *Phaedo*. They are often mentioned in literature as Pythagoreans. Sedley (1995, p. 13) notes that, since the Pythagoreans had a doctrine on the immortality of the soul, the fact that two Pythagorean students are confused allows Plato to 'make it his own – to appropriate it to his own "Socratic" heritage'. He incorporates the theories of Forms and recollection to this end. There are several others present, including members of Socrates' family, such as Xanthippe, his wife. There are also some sophists, including Aeschines, and philosophers, for

example, Euclides of Megara. The people present represent a wide cross-section of society, including philosophers and non-philosophers.

There are three arguments for the immortality of the soul. The first notes that there is an obvious process whereby things go from a living state to a dead one. Since other processes are reversible Socrates concludes that 'the living come to be from the dead, and the souls of the dead exist' (72d). However, there is no necessity for a soul to retain intelligence gained in a previous life. This is dealt with in the second argument which depends on the theory of *anamnesis.* Socrates' use of *diagramma* in this argument will be explored in the next section. He gives several examples of recollection coming after a sense perception. Sometimes seeing someone, or something, can remind us of a person we know. Socrates mentions a lyre which can remind us of its owner, or seeing Simmias can bring to mind his friend Cebes (73d). He then mentions the notion of equality, which we understand although it can never be perfectly realised on earth. It must, therefore, come from some latent knowledge, gained before birth. Sense perception of apparently equal objects reminds us of this pre-birth knowledge (74a ff). Only sense perception is needed so there is no necessity to have someone using questions to bring the original knowledge to mind, as happens in the *Meno*, where *anamnesis* is also discussed. The dubious assumption in the *Phaedo* is that the thing, or person, recollected must have been known before birth and so the soul must have also existed before birth and hence be immortal.

It is the third proof for the indestructibility of the soul which refers to the theory of the Forms (102a ff). Bluck (2000, p.18) suggests that Plato considered this last argument to be convincing, a view shared by others. Sedley (1995, p.17), for example, bases his belief on the fact that 'it is the foundation for Socrates' optimistic acceptance of his own death'. There is a final warning. If souls are immortal we must care for them and a myth is related about the world to which souls go when we die (107a ff). Socrates concludes that a good man 'should be of good cheer about his own soul' (114d).

The argument which contains the reference to *diagramma* is concerned with *anamnesis* but there is considerable confusion about this notion, both within the dialogue and in commentaries on it. Initially, in the arguments found in the *Phaedo*, it appears to work for everyone. Then Simmias doubts if anyone other than Socrates could gain knowledge by recollection, saying that 'by this time tomorrow there will be no one left' (76b), a reference to the fact that Socrates is about to die. Scholars through the centuries have professed confusion about some aspects of *anamnesis.* One example is given by Hackforth (1972a, p.197) who presents the comments made by Strato, the Head of the Peripatetic School in

around 287. His writings are preserved by Olympiodorus, in the sixth century CE, and show that Strato cannot understand the theory of recollection. He wonders, for example, why knowledge of how to play a musical instrument does not come through recollection, but always needs practice. Gosling (1965, p.151) calls the passage on recollection 'notorious in discussions of the Theory of Forms' and declares that some authors, including Bluck, had some 'muddling additions about meaning' in their commentaries on the *Phaedo.* Despite the attendant problems in interpreting this dialogue, my specific purpose here is to focus on how the reference to *diagramma* is relevant to the study of the acquisition of knowledge.

The use of *diagramma* in the *Phaedo*

Cebes reminds Socrates that he has often mentioned that 'learning is no other than recollection' (72e). This implies that we must have previously learned what we now recollect which 'is possible only if our soul existed somewhere before it took on this human shape' (73a). Simmias claims to not remember the proofs for this and asks Cebes to remind him. Cebes says:

> ... when men are interrogated in the right manner, they always give the right answer of their own accord, and they could not do this if they did not possess the knowledge and the right explanation inside them. Then if one shows them a diagram [*diagrammata*] or something else of that kind, this will show most clearly that such is the case. 73a-b

Most translators give the same wording for *diagrammata.* It is interesting that Socrates is not the speaker when the geometrical reference is made but he gives his support to the sentiment which Cebes has expressed.

The phrase 'something else of that kind' has given rise to several different interpretations. White (1989, p.82) notes that some commentators assume that it refers to the *Meno*, where geometrical diagrams are used to demonstrate recollection. However, White refers to a later reference by Socrates to pictures (73e) which are compared to other versions of the same thing and White suggests that the pictures are the 'something else of that kind', which he translates as 'anything of that sort'. Hackforth (1972a, p.67, note 1) reaches a similar conclusion by looking at a different dialogue. He refers to the *Republic*, 510d, where Socrates mentions visible figures which a geometer uses to aid his thinking. Hackforth supposes these can be either diagrams or models of geometrical shapes and so suggests that the 'something else' is a model. A few commentators enter into the problem of the true meaning of the word *diagramma*, for example Taylor (1963, p.186, note 2) says that *diagramma* can mean

geometrical proofs but he refers to Xenophon, *Memorabilia* 4, 7.3 where it appears to mean a demonstration.

Although the *Phaedo* is concerned with the immortality of the soul, the argument in which *diagramma* is found is based on the theory of recollection. Since this is an account of how we can be reminded of knowledge we once had, before birth, this gives another instance of *diagramma* being imbedded in a discussion about the acquisition of knowledge. It is noticeable that Socrates assumes the theory of recollection, with no explanation, and all those present seem familiar with it. This approach contrasts with the account in the *Meno*, where Socrates takes great pains to convince his audience that recollection can be demonstrated. Burnet (1977, p.53) suggests that Plato had no need to prove the existence of *anamnesis* in the *Phaedo* since he had already done so in the *Meno.* Grube (1977, p.22, note 13) also sees the reference to a diagram as a reference to the diagrams found in the Meno, where Socrates demonstrates *anamnesis*. Neither of these authors record that the word *diagramma* is not used to describe the diagrams found in the *Meno*. In addition, they do not mention that *diagramma* appears elsewhere in Plato's texts. I believe that the connection with the other discussions is important. We are now aware that we should not accept knowledge from others, without checking it carefully, but the *Euthydemus* revealed that we can discover knowledge for ourselves. The *Phaedo* is offering a suggestion as to how we can do this, by *anamnesis*.

It is clear in this dialogue that Plato is referring to the Forms, and their importance in his arguments would suggest that the knowledge that is being sought is philosophical knowledge. It is, therefore, possible that *diagramma*, a specialized geometrical term, is being used to single out the philosophers as the main audience for Socrates' pronouncements. There is a long list of people present at Socrates' death who play no part in the discussion. Some of these are known to be eminent philosophers but at 59b it is mentioned that some others are present. There is also the possibility that many 'ordinary' men would want to read about Socrates' last day, when the *Phaedo* was published and this is also true today. If only philosophers have any chance of gaining the supreme knowledge that includes knowledge of the Forms the use of the technical term *diagramma* could again be designed to warn general readers that this discussion is not for them.

***Diagramma* and recollection**

The use of geometry in the *Phaedo* argument about the immortality of the soul, based on the theory of recollection, has provoked some scholarly debate. For many writers, the main

purpose found for it is that it ties the discussion to the one found in the *Meno* since that is based on geometrical diagrams. I consider that this deduction fails to take into account the fact that the *Meno* does not contain the term *diagramma.* If Plato wanted to bring to mind the *Meno* he would surely have used one of the terms found there when he refers to diagrams, for example *schēma*.

There are several comments in the literature about the aptness of a geometrical reference. Dilman (1992, p.102) voices one of these. He says that Socrates is considering the dependence of the world of sensible things on the world of mathematics but he does not recognise that there is a dependence which goes the other way. We use mathematical language to describe our activities in the sensible world and 'the reality of mathematics depends just as much on the reality of the empirical world'. Sprague (2007, p.132) argues that the reference to 'showing people a diagram' means that Plato is talking of the soul as rational rather than moral. Ancient commentators on the *Phaedo* also puzzled over the reference to geometry. Westerink (1976, p.160) gives the commentary of Olympiodorus on line 73b. He says that geometry is chosen because 'there the road is a single and narrow one', presumably meaning that there is no room for errors. I offer a different suggestion.

I maintain that *diagramma* works as a reminder of other discussions, with the references to *anamnesis* filling the gap left in the *Euthydemus*, where no method of discovery is given. This view is contrary to those expressed in many articles. One notable example is in Frede (2001, p.265), who points out that Plato does not use the concept of *anamnesis* in the final argument for immortality of the soul in the *Phaedo.* She concludes that this is because 'he hardly ever expresses the same thought twice, and certainly not in the same words. That is why he is often so hard to understand and his intentions are so hard to "tie down"'. My contention is that the use of 'the same words' is what enables us to gain a greater understanding of Plato's philosophy. Having decided that knowledge must be discovered, as reported in the *Euthydemus*, Plato is now providing a means to make such discoveries since *anamnesis*, according to his account, allows us to re-discover the knowledge we had before birth.

Anamnesis is really the only aspect of Plato's advice on acquiring knowledge which would not be accepted in modern times and there are problems within the dialogues about the part it can play. Its demonstration, in the *Meno*, has drawn much criticism, since Socrates appears to be leading the slave boy to achieve the knowledge which he claims comes from *anamnesis*. It is in the *Meno* that Plato wrestles with the problem of how anyone can gain knowledge, when they do not know the object of the search. Meno asks Socrates: 'How will you aim to

search for something you do not know at all? If you should meet with it, how will you know that this is the thing that you did not know?' (*Meno*, 80d). Socrates summarizes the problem:

> Do you realize what a debater's argument you are bringing up, that a man cannot search either for what he knows or for what he does not know? He cannot search for what he knows – since he knows it, there is no need to search – nor for what he does not know, for he does not know what to look for. 80e

He proceeds to demonstrate *anamnesis*, and it may be that Plato saw this as the only solution to the problem, although, as Sedley points out (2004, p.29), Socrates is critical of *anamnesis* in the *Theaetetus*, at 197e.

The search for knowledge beyond our experience must start with something which is familiar. This is the world of the senses. *Anamnesis* provides a link between things which can be heard and seen and the abstract world which such things can bring to mind, giving knowledge of otherwise unknown concepts. I claim that there is a parallel between the use of *diagramma* in a discussion about *anamnesis* and the importance of the word with regard to separating philosophers from non-philosophers. An example demonstrates this. A geometrical diagram which contains a circle can lead a geometer to an understanding of a perfect circle. This can be taken further by a philosopher who can comprehend the notion of the Form of Circle. The use of the term *diagramma* does not restrict the further knowledge to geometrical concepts but demonstrates how, starting with the sensible, imperfect and transitory world, the philosopher can use *anamnesis* to 'recall' forgotten knowledge of all the Forms.

The concept of *anamnesis* is strange to modern philosophers but this does not detract from its relevance for Plato's account. It is also possible to suggest alternative, but equivalent, methods for philosophers to use. A questioning mind, trained in geometry and dialectic, will not stop with the things sensed in this world but will use them as stepping stones to further knowledge. For Plato, *anamnesis* was a way to explain how this could be done in practice. Perhaps, as a substitute, we can accept Newton's explanation of how he entered into new realms of knowledge: 'If I have seen further, it is by standing on the shoulders of giants' (quoted by Hawking, 2002, p.ix). Plato imagined recalling knowledge known before birth and perhaps we can recall knowledge discovered by those who lived before we were born.

Unfortunately, if we accept *anamnesis* as part of Plato's belief about the acquisition of knowledge, there is a further problem. He does not give any indication in the *Phaedo* of how recollection in general can be triggered, or why it would work for philosophical knowledge. We are, therefore, left with no clear idea about how knowledge of the Forms can be gained. It appears that the passages examined so far do not present a complete picture of how Plato

thought that a philosopher can acquire knowledge. I show that in the final text where *diagramma* is found there is further clarification of this topic.

5.3 The *Republic* and reason

Apart from the disputed *Epinomis*, the final reference to *diagramma* comes in the *Republic*. The value of taking note of this unusual word is demonstrated by its association, again, with how to acquire knowledge. This allows the final piece of Plato's advice to be given. After exploring its use in the *Republic* I give an overview of how philosophers can attain knowledge, drawing on all the passages which have been discussed.

The arguments in Book 7 of the *Republic*

The *Republic* has already been considered in Chapter Two, when the distant object analogy was examined. The word *diagramma* appears in Book 7 and I restrict my analysis here to that book. The main characters in the dialogue were listed in the earlier section on the *Republic*. Of particular interest is the lack of philosophers among those present, although much is written in Book 7 about the education of philosophers. The only one explicitly stated to be a philosopher is Polemarchus who in the *Phaedrus*, at 257b, is said to have been converted to philosophy. Others are politicians, such as Clitophon (Nails, 2002, p.103), or soldiers, such as Adeimantus, who fought at Megara (Nails, 2002, p.3).

Book 7 runs from 514a to 541b, with *diagramma* appearing at 529e, about halfway through the book. At the beginning is the description of the prisoners in the cave. A person who is freed is initially unable to see in the light. As his eyes become accustomed he can see shadows, then reflections in water and then 'the things themselves' (516a). Eventually he would be able to see the sun (516b). If he returned to the cave he would initially be unable to see in the darkness and the cave dwellers would ridicule him (516e-517a). Socrates matches the ascent out of the cave with 'the upward journey of the soul to the intelligible realm' (517b). In this realm, the Form of the Good 'controls and provides truth and understanding' (517c).

The proposal that it is philosophers who can attain true knowledge and then return to teach others, leads to a discussion about the best way to educate such men. Socrates reminds Glaucon that they had earlier discussed the initial stages of education, where physical training and music were to be introduced (521d, referring to 376e ff). These are now rejected: physical training because it is 'concerned with what comes into being and dies, for it oversees the growth and decay of the body' 521e) and music 'is just the counterpart of physical

training' 522a). However, they decide to consider 'one of the subjects that touches all of them' 522b), namely number and calculation (522c). A discussion about the importance of numbers leads to the conclusion that a knowledge of them must be compulsory for both soldiers and philosophers and the guardian of a city must be both soldier and philosopher (525b). Other subjects are then discussed, namely geometry, solid geometry and astronomy. The latter 'compels the soul to look upward and leads it from things here to things there' (529a) but a correct learning of astronomy should not depend on observation by the senses. The next passage refers to *diagramma* and will be examined later.

It must be decided who should be taught all these things. Various admirable qualities for students are discussed, for example they must be 'keen on the subjects and learn them easily' (535b). When suitable students have been chosen they must follow a specified curriculum. Arithmetic, geometry and physical training must be mastered before a student can be chosen to proceed further. Only mature people can be taught to argue correctly since young people act as if it is a game, refuting and being refuted until 'themselves and the whole of philosophy are discredited in the eyes of others' (539c). However, an older person will 'engage in discussion in order to look for the truth' (539c). It is only after the age of fifty that a man will spend 'most of his time with philosophy' (540b) but he must also spend some time in ruling the city and educating its citizens, before he departs to 'the Isles of the Blessed' (540b).

It may appear as though the advice given to philosophers searching for knowledge is already complete in the dialogues already explored. The *Euthydemus* revealed that knowledge must be discovered and then explored using dialectic and the *Phaedo* showed how to discover the knowledge, by *anamnesis*. However, an assessment of the use of the word *diagramma* in the *Republic* reveals that there is an important aspect of the search for knowledge that has not yet been covered, *where* the knowledge lies. I show that the passage containing *diagramma* reveals this.

The use of *diagramma* in the *Republic*

Socrates says:

> We should consider the decorations in the sky to be the most beautiful and most exact of visible things, seeing that they're embroidered on a visible surface. But we should consider their motions to fall far short of the true ones – motions that are really fast or slow as measured in true numbers, that trace out true geometrical figures [*schēmasi*] that are all in relation to one another, and that are the true motions of the things carried along in them. And these, of course, must be grasped by reason and thought, not by sight. .... Therefore, we should use the embroidery in

> the sky as a model in the study of these other things. If someone experienced in geometry were to come upon plans very carefully drawn [*diagrammasin*] and worked out by Daedalus or some other craftsman or artist, he'd consider them to be very finely executed, but he'd think it ridiculous to examine them seriously in order to find the truth in them about the equal, the double, or any other ratio. 529c-e

It is interesting that when Socrates mentions 'geometrical figures' the Greek word used is the plural of *schēma*, a commonly occurring word, with the *TLG* listing 261 uses in Plato. There are several translations but the usual ones are figure, where it can refer to the figure of a person, for example at *Statesman* 268c, or shape, for example at *Timaeus* 44d, where it refers to the shape of the universe. In the *Meno* both these translations are used, with connections to geometrical shapes or figures.

I suggest that this supports the use of *diagramma* to imply more than just a diagram being viewed. The 'plans carefully drawn', where *diagrammasin* appears, relate to diagrams where an attempt will be made to use them to find out facts about ratios, for example. It is possible that *schēma* is used to refer to the world as experienced by the senses alone but d*iagramma* implies something more, involving reasoning which can lead to an understanding of the intelligible world of the Forms. I explore this below.

Daedalus was a legendary figure who appeared in many of the most well-known myths, such as the flight of his son Icarus, who used wings made by Daedalus. He was also involved in making the labyrinth of the Minotaur. Radice (1977, p.97) writes that his name 'became synonymous with uncanny skill and ingenuity'. Daedalus could thus be expected to produce miraculously accurate diagrams but Plato insists that even these could not perfectly represent equality or ratios. A true 'double' or 'equal' can only be seen in the mind. This can be extrapolated to all philosophical knowledge, which can never be found in the ordinary world. Just as a carefully drawn geometrical diagram can hint at such a property as equality so examining something using sensory perception can hint at the true thing, but can never allow it to be known completely. This is the final piece of advice for philosophers who are searching for ultimate knowledge. It cannot be found in the world around us but only, through reason, in the intelligible world.

In the following passage Socrates explains what he means by expanding on the reference to the skies (530a-b). When an astronomer looks at the stars he will think that they have been organised by their creator in the best way possible but ratios, such as that of night to day, are not necessarily always the same. Plato would know that the ratio of night to day varied as he could observe this over the course of a year, although the change would not have been as great in southern Greece as in more northern countries. Astronomy should be approached

the same way as geometry, 'by means of problems' (530b). Socrates continues with examples drawn from the study of harmonics. This leads eventually to an assertion that only by 'argument and apart from all sense perceptions' can someone find 'the being itself of each thing' (532a). Here, I prefer the translation of Emlyn-Jones and Preddy (2013). They give: 'the true goal of the intelligible world' is reached 'by dialectical methods through reason and without all the sense perceptions'. Plato wants philosophers to search for immutable knowledge of the Forms. He is telling them that such knowledge cannot be found in the ordinary world around us. However, the reference to a geometrical diagram suggests that sensory perception can be used as a starting point, just as the diagram can hint at the notion of equality. Only dialectic can lead to the ultimate knowledge that has been discussed and then only to 'someone experienced in the subjects we've described' (533a). These subjects are presumably calculation, geometry, solid geometry, astronomy and harmonics. The reference to dialectic reinforces the notion that the passages associated with *diagramma* are aimed at philosophers, since they are the ones who are chosen to study dialectic.

***Diagramma* and reason**

It appears that just looking at a geometrical diagram does not give much information. This reinforces what was said in the *Euthydemus*. The diagram must be *used* to uncover truths beyond the scope of the actual lines and curves. No drawn diagram can exactly demonstrate, for example, two lines in some precise proportion but, if reason is applied to a diagram, the various facts about proportional lines can be deduced. Diagrams are seen as a poor imitation of geometrical truths and this notion can be extended to suggest that anything we see in the world is merely an imitation of the true object, the Form. However, only philosophers can know the Forms whereas anyone can look at a geometrical diagram. In addition to connecting this discussion with those in the other texts where it appears, *diagramma* here acts to restrict those who can gain the extra knowledge, through reason. Anyone who is experienced in geometry will realise that a diagram cannot reveal everything but it is the philosopher who will apply reason to other things apart from geometrical facts.

Socrates criticizes geometers who 'dream about what is' (533b) and use hypotheses 'that they cannot give any account of' (533c). By contrast, the dialectician removes the hypothesis and proceeds 'to the first principle itself, so as to be secure' (533c). It is interesting that Annas (1981, p.293) concludes that Plato thinks, with regard to the *Republic* and dialectic, that 'what matters is that *you* are moved ...'. This brings to mind Plato's advice in the *Lesser Hippias, Cratylus* and *Theaetetus*, where he is against gaining knowledge from others. The texts explored in this chapter have indicated that philosophers must discover, use and reason

about the knowledge for themselves. The discovery and use may depend on others but the final stage, of looking to the intelligible world, must come from within.

The arguments given in the *Republic* provide the final piece of advice on acquiring knowledge, associated with Plato's use of *diagramma* in non-spurious dialogues. The stress on using reason could also be applied to the passages examined in the previous chapter. Reason must be used to assess whether a lie is being told, whether it is deliberate or unintentional, and reason is needed to check premises. It is appropriate, therefore, to present here the completed discussion from all the dialogues, as it might have been absorbed by the philosophers of Plato's day.

I separated warnings from more positive advice but the order at which I looked at the dialogues is almost certainly not the order in which they were written. I now show how a contemporary of Plato's might have received all the advice, taking the order given by Kenny (2004, pp.39-41). He summarizes the various studies which have been used to provide a proposed chronology for Plato's dialogues. Although there is no absolute agreement, he states 'there is good reason to accept the general consensus that thus divides the Platonic dialogues into three groups, early, middle, and late' (p.40). I give the proposed order of those dialogues about which there is general agreement. Although the position of the *Cratylus* is unknown Kenny places the *Euthydemus* and *Lesser Hippias* in the early section. The *Phaedo* and the *Republic* fall in the middle section and, unusually, there is general agreement about their order within the section, with the *Phaedo* coming first. The *Theaetetus* appears to come between the middle and later dialogues. This gives the following order: the *Euthydemus* and the *Lesser Hippias* (order unknown); the *Phaedo*; the *Republic*; the *Theaetetus* with the *Cratylus* position unknown. It must be noted that not all recent scholars agree with the placing of some of these dialogues, for example Kahn (2000b, p.94) suggests that the *Euthydemus* could have been written at around the time of the *Phaedo*, although earlier in his article he defends what he calls traditional chronology, which places it in the earlier dialogues (p.89). Kenny's order is similar to what Altman assesses as the 'reading order of Plato's dialogues' (Altman, 2010, p.44), except that Altman places the *Phaedo* at the end, which does not agree with most attempts to provide a chronology.

I have stressed throughout that my work does not depend on an accurate chronology of the dialogues but I offer here a possible sequence for the passages associated with the use of *diagramma*. The discussion would start in the dialogues of the *Lesser Hippias* and the *Euthydemus*, and their order is immaterial since the advice in each complements the other. Philosophers learn here that they must not look towards so-called experts, such as the

sophists, for help since they are in the best position to tell lies about their specialist subjects. Instead, seekers after knowledge must discover it for themselves and then use dialectic to further their understanding. In the *Phaedo* we are told how the knowledge can be discovered, through *anamnesis*, and in the *Republic* Plato stresses that ultimate knowledge must not be sought in the world of the senses but, through reason, in the intelligible world. The advice follows such a logical path that it could act as further evidence for the chronology of these dialogues. The *Theaetetus* then provides, almost as an afterthought, further advice. Looking back on the path to knowledge which has been presented it may appear that renowned philosophers could provide help with the journey, but Plato shows that even they may be mistaken. Philosophers must at least start the journey to true knowledge alone although, since dialectic is prescribed, they can receive help through discussions. The advice in the *Cratylus* is apt wherever it comes, since a philosopher must continually question carefully any assumptions made by others.

Noting the appearance of the word *diagramma* has allowed a very much enhanced and augmented assessment of Plato's advice about how to obtain knowledge. This advice is so thorough that many of the points, with the exception of the reference to *anamnesis,* could be of benefit today. The work on *diagramma* thus offers the possibility of new insights into how to acquire more knowledge. In addition, with an overall perspective of the discussion as it ranges across several dialogues, contrasts and comparisons can be made within Plato's work.

5.4 The value of *diagramma* in Plato's dialogues

Since all the occurrences of the word *diagramma* in texts accepted as being by Plato have been examined I assess the case of the disputed *Epinomis*, which also contains *diagramma*. I show that my work on the use of the word supports the general opinion that this was not written by Plato, although the dialogue is often taken to be Book 13 of the *Laws.* I then examine the meaning of *diagramma* in the light of my findings and consider a possible reason for why it is generally ignored. I end with an assessment of the enhancement which *diagramma* gives to our understanding of Plato's philosophy.

The *Epinomis* and its authenticity

Although the *Epinomis* is generally accepted as being spurious there are some conflicting views. I examine the use of *diagramma* in the dialogue using the same approach as I have used for the other dialogues where it appears, before showing that there is support for the

spurious nature of the *Epinomis*. I also speculate about the possible author, basing my suggestion on the rarity of *diagramma* in extant ancient texts.

As in the *Laws*, the main speaker of the dialogue is an unnamed Athenian. The only other people present are Megillus, of Sparta, and Clinias, of Crete. Little is known of them, but Nails (2002, p.198) suggests that Megillus could be the Spartan peace commissioner since someone of that name is chronicled as holding this role. Clinias is known to be a citizen of Cnossus (*Laws*, 629c). Taylor (1963, p.465) notes that some writers claim that Plato may have modelled the Athenian on himself but he admits that there is no evidence for any 'deliberate intention of self-portraiture'. In *Laws,* it is stated that the three men are on Crete, walking from Cnossus to the cave and shrine of Zeus (*Laws*, 625b). They agree to pass the journey 'in a discussion about constitutions and laws' (*Laws,* 625b). At the end of Book 3 of *Laws* the purpose of Clinias' journey is revealed. He tells how Crete is attempting to establish a colony and has given responsibility for this to the Cnosssians who have delegated the task to Clinias and nine colleagues. They must provide a legal code based on local and foreign laws. He hopes that the discussion he is having with his travelling companions will give him a 'framework for the future state' (*Laws*, 702a-d). It is possible that the *Epinomis* was written to follow on directly from *Laws*, in which case the debate is continuing as the journey continues. However, there is nothing to confirm this and the intention may have been that the men met up again later. The Athenian's companions are not known as either mathematicians or philosophers, so that the reference to *diagramma* would not be fully appreciated by either of them.

The word *diagramma* appears in the *Epinomis* at 991e. The dialogue runs from 973a to 992e which means that the term appears near the end. The discussion from the start concerns the subjects which need to be taught in order to achieve wisdom (*sophia*). The Athenian considers several skills such as carpentry, hunting, generalship and medicine but each is rejected (974d ff). He decides to approach the problem by considering the gift of number, since a loss of this 'would render humans the most unintelligent and senseless of living things' (976d). He must then decide who gave the concept of number to man and suggests that it was a god, the one which we call 'Cosmos or Olympus or Heaven [Uranus]' (977b). This was so that we can learn to count, for example by tracking the path of the moon over the half monthly cycle of fifteen days, from New to Full (978d). Further astronomical data is discussed and Lloyd (1956, p.207) notes that the *Epinomis* is the first surviving Greek text to list the five known planets, Mercury, Venus, Mars, Saturn and Jupiter, to give the names by which we know them.

The analysis of the heavenly bodies is complicated and solid geometry is mentioned (990d), before there is a reference to the word *diagramma*:

> To the person who learns in the right way it will be revealed that every diagram [*diagramma*] and complex system of numbers, and every structure of harmony and the uniform pattern of the revolution of the stars are a single thing applying to all these phenomena. And it *will* be revealed to anyone who learns correctly, as we say, fixing his eye on unity. To one who studies these subjects in this way, there will be revealed a single natural bond that links them all'. 991e

Taylor (1956, p.252) translates *diagramma* as 'geometrical constructions'. The first thing that comes to mind on reading this is the difference between the use of *diagramma* here and in the other passages which I have explored. In those texts it appeared as an example to demonstrate a point being made about the acquisition of knowledge. Here it appears within a passage devoted to the unity of knowledge. At the start of the text Clinias declares that the purpose of the dialogue is to examine 'what a mortal must learn in order to be wise' (973b). It should be noted that Clinias speaks of *what* we should learn, not *how* we should learn. The other passages where *diagramma* appears are all concerned with *how* to gain knowledge.

The authorship of the *Epinomis* was under suspicion from early times. Taran (1975, pp.115-123) draws together all the extant ancient texts which refer to the matter and, like several other authors, quotes from Suidas where it is said that some philosopher 'divided the *Laws* into twelve books, and that he himself is said to have added the thirteenth' (p.124). Taren notes that the *Epinomis* was referred to as the thirteenth book of the *Laws* as early as the second century CE. Comparing several texts Taran concludes that the unnamed philosopher is Philippus of Opus (p.124).

In modern times, a glance at the literature reveals contrasting views. Taylor (1929, pp.235 ff) examines a doctorial thesis by Müller in which he attempts to show that the *Epinomis* was not written by Plato. Taylor considers some single words and short phrases cited by Müller as support for his hypothesis and, for each example, Taylor explains how Plato could have used it, or does use it elsewhere, even if rarely. I find it surprising that the word *diagramma* is not mentioned by either Müller or Taylor. Later Lacey (1955, p.81) mentions 'the vexed question of the authorship of the *Epinomis'* but he assumes the author is Plato, 'which the balance of evidence seems to support'. In 1971, Morton, Winspear, Levison and Michaelson summarised findings made by applying computer analysis to several ancient texts, including the *Epinomis.* Their final conclusion was that the *Epinomis* was 'probably not by Plato' (p.100). However, a comparison with a letter known to be from Speusippus to Philip of

Macedon shows enough similarities with the *Epinomis* for Winspear, the classical consultant on the team, to suggest that the *Epinomis* was written, or revised, by Speusippus (p.100).

More recent writers often refer to Taran's analysis of 1975. Dillon (2003) agrees with Taran's conclusions and states that the authorship of Philippus of Opus is now 'virtually the consensus of scholars' (p.178). Dillon notes that the 'main claim to fame' of Philippus is that he edited Plato's final work, *Laws* (p.182). However, only a few years later, Altman (2012, p.83) sets out to argue against Taran's claim since he believes that the *Epinomis* was written by Plato. Lloyd (1956, p.217) draws a conclusion which has not been reliably overturned when he states 'it seems fair only to deny that we have enough knowledge on which to base a definite verdict'. In all these discussions there is no exploration of the use of the word *diagramma*. Lacey (1955, p.99) quotes the passage which contains the word, which he translates as 'geometrical figure' but he does not refer to it in his assessment of the text, only mentioning that the passage refers to 'the mathematical unity behind widely diverse phenomena'. I suggest that an analysis of the use of the word *diagramma* in the *Epinomis* can throw some further light on the vexed question of authenticity.

The anomalous use of *diagramma* described above provides some support for the scholars who believe that the *Epinomis* was not written by Plato. In addition, within the context of the passage quoted (991e) the word could be taken to mean merely 'diagram', whereas I have argued that in the other dialogues where it appears the connection with constructions or theorems is implied. However, the word appears so rarely in ancient texts that an explanation must be sought for why Philippus, or some unknown author, used it at all. Sedley (2003) offers a suggestion which could act as a basis for such an explanation. He notes that the *Epinomis* has a 'heterogeneity of styles' and he proposes that Philippus compiled it partly from authentic material left over from other works by Plato, such as *Laws,* and partly from his own 'somewhat appalling literary efforts' (p.14). I suggest that this would explain the presence of a rarely found word, *diagramma*, being used in a different context to that in the other Platonic dialogues where it appears. Plato could have rewritten some text which contained *diagramma* and abandoned his earlier version. In this case Philippus could have appropriated the 'authentic material' for his own use. Although no firm conclusion can be reached about the authenticity of the *Epinomis* I argue that an analysis of the use of *diagramma* can play a part in the discussion.

More on the meaning of *diagramma*

Throughout my work on *diagramma* I have taken it to have a connection with constructions and theorems and I offered some support for this view from ancient and modern writers, including Plato. I argued that Plato's choice of the word *diagramma* in connection with the acquisition of knowledge implies that it means something more than a diagram, since he is critical of geometrical diagrams which merely show figures. I suggest that my work offers an enhanced understanding of the specialized meaning of *diagramma* and I demonstrate this in more detail by comparing what I designate as a 'figure', the lines and curves of a drawn geometrical diagram, and *diagramma*, with its extra implications.

I consider the example of Pythagoras' theorem. A figure of a right-angled triangle has the disadvantage that it depicts a particular triangle, say one with sides of 3cm, 4cm and 5cm. Although it is possible to imagine it as a representation of any right-angled triangle, if no assumptions are made with regard to the actual lengths, the figure by itself gives no information about a general relationship between the lengths of the sides. The construction of lines and the application of geometrical reasoning are needed to demonstrate Pythagoras' theorem[15]. My claim is that *diagramma* refers to a 'figure' plus the added associations with constructions and proofs. This has a significance for the acquisition of philosophical knowledge.

Just as someone who sees, or draws, a geometrical figure can gain some knowledge from it, for example that in a right-angled triangle the hypotenuse is the longest side, so anyone can gain ordinary knowledge in several ways. An apprentice carpenter can gain knowledge of carpentry, for example how to construct a table, by watching an expert make one. However, such knowledge is not eternal and unchanging and new construction methods may be devised. Philosophical knowledge of the Forms *is* eternal and unchanging and so correlates with the knowledge demonstrated by the proof of a theorem. To refer to a 'diagram' is to lose this connection with the unchanging knowledge which *diagramma* implies.

However, most translators give 'diagram' or 'geometrical diagram' and I suggest that this provides a reason why *diagramma* has not produced much interest in academic writings. The question remains of why the word 'diagram' is given as a translation so often, without any consideration of a possible deeper meaning. It is possible that it has not been more fully explored by modern scholars because of its similarity to the English word 'diagram'. This

[15] See Appendix for a version of a proof of the theorem.

word describes, at a superficial level, what is being discussed in the passages in which it appears in Plato's texts. Diagrams are involved in all of the accounts and there is no immediately obvious reason to go beyond the word which mirrors the Greek so closely.

An obvious criticism of the foregoing explanation is that Plato is studied in many countries where English is not the first language. Scholars in such countries could have identified *diagramma* as an unusual word with no obvious link to any modern form of the word and in such cases I would have expected to find some analysis of this. Although I have restricted myself to English writings, important work in other languages is normally translated and made available in English and so would be available for comment in accessible journals or books. However, a search of an on-line translation site reveals a surprising result which could explain the lack of references to the word. A check of languages which use Roman script shows that there is a word which corresponds to our 'diagram', and which is similar to the Greek *diagramma*, in a whole range of languages. The form 'diagram' is used by the Danes, Dutch, Hungarians, Norwegians and Poles in addition to English speakers, including those in the USA. Italians and modern Greeks use the word '*diagramma'*, and there are only slight variations adopted by the French (*diagramme)*, Germans (*diagramm*) and Finns (*diagrammi*). The Portuguese and Spanish use *diagrama*. Recognisable as being close to the Greek *diagramma* are the words used by Romanians (*diagramă*), Swedes (*diagrammet*) and Turks (*diyagram*). Although users of most non-Roman script languages, such as the Chinese, use different words, the Japanese use a word which when transliterated is *daiagurama*. Scholars from all the countries where there is a word for a diagram which is almost the same as the Greek *diagramma*, could use, not entirely incorrectly, their version of the word for diagram, and see nothing unusual in the Greek term. A possible explanation for the closeness of the spellings across so many languages is given by an on-line etymological dictionary which states that the first recorded use of 'diagram' in modern English is in the 1610s CE. It is given as coming from the Latin *diagramma* and the French *diagramme*, with the following definition: 'a geometric figure, that which is marked out by lines'. It is likely that it is from these sources that so many similar words have evolved. Such widespread similarity with the Greek *diagramma* could explain the lack of comments about this rare word.

Insights given by *diagramma*

In this section I summarize my findings with regard to Plato's use of *diagramma* and I show how they can be correlated with the insights obtained from my analysis of the distant object analogy. Taking note of the use of *diagramma* has further added to our understanding of Plato's philosophy. Tracking the occurrence of the unusual word allowed me to join together

several passages from different dialogues to give a complete picture of how best to obtain knowledge. It could be argued that a single word is less likely to be noticed than the distinctive distant object analogy which I examined earlier but I offer a reason why this is not necessarily the case. I have claimed that the word is both specialized and rare and so would stand out for those who understood it. I have suggested that these are philosophers.

The evidence for the use of *diagramma* as a specialized term, aimed at people who understand geometry, is supported by its absence from the *Meno.* It may appear strange to use as evidence the lack of a word, especially one that is only rarely found, but the *Meno* is a dialogue where *diagramma* might be expected to be found. It contains many references to geometrical diagrams, as they are used to demonstrate *anamnesis*. I have argued that it is absent because the assumption is that the slave boy is ignorant of any geometry and to use an advanced geometrical term would undermine the argument. Although this gives further support to the notion of *diagramma* as what Netz (1999, p.276) describes as jargon, it raises the question of who the intended recipients of the use of such a word are in the other dialogues.

If Plato is aiming his words at philosophers it is first necessary to consider the people present with Socrates in the dialogues. The following is based on information researched by Nails (2002). She lists characters alphabetically, distinguishing between speakers and those who are merely present. Forty eight people are named for the dialogues which contain the word *diagramma*, including the *Epinomis.* Of these only thirteen are known to have any connection with geometry or philosophy, including sophists. There are also some unnamed characters, for example in the 'group coming toward us' (*Theaetetus,* 144c), but it is unlikely that these were considered to be important by Plato, since they are unnamed, and so are not likely to be philosophers. This means that there are very few philosophers present in the discussions with Socrates, most of his audience having no connection with geometry or philosophy. It should be noted that Socrates is not present in the *Epinomis* but, unlike the distant object analogy case, there is one participant who is present in more than one dialogue containing *diagramma*. Euclides is in the *Phaedo* and *Theaetetus.* This is such a small percentage of the named characters, that I suggest that it is the philosophical reader whom Plato is targeting. The members of his Academy, other philosophers of his day and modern philosophically minded people can all benefit as they read the advice given in the six non-spurious dialogues containing the word *diagramma.* The use of a specialized term designed to be fully understood only by philosophers provides value in two additional ways. Firstly, it separates philosophers from non-philosophers and, secondly, it implies that the knowledge

under investigation is philosophical knowledge, or knowledge of the Forms. In some of the dialogues under consideration a word which indicates 'Form' is used but in the others it is not. This means that the use of *diagramma* offers support for those who claim that the Forms are implied in some dialogues even though they are not specifically mentioned in them.

I have shown that Plato's use of *diagramma* reinforces the importance which I claimed for the distant object analogy. I chose *diagramma* as an example of theoretical geometry but there are several features with regard to Plato's use of it which are similar to those found in my other investigation. In both cases the philosophical discussions associated with the geometry are continued across several dialogues. Using the geometrical reference to track all the relevant passages reveals a more complete analysis of each of the topics, the estimation and choice of pleasures and pains, and the acquisition of knowledge. I have argued that both references allow Plato to separate philosophers from non-philosophers, since only the former fully understand the geometry and want to learn from the philosophical discussions. However, there are some aspects connected to both the distant object and *diagramma* which non-philosophers would understand. I have mentioned that the repetitions of a geometrical reference could act as a reminder of the other texts where it appears and this is the case for both the examples which I have explored. I have suggested that Plato was aiming his work at his readers, rather than Socrates using the examples for the benefit of the participants in the dialogues. This can be connected to Plato's views, given in the *Phaedrus*, that writing can act as a reminder. I explore this concept more fully in the next chapter, when I discuss cross-dialogue connections. I also examine other topics of interpretation which I show can be better understood by taking note of Plato's use of geometry.

Summary of Chapter Five

In following my overall aim, to show that Plato's use of geometry has an importance which has been overlooked by those who ignore his geometrical references, I examined the final occurrences of the term *diagramma.* In the *Euthydemus, Phaedo* and *Republic* there is some positive advice about how knowledge can be gained. It can be discovered and *anamnesis* offers a way to do this. However, reason must be applied and the use of dialectic is also mentioned, giving further evidence that Plato is addressing philosophers. I have shown that these are likely to be the readers of the dialogues.

When examining the final use of *diagramma*, in the *Epinomis,* I was able to use the rarity of the term for a further purpose. Since the use of the word does not follow the pattern set in

the other dialogues it is possible to use the anomaly to substantiate the views of those who claim that the *Epinomis* is a spurious dialogue. There is also some support for the hypothesis that the writer was Philippus of Opus.

I have argued that *diagramma* should not be translated as 'diagram' without some fuller explanation, and I have stressed that taking note of its use offers insights into Plato's philosophy. I examined possible reasons why there is little interest in the word, and found that in many languages a word for a diagram exists which is similar to *diagramma*. I ended by summarizing the insights given by noting the presence of *diagramma* in the six non-spurious dialogues explored.

These conclusions indicate the importance of considering Plato's use of geometry, demonstrated by examining, in Chapters Four and Five, the word *diagramma* following the analysis, in Chapters Two and Three, of the distant object analogy. My work shows that a contemplation of Plato's geometry can reveal aspects of his work which are missed by those who omit the geometrical references from their commentaries and other writings. Thus, I obtain a positive answer to the first part of my original question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how? I have provided several instances to demonstrate how the philosophy is enhanced and in the next chapter I add to these as I explore the impact of my findings on various theories which concern the best way to interpret Plato's dialogues.

Chapter Six

The geometrical references applied to Platonic interpretation

6.1 Cross-dialogue connections 146

Some current views 147

An assessment of the opposing theories 149

The repetitions as reminders 151

6.2 The reception of the dialogues 153

Plato's audience outside the Academy 153

Geometry and discrimination 156

Some roles for the dialogues 159

6.3 Form and content 161

Plato's use of the dialogue form 161

Socrates and the geometrical content 163

Plato's other geometrical references 165

Summary of Chapter Six 168

In this chapter I look at the relevance of Plato's geometry for some theories of dialogue interpretation, reinforcing the importance of taking note of his geometrical references. I use several aspects of my findings and apply them to some current views. In my work so far the thoughts of other scholars have been in the background. I have drawn on their writings but I have concentrated mainly on my assessment of the content of the dialogues and how Plato made use of the geometrical references. I now bring other authors to the fore as I examine the implications of Plato's geometry for modern interpreters of his dialogues. Much has been written on every aspect of Platonic interpretation and there is a wide variety of critical resources available on which I can draw for my analysis. When I seek to link my findings to existing interpretive theories of Plato's work, I find some are more compatible than others. While it is important to present both sides in any debate I give greater weight to those authors who are sensitive to the historical and cultural context of the dialogues and offer integrative readings, as I have done. However, I make an exception in the case of those who argue for taking each dialogue as a separate entity. Since I have stressed the value of the

repetition of each geometrical reference across several dialogues I present a full account of the view that each dialogue is autonomous in order to demonstrate that my work refutes this. I also look at some authors who suggest that there are cross-dialogue references and I show how my work offers support for this theory. I then explain my claim that the geometrical repetitions fit with Plato's views about writing.

I then consider the initial reception of the dialogues. I use Plato's choice of different types of geometrical examples to explore how he could connect with a wide audience. I claim that the use of the distant object analogy gives some support to the theory that the dialogues were available outside the Academy. I also suggest that the use of the term *diagramma* demonstrates an awareness, by Plato, of some of the difficulties which would result from reaching a wider audience, if he really wished to restrict some of his thoughts to philosophers. There are also various purposes or uses which the dialogues may have had for both those outside the Academy and those within. I consider them as a teaching aid and analyse some possible lessons which can be learnt from the use of the geometrical examples. I then look at another role for the dialogues, as an advertisement for the Academy, and assess this in the light of the geometrical content.

In the final section I consider the form of the dialogues in order to examine the role of Socrates, offering the suggestion that the use of geometrical examples in the discussions indicates that they originate from Plato rather than Socrates. Finally, I assess some further geometrical references found in the dialogues. In order to give a thorough account I have restricted my earlier discussions to two examples, the distant object analogy and the term *diagramma*. Although I have shown that taking note of their occurrence enhances our understanding of Plato's philosophy there would be limited impact if only two geometrical examples were used in the dialogues. I examine some of the many other geometrical references and, although I cannot analyse them fully, I indicate how they could provide material for further investigation.

6.1 Cross-dialogue connections

When assessing interpretation theories some writers consider Plato's use of the dialogue form, and assess the consequences of this method of communication. For many years a popular notion was that such a form would not allow any cross-dialogue referencing to be possible. I look particularly at the views of Clay (1988). However, a few authors have announced that they accept some connections between dialogues, and there have been investigations into how Plato could have used more than one dialogue to complete a

discussion. I have already referred to the works of Miller (1986) and Kahn (1996 and 2003). I give here the views of those who think that each dialogue should be treated as a separate, complete, entity and then record statements by those who disagree with this stance. I present both sides without critical comment before I assess the viability of each in the light of my findings. I then show that the repetition of the geometrical references substantiates a claim about Plato's views on readers, based on his account of writing in the *Phaedrus.*

Some current views

I examine first the writers who consider how the dialogues were initially received and show that there is no consensus of opinion about this. Charalabopoulos (2012, p.11), for example, declares that the original audience took each dialogue to be separate: 'Each dialogue should be read *primarily* as an independent, integral piece of dramatic literature, appreciated on its own terms, since that is how the original public first came to know it'. However, Howland (1991, pp.194-5) claims that traditionally the dialogues were taken to form a well-ordered arrangement of interconnected conversations which gave different perspectives on the same issues. Presumably Charalabopoulos is thinking of the fact that there was a space of time between the writing of any two dialogues and Howland is considering a later period when scholars had access to the whole corpus. Regardless of this, both views persist in modern interpretations of the dialogues.

I start with the views of a representative sample of modern authors who are adamant that there is never a connection between different dialogues, beginning with Clay (1988) who gives a thorough criticism of the possibility of cross-dialogue references. He states (pp.145-6) what he calls a law of Platonic writing: 'No Platonic dialogue can ever explicitly harken back to an earlier Platonic dialogue'. He explains that he means that when one dialogue points back to another it does so as a conversation referring to an 'earlier, unrecorded conversation'. He specifies that when a conversation in one dialogue does refer to an earlier recorded conversation, 'Plato is at pains to dissociate the two'. He later (p.147) admits that it would have been possible to have Socrates, or some other character, refer back to an earlier recorded conversation but he insists that 'Plato avoids even this'. After some further discussion Clay (pp.155-6) cites Monro's 'Law' concerning Homer: that the *Odyssey* never refers to any incident in the *Iliad,* and he concludes 'nor does any Platonic dialogue ever depend on an argument established in another'.

Other authors take a similar view, and I summarize a cross section of these, taken from the last few decades. Bowen (1988, pp.64-5) looks at the approaches to Plato's dialogues taken

by philologists and philosophers. He notes that the former take each dialogue to be a literary unit and the latter think the dialogues are presented as philosophically independent, with 'no footnotes or cross-references that make the arguments interdependent'. Arieti (1995, p.121) states emphatically that 'It also seems to me a fundamental error in interpretation to require the understanding of one dialogue to depend on what is said in another'. Gill (2002, p.155) states that each dialogue is differentiated 'by differences in the *personae* (apart from Socrates, in those dialogues where he is the main speaker), context, subject matter, and sometimes mode of dialectic'. He claims that there is a 'complete absence of explicit references between dialogues', although he excludes from this statement those dialogues presented as being sequential. He suggests that any apparent allusions to other dialogues 'are generalized and do not refer to a specific text'. Such opinions, expressed in the last few years, reiterate views about the separation of the dialogues which have been held since the nineteenth century.

Some other writers are less convinced that the dialogues can *only* be read independently, and note that there are some cross-references. Nails (1995, p.25) discusses the views of contextualists, who concentrate on Plato's literary skills, and she notes that some look at the aesthetic unity of each dialogue. They take the dialogues to be 'autonomous, containing almost no cross-references' (p.25, note 36). This is similar to Blondell's view. She claims (2002, p.5) that in Plato there is a 'paucity of cross-references in his dialogues'. Other writers take the view that after considering one dialogue there is some value in looking at others. Morgan (2002, p.184) suggests that we should start with localised readings, before allowing that 'Plato challenges us to move from local to systemic readings'. Weingartner (1973, p.13) initially states that 'the dialogue form has no room in it for footnotes directing the reader to the author's previous writings'. He then admits that the form does allow the author to indicate that what he says in one dialogue calls for knowledge that must be gained from another work. He concludes: 'The reader of a Platonic dialogue must thus be sensitive to the cues Plato weaves into his compositions and must pursue them'. Rowe (2007, p.23) thinks that we should always begin by trying to understand a dialogue by itself but he then suggests that a full understanding is only achieved by cross referencing to other dialogues. These writers show an awareness of the possibility of interpreting each dialogue singly but they admit that Plato may be best understood by taking several dialogues together.

Other scholars take a more positive attitude about the use of cross-references in the dialogues. When Nails (1995) gives her own views (p.223) she suggests that Plato picks up some themes repeatedly across the dialogues so that premises and conclusions accepted in

one dialogue may be refuted in another. She claims that these are not oversights but 'instances of genuine philosophical problems that deserve to be argued further'. There are also scholars who give support to cross-dialogue references but appear reluctant to take such a stance. Sprague (2000, p.6) states 'I might as well confess to a conviction that all the dialogues of Plato should, as much as is humanly possible, be read in the light of all other dialogues'. The way that Sprague expresses herself shows how ingrained is the notion that each dialogue is a separate entity. However, there have recently been some works which have taken specific examples of cross-dialogue references rather than confining their remarks to generalities.

One author who demonstrates connections between dialogues is Miller (1986, pp.168 ff), as I mentioned earlier when describing my method of approach with regard to Plato's geometry. He later (1988) offers a criticism of Clay (1988), and the views which I stated above, by again considering a connection between the *Parmenides* and the *Republic* (pp.162-4). He suggests that the *Parmenides* does not lead away from the teaching of the *Republic* but offers a new level of insight into it and he wonders (p.164): 'if the sum of the dialogues, ostensibly a plethora of isolated reflections, can be construed instead as a set, or sets, of fresh beginnings that lead the responsive reader, in measured stages, into philosophy'. His work is followed by that of Kahn (1996 and 2003), also discussed earlier. He shows that new insights can be obtained when discussions are tracked across several dialogues and states that the reason for his work is to refute the theory concerning 'the absence of cross-reference between dialogues' (2003, p.299). He concludes (p.312) that 'their literary autonomy does not signify their philosophical autonomy'. These authors hint that there is a possibility that Plato made a conscious decision in presenting discussions which are spread across several dialogues.

An assessment of the opposing theories

To say that in a series of dialogues there can never be any explicit references across different dialogues is, in one way, correct. A character in the *Laws*, for example, could never say words such as 'let us refer to a conversation that occurred in the dialogue entitled *Gorgias*'. This contrasts with the situation where a philosopher writes a book, explaining various doctrines, and refers to earlier works, perhaps to adjust previous views. If Plato had chosen to write prose detailing his doctrines, as did later philosophers such as Kant, then he could say, in a book entitled *Laws*, something like 'as I said in my earlier book, *Gorgias*...'. In this way he could connect together different parts of a discussion, allowing him to offer new thoughts on a topic. I suggest that my work demonstrates that he achieved such cross-dialogue

references implicitly. I will explain this as I examine the views given above for and against such referencing.

My first criticism is directed at those, such as Bowen (1988, pp.64-5), who state that scholars are able to find few, if any, repetitions which can be connected by footnotes directing a reader across dialogues. Both the geometrical examples which I have studied demonstrate the possibility of such footnotes. With regard to the distant object analogy, several versions of the texts do give footnotes which cite the other occurrences of the same analogy. However, authors appear not to follow these up to investigate why Plato would include such repetitions. My findings indicate that the discussion regarding the measurement of pleasures and pains is greatly enhanced when all the texts containing the analogy are taken together. Omissions in one dialogue are addressed in another and it is only by considering the three different references to the distant object that a complete assessment of the problem of different sized pleasures can be made. A similar situation exists with the repetition of *diagramma*, since Plato's advice on how to acquire knowledge is incomplete until all the non-spurious dialogues where it appears have been considered. However, with regard to footnotes, the *diagramma* case is rather different. One word could not normally appear in a footnote giving other locations of the same word, since the number of references would rapidly exceed the amount of Plato's original text. My argument is that *diagramma* is so rare that the few other occurrences of it should be cited. Without any reference to these uses of *diagramma* we have a situation where the scarcity of the word has not been explored by writers, and the usual translation of 'diagram' masks the fact that it is a specialized word. My work shows, therefore, that the footnotes which are present, for the distant object analogy, should be investigated thoroughly and further footnotes should be provided, for *diagramma*. I am not limiting my appeal for more notice to be taken of the repetition of geometrical references to these two examples. I show in my final section that Plato repeats many other terms taken from geometry and although there is no space to explore these I suggest that they would also benefit from cross-dialogue citations.

Another problem comes from the fact that the people present in one dialogue are rarely present in another. This is mentioned by Gill (2002, p.155), for example, in support of the theory that there are no cross-dialogue references. I argue that this only prevents such references by Socrates, or some other speaker, within the context of the scenarios of the dialogues. Few scholars note that the reader is always present for all the dialogues. This enables Plato to use cross-dialogue examples for the benefit of his audience. I will return to the question of whether his expected audience were readers or listeners in the next section

of this chapter, but for convenience I will refer here to readers. There may be some dialogues which stand alone, with no connections to any of the others, but my work indicates that this is not always the case, since I have explored geometrical repetitions across several dialogues. I hold that my findings demonstrate how the reader gains from cross-dialogue references, offering support for the theories of writers such as Miller and Kahn. In addition I suggest that Plato included memorable geometrical examples to ensure that his readers found the intended connections between the dialogues. This conforms to his views on writing, found in the *Phaedrus*, as I show in the next section.

The repetitions as reminders

There has been much discussion, in modern scholarship, about whether Plato's dialogues were initially heard or read. Plato lived in a time of transition between oral and written transmission of texts and there are several references in Plato's dialogues which appear to support the suggestion that it was common for texts to be read to an audience. One example is in the *Theaetetus* where Euclides and Terpsion listen while a slave re-enacts a drama of Socrates' earlier conversation. The author of the work, Euclides, does not read it himself and Tarrant (1996, p.133) suggests that such a presentation to an educated company might have been common. However, Plato mentions books being bought, for example in the *Phaedo* **(**98b) where Socrates mentions acquiring and reading the books of Anaxagoras. Since Plato was aware of the sale of books written by earlier philosophers it is likely that he would expect his own works to be read. I argue that my work on cross-dialogue references supports his views on writing.

Although he reports that some people claim that the new invention of writing will allow readers to have better memories, Plato fears that writing can only serve as a reminder. He describes, in the *Phaedrus*, how an ancient king of Egypt, Thamus, also called Ammon, is told of writing by its inventor, Theuth. The claim is made that it will 'make the Egyptians wiser and will improve their memory; I have discovered a potion for memory and for wisdom' (274e). However, Thamus, criticizes the new invention of writing:

> In fact, it will introduce forgetfulness into the soul of those who learn it: they will not practice using their memory because they will put their trust in writing, which is external and depends on signs that belong to others, instead of trying to remember from the inside, completely on their own. You have not discovered a potion for remembering, but for reminding; 275a

This could be a criticism of a view given by Aeschylus, in the fifth century. In *Prometheus Bound* (460-1) Prometheus states that he invented 'combining letters into written words, the

tool that enables all things to be remembered'. Plato is clear that writing is not 'a potion for remembering'.

It is easy to appreciate how, in the ancient world, people could welcome the presence of the written word as a way of remembering knowledge gained. Plato demonstrates a changing attitude by his use of a clever play on words in his analysis of writing, as Derrida (1981, p.70) notes. Writing is said to be a *pharmakon* (*Phaedrus*, 274e) and *pharmakon* can mean both medicine and poison. Derrida continues (p.100) to say that the ambiguous word is well chosen because writing is first proposed as an aid to memory but then shown to diminish it. Miller (2007, p.148) assesses Derrida's work and explains that the ambiguity is usually lost in translation. This is because the word is either translated in two different ways so that the original wording is lost, or it is translated in the same way each time so that the use of an ambiguous word is not obvious. However, it is clear that Plato has reached a conclusion that, once writing becomes more widespread, people will lose the excellent memories which a pre-literate culture possessed.

It is thus possible that Plato feared that his readers would not easily remember a previous discussion whilst reading a later account, written some years after the first. I claim that my work highlights the distinction between 'remembering' and 'reminding'. It is likely that the repetition of a distinctive example would act as a reminder that an earlier argument existed. The reference to the false perception of a distant object stands out more than one discussion, among many discussions, found in the dialogues. The reader could then either bring to mind the earlier philosophical argument or search through the texts until the correct one is found. *Diagramma* serves a similar purpose. I have argued that only philosophers would fully understand it, but it would remind people who were comfortable with the word of its earlier occurrences. I see this as one reason why Plato uses geometrical repetitions, to include something which stands out from the surrounding text, in order to *remind* his audience of the other discussions associated with them. Strauss (1964, p.60) states that: 'nothing is accidental in a Platonic dialogue; everything is necessary at the place where it occurs'. The geometrical repetitions, placed next to specific discussions, can act as reminders to help a reader to notice arguments carried across several dialogues, thus demonstrating Plato's belief that writing serves as a reminder rather than as a method of improving memory.

After the comments about writing Socrates criticizes those who,

> ...thinking that writing can yield results that are clear or certain, must be quite naive and truly ignorant of Ammon's prophetic judgment: otherwise, how could they

> possibly think that words that have been written down can do more than remind those who already know what the writing is about? 275c

Whether or not the use of books affects memory ability, it is clear that Plato thinks that writing can be used to *remind* a reader of other works. The use of repetition of the geometrical references, reminding a reader of previous discussions which are pertinent to the passage being read, then implies that Plato realised that his books might be read, rather than only heard. However, throughout this analysis I have avoided speculating whether the contemporary readers were restricted to the Academy or not. In the next section I explore opinions about the intended audience for the dialogues and demonstrate that my findings can be used to clarify some of the views which are currently held.

6.2 The reception of the dialogues

Hershbell (1995, p.29) states that 'it seems impossible to ascertain for whom Plato's dialogues were composed'. However, whatever Plato's intentions may have been, I show that he was aware that people outside the Academy could have access to his work. I then argue that his choice of different types of geometrical examples allows him to be understood by a wider audience than the Academy alone. There is an accompanying problem to be examined if, as many scholars have claimed, he wanted to restrict some of his ideas to those who were worthy enough, and able enough, to grasp his thoughts. I discuss different theories about how Plato could exclude some people from understanding his doctrines, and show how his choice of *diagramma* offers support for one of the proposals, the theory of hidden doctrines. If the dialogues reached a wider audience than the Academy they may have had additional purposes and I show how the geometrical examples could play a part in these.

Plato's audience outside the Academy

I have indicated, in the previous section, that Plato was aware that books could be bought and read privately in the Athens of his time. There are references in other ancient texts about people outside the Academy reading a dialogue and Riginos (1976) gives several anecdotes that support this view. One example (p.180) is first recorded by Callimachus, in the third century BCE, and describes how the doctrine of the *Phaedo* led Cleombrotus of Ambracia to commit suicide. There are also further references in Plato's work about books being bought, for example when Socrates talks about the books of Anaxagoras in the *Apology* (26d-e). He says that they are available to buy 'from time to time for a drachma, at most'. Morgan (1999, p.48, note 10) states that a drachma would be approximately one day's wage

for a labourer so that not many people would be able to afford to purchase such books. However, those with sufficient means could buy them and such people would not necessarily be members of the Academy.

Those who could not afford to buy books, or had no desire to do so, could still have access to their contents. In addition to the *Theaetetus*, where Euclides says that a slave will read the account of the conversation between Socrates and Theaetetus (143b), there is considerable evidence that oral transmission of texts continued for many years after private reading began. Thomas (1992, p.4) notes that, until the second century CE, even if a text was written it was read aloud. Hershbell (1995, p.31) goes further and claims that the oral-aural culture remained, historically, until book printing in the fifteenth century CE triggered more widespread reading. I look next at some documentary evidence for the continuation of oral transmission of books in Plato's Athens.

There are many textual references which support the reading aloud of texts to small groups in the fourth century BCE, and I present first a few which refer to Plato as being part of the audience. Diogenes Laertius (3.35) says that the enmity between Plato and Antisthenes was brought about when Antisthenes invited Plato to be present while he 'read something that he had composed'. Plato was told that it would be about the impossibility of contradiction and asked how Antisthenes could write on this since the argument refutes itself. Riginos (1976, p.168) quotes Cicero Brutus, in the first century BCE, who says that Antimachus held a reading of his poetry but everyone, except Plato, left before he had finished. Statements such as these indicate that some, at least, of Plato's dialogues were likely to have been heard initially, and Plato would be aware that this could happen. However, the audiences described in these passages were small and it is possible that the readings were confined to a close few, such as chosen members of the Academy.

Other writers suggest a different type of oral presentation, which would reach a wider audience. There are claims that Plato intended the dialogues to be performed in a manner similar to the plays with which he would have been familiar. Ryle (1994) thoroughly explores this notion. He suggests (pp.28-32) that the dialogues were recited to audiences as dramas, and considers the possibility that Plato took the role of Socrates. He arrives at the conclusion (pp.33-4) that the audience could have been people at the Olympic Games. Although Ryle offers support for his theory, such as the approximate uniform length of the middle dialogues to conform to rules at the Games (p.39), his highly speculative theory about the dialogues has drawn much criticism. Sayre (1992, pp.224-6) notes that there are other writers who also think that the dialogues may have been intended for performance, and cites Miller (1986),

but concludes that there is no evidence for the performances that these authors suggest. Waugh (1995, p.73) withholds any decision about Ryle's proposal concerning public performances but states 'it is hard to deny his claim that Plato saw that the way to teach people how to think and speak philosophically was to compose dramatic examples of such discussions that they could hear and in which they could, at least vicariously, participate'. I conclude that the possibility exists that the dialogues could have been heard in some way by large groups of people.

The availability of books for those who could afford them, or borrow them, and the possible readings of texts to groups both imply that non-Academy members had access to Plato's dialogues. Based on his knowledge that the works of writers such as Anaxagoras were widely available years after they were written, Plato would be aware that, in addition to non-Academy members reading the dialogues in his lifetime, there could be readers in the future. These buyers of the dialogues would be unknown to him and outside his sphere of influence. He would be unable to specify who should have access to his work and the dialogues would, therefore, be available to both philosophers and non-philosophers. We might, therefore, ask the question: did Plato specifically compose some parts of his work for non-philosophers?

Several authors express a belief that Plato wrote for more than one audience. Strauss (1964, pp.52-3) declares 'We may conclude that the Platonic dialogue says different things to different people – not accidently, as every writing does, but that it is so contrived as to say different things to different people'. Kahn (2000a, p.192) makes the same point when he suggests that Plato writes at different times 'for different audiences and with different aims'. Thesleff (1993, p.37) makes a similar point when he says 'we may take it for granted that many of Plato's dialogues were addressed at the same time to an initiated and a less initiated audience'. Rather than taking it 'for granted', I claim that the two geometrical examples which I have examined demonstrate the two different attitudes which Plato adopts towards his audience.

I consider first the distant object analogy. It may not be the best one to use in each dialogue but it gives a clear picture of the way in which pleasures and pains can appear to be smaller than they really are. This is something in which everyone has an interest, whether they are philosophers or not. Robb (1994, p.234) makes a valid point when he states: 'Like every great writer, Plato must conform somewhat to his audience's expectations'. Associated with the discussion on pleasures and pains, of universal interest, is an analogy which is clear to everyone, since we are all aware that distant objects look deceptively small. This is not something that needs philosophical insight, or mathematical training. Herrmann (2007,

p.278) states 'Any author who intends to communicate his thoughts can only ever hope to succeed if, to some degree, he speaks the language of his audience'. Although philosophers would also be interested in choosing pleasures and pains correctly the use of such a simple example implies that Plato intended some parts of his work to be understood by non-philosophers, presumably outside the Academy.

There is another aspect to the use of the distant object analogy, which may have been part of Plato's intentions when he chose it. The general public of his day would not know how to calculate the true size of the distant object, since such a calculation depends on discoveries in geometry made within Plato's lifetime and therefore 'cutting edge' mathematics. The method possible in fourth century Athens, without using as yet undiscovered trigonometry, depends on proportion[16]. This had only recently been thoroughly explored by Eudoxus and Plato would be aware of the possibility of a calculation, whether or not he himself understood it. It is possible that the achievements of Eudoxus were recognised within Athens so that the non-philosophical reader of the dialogues would be aware that a calculation of the true size of a distant object might be possible, and perhaps infer that philosophers could also assess the true size of distant pleasures and pains. However, such readers would realise that these feats were impossible for anyone without philosophical training, with the associated lessons in geometry. This means that, although the use of the analogy gives some understanding to non-philosophers, a full understanding is withheld from them. I next consider how Plato may have wanted to restrict some parts of his work more fully and how his use of geometry aids this.

Geometry and discrimination

The passages associated with the analogy, for all to understand in part, contrast with the more specialized language of geometry, such as the term *diagramma*, which, I argued earlier, would only be grasped by philosophers. I show here that this supports the notion that Plato wanted to restrict some of his thoughts to those whom he considered worthy to receive them. This view is found in ancient times and has been adopted by some modern scholars. Within this group there is a division between those who think that Plato did not reveal his doctrines in the dialogues and those who claim that he included them but wrote in such a way that the doctrines are hidden. In the latter case it would be necessary for the intended readers to uncover them in some way. I argue that the use of the word *diagramma* allows

[16] As shown in the Appendix.

this and so demonstrates that there was no necessity to withhold the doctrines from the dialogues. I start by giving a brief description of both theories.

I present, first, an account of the 'unwritten doctrine' theory. Tigerstedt (1977, p.65), in his survey of Platonic interpretation, notes that for many Neoplatonists, such as Plotinus, Porphyry and Iamblichus, Plato's philosophy was comparable to a mystery religion, revealed to Plato by the gods and, through Plato, to an elected few. Some modern scholars follow the Tübingen School esoteric hypothesis which claims that Plato only expressed his doctrines orally, to members of the Academy. Supporters of the theory look to Aristotle for support since he claims that Plato's account of the Forms 'differs in the *Timaeus* and in what are known as his *Unwritten Teachings*' (*Physics* 209b13-16). This is cited as evidence that there were doctrines known to members of the Academy, including Aristotle, that are not in the dialogues, and Plato is deemed to have wanted to keep such doctrines secret from some of his readership by not writing them down. If the doctrines were given orally to members of the Academy then we can only access them through comments made by later scholars, starting with Aristotle.

I find it difficult to support a suggestion that forces us to give more credence to what are effectively secondary sources, the writings of Aristotle and others on Plato's philosophy, than to Plato's original work. Aware that his books could be read many years after his death it would be strange if Plato decided to produce copious work which did not give his doctrines and ideas. This does not prevent him from deciding that some thoughts should only be given to a chosen few.

If Plato wished to restrict knowledge of his doctrines he would be faced with a problem. He shows an awareness that he could not apply any restriction on access to his work when he states, in the *Phaedrus* (275e), that once a discourse has been written down it 'roams about everywhere, reaching indiscriminately those with understanding no less than those who have no business with it, and it doesn't know to whom it should speak and to whom it should not'. Plato realises that if his books are available to anyone who can afford them he cannot determine who will read them and he must find a way to discriminate amongst his readers.

One method of discrimination is described admirably by Wittgenstein, in *Vermischte Bemerkungen* (1977, p.237), cited by Szlezak (2003, p.204) who gives a translation:

> If a book is written only for a few, this in itself indicates that only a few understand it. The book must automatically create a division between those who understand it and those who do not understand it...

> If you want certain people not to enter a room, you must put on the door a lock to which they have no key. But it is senseless to speak of this to them, unless your aim is to make them admire the room from outside!
>
> It is more decent to attach to the door a lock that is noticed only by those who can open it, and not by the others.

Szlezak (2003, p.205) describes the following characteristics which Wittgenstein's attitude has; there are certain insights which can only be grasped by certain people, there is a desire to exclude all except these and there is a belief that such an exclusion is feasible for an author. The doctrines would be hidden from some but not others. Szlezak later (2003, p.215) translates the views of Hegel, in *Werke* (1971, pp.76-7). He claimed that a doctrine was not secret in itself but 'hidden from those who have not sufficient interest in it to exert themselves'. My work offers support for such views since a geometrical reference, and in particular the word *diagramma*, would act as the lock mentioned by Wittgenstein. It fulfils his requirement that it can only be opened by the intended recipients, philosophers, and the fact that it is so often ignored by scholars, supports the notion that it 'is only noticed by those who can open it'. Using the term *diagramma* could mean that the arguments associated with such an unusual word would be ignored by those who were not the intended recipients of the advice given, on attaining knowledge. Thus, *diagramma* allows Plato to write for the few, the philosophers, who can understand his work.

With regard to the Tübingen School theory of interpretation, the use of *diagramma* makes the notion of having no doctrines written in the dialogues redundant, since such doctrines could be hidden from those unworthy to know them by using specialized terms. This would mean that when Plato uses a word such as *diagramma* he is targeting philosophers. Aristotle's reference to 'Unwritten Teachings' can still be accepted since it is likely that Plato had many oral discussions with members of the Academy. However, there was no necessity for Plato to exclude *all* doctrines from his dialogues if he could restrict sections of his work to a chosen audience. An implication of such a restriction is that he knew that his work could be accessed by non-philosophers, as does his use of the distant object analogy, which has some relevance for everyone.

Whether non-Academy members had access to the dialogues within Plato's lifetime is unknown but it seems likely that they did so, and Plato knew that texts were available long after an author's death. This means that non-philosophers, contemporary and those of the future, could access his work in addition to philosophers. I have considered two very different geometrical references that Plato used in his dialogues and shown that each is necessary in order to distinguish between these two audiences for the texts. I conclude that

some of his geometrical examples, such as the distant object analogy, were aimed at anybody who had access to his dialogues and the associated topics were those which would be of interest to everyone. However, the advanced geometry connected with the discussion means that a full understanding is withheld from the general public. Other references, such as *diagramma*, are associated with passages which would only be understood by those considered worthy of the insights they revealed, since they involved philosophical knowledge. Thus, Plato's use of geometry has fulfilled the purpose of allowing his dialogues to be appreciated by non-philosophers and philosophers and it has allowed some discrimination to take place. In addition, the awareness that there could be a wide audience for his work means that Plato could plan other uses for his dialogues, applicable to either Academy members or the general public. I examine these next.

Some roles for the dialogues

I start with the view that the dialogues were teaching aids. Originally this role was possibly for Academy members, although there is no doubt that readers through the ages have made use of them for this purpose. Since ancient times writers have looked at different aspects of the dialogues to see how they fulfil a teaching role and I cite one ancient author and one modern one to demonstrate this. A suggestion given in the *Anonymous Prolegomena to Platonic philosophy* (4: 15. 1-40), which Westerink (1962, p.xlix) dates to the early sixth century CE, may not be readily accepted by us. It is said that Plato used several speakers, 'otherwise, if it is always one and the same person teaching us, we might, so to speak, doze off'. On a more serious note, it is said that the dialogues 'reproduce dialectical disputation'. Many modern scholars would agree with this, and Frede (1992b, p.219) states unequivocally:

> The dialogues are supposed to teach us a philosophical lesson. ... A good part of their lesson does not consist in what gets said or argued, but in what they show, and the best part perhaps consists in the fact that they make us think about the arguments they present. For nothing but our own thought gains us knowledge.

This suggests that the type of teaching in the dialogues is not necessarily restricted to presenting facts. The use of *diagramma* supports such a notion since the associated discussions offer descriptions of how to achieve true philosophical insight. The knowledge that is gained when *diagramma* is mentioned is not factual but it is a basis for obtaining further knowledge. Instead of *giving* knowledge, in these instances, Plato is explaining how to *acquire* knowledge. Thus, the dialogues may have been designed as teaching aids but they were not necessarily designed to teach a series of theoretical facts.

There is also a view that the dialogues were text books. Robb (1994, p.236) suggests that students within the Academy could detect the errors of the interlocutors. My work on geometrical examples offers support for such a theory since, as the references are repeated across several dialogues, the associated discussions are also divided into separate parts. Considering the use of the distant object analogy, it is accepted that the dialogues containing it were written several years apart, with the probable dates as given earlier. This means that there would be time, after receipt of the *Protagoras,* for discussions which would reveal such omissions as I have noted above. The use of the same distinctive analogy, in the *Republic,* would signal a return to the same argument but leave some scope for further discussions among the Academy members before the *Philebus* appeared. The extended discussion across the dialogues thus mirrors the many conversations within the dialogues, where faults in arguments are gradually revealed. The same can be said of the advice associated with each use of the term *diagramma*, which would be fully understood by members of the Academy. The time spans between the production of the various texts would allow questions to be asked about further pitfalls in the search for philosophical knowledge. I conclude that Plato's use of geometry aids the teaching of a philosophical way of thinking.

Considering a wider audience than the Academy for the dialogues, a different proposition is that the dialogues could have been intended to advertise the Academy. Tarrant (1996, p.135) observes that much of the philosophical prose written before Plato's time was intended to advertise, to gain pupils, and a quote from Isocrates supports this. In *Antidosis* 87 Isocrates says that he has reaped rewards from his works since 'the writing and publication of them has won me distinction in many parts of the world and brought me many disciples'. I give some examples from authors, ancient and modern, in support of the notion that Plato's dialogues also acted as an advertisement for the Academy, before showing the value of the geometrical examples for this role.

Illustrative of stories in ancient texts which support the notion that Plato's dialogues served to attract pupils are those recorded by Riginos (1976). In one it is told how Axiothea read the *Republic* and then set out from Arcadia for Athens in order to become a follower of Plato, concealing that she was a woman (p.183). This is first found written in the fourth century CE, in Themistius' *Oratio* (23.295c). Also from Themistius' *Oratio* (23.295c-d), Riginos (p.184) gives the story of the Corinthian farmer who became a Platonist, leaving his fields and vineyards, when he read the *Gorgias*. Such stories suggest that the dialogues served to attract students to the Academy, whether that was Plato's intention or not. Arieti (1991, p.7)

suggests that fathers could compare the writings of Isocrates and Plato in order to choose a school for their sons.

Arieti is later (1995, p.123) very specific about the type of advertisement the dialogues could give to prospective students. He considers the 'intellectual milieu' of Plato's Athens and notes that there were many schools of rhetoric. This was because good speech making was essential in many areas of life. Several of the dialogues, such as the *Symposium* and *Phaedrus*, contain oratorical competitions and these would demonstrate that the Academy could teach rhetorical ability. If the rhetoric mentioned by Arieti could attract students to the Academy, to learn how to make speeches, then I suggest that the geometrical references could serve a similar role.

The geometry itself would make readers aware that there were mathematicians associated with the Academy. It is tempting to think of fathers, influenced by the geometry they read in the dialogues, sending their sons to learn at the place in which the texts originated. It is also likely that a geometrical example, such as the use of *diagramma,* would appeal to fledgling philosophers, and they would know that they could gain much by attending the Academy. As with most references to Plato's school, this is only speculation, and it may be that such a rare term would deter a prospective student, although the aptitude of such a student could, perhaps, be questioned. However, the use of geometrical references would indicate to worthy students that the Academy offered geometrical training alongside philosophy.

6.3 Form and content

I move now to a consideration of Plato's use of the dialogue form. I start by considering the problem of Plato's reliance on Socrates to provide most of the discussions in the dialogues, which makes it uncertain whose views are being recorded. A brief examination of some alternative theories in the first part of this section lays a foundation for my claim in the second part that the geometrical references, which illustrate the discussions, are unlikely to have originated with Socrates. I offer support for this by considering the geometry of Socrates' time. I then examine briefly some of the other geometrical content of the dialogues, to demonstrate that there is the potential for much further analysis on the use of Plato's geometry to enhance our understanding of his philosophy.

Plato's use of the dialogue form

The author known as Alcinous, usually dated to the second century CE, started his *Handbook of Platonism* with: 'The following is a presentation of the principal doctrines of Plato' (152.1).

Unfortunately, later scholars have disagreed over how Plato's doctrines are presented in the dialogues, and attempts to understand this have formed the basis for many different theories of interpretation. Since Plato never includes himself in any of the discussions he records, a situation sometimes referred to as 'Platonic anonymity' (Press, 2007, p.146), we cannot know whose doctrines are being presented. This problem with Plato's choice of the dialogue form is one for which my work has some relevance. Before explaining how this is the case, I present some of the conflicting theories.

On one side of the debate concerning the authorship of the doctrines found in Plato's text, is the suggestion that Plato uses the dialogue form, with Socrates as the main speaker, to record the views of the historical Socrates. However, when Taylor (2002, p.82) reports that some modern writers agree with this he observes that most stress the fictional re-creation of the actual words put into Socrates' mouth. Against this is the theory that all the doctrines belong to Plato but he uses Socrates to deliver them, often termed a 'Mouthpiece Interpretation'. Sedley (2002, p.38) relates such a notion to the views of ancient commentators. He suggests that they had no problem with the relationship between Plato as author and Socrates as main speaker. This is demonstrated by Diogenes Laertius (3.52) who claimed that Plato's 'views are expounded by four persons, Socrates, Timaeus, the Athenian Stranger, the Eleatic Stranger ... even when Socrates and Timaeus are the speakers, it is Plato's doctrines that are laid down'. There is also a middle ground, where some of the doctrines are seen as Socrates' and some as Plato's. This 'Moderate Mouthpiece Interpretation' (Corlett, 2005, p.18) has many variations, with some scholars suggesting that Plato started by recording Socrates' beliefs but gradually, over his lifetime, introduced his own doctrines. One problem with attempting to verify this is our lack of knowledge of the chronology of the dialogues. Unfortunately, arguments can be made for, or against, any of the interpretations concerned with the Plato/Socrates debate and there is no universal agreement about the best way to look at the dialogues.

Instead of attempting to assess the ownership of the doctrines found in the dialogues, some writers critically examine comments made by the Socrates of the dialogues. They offer explanations as to why Plato would invent them, demonstrating support for some version of a 'mouthpiece' interpretation. One example of this approach is given by Wolfsdorf (1999, pp.18-19). He explores discrepancies between what Socrates says in the dialogues and what Plato knows to be true. He cites the *Phaedrus,* 278e-279b, where Socrates talks of Isocrates who, he says, is young but will become skilled in rhetoric as he becomes older. Socrates then suggests that 'a higher, divine impulse' will lead Isocrates to more important things, 'for

nature, my friend, has placed the love of wisdom in his mind'. This implies that he will become a renowned philosopher. Wolfsdorf notes that the dramatic date for this dialogue must be before 399, and probably before 415. Since Isocrates was born in 436 he would be a young man at the time when Socrates is supposedly speaking. However, when Plato wrote the dialogue, around 370, Isocrates would be old and had not taken up philosophy, as Plato would know. Wolfsdorf claims that commentators do not say that 'Socrates is a faulty prophet' but that Plato 'wanted to take a jab at Isocrates' and used the Socrates of the dialogue to do it. This type of analysis concentrates on comments made, rather than doctrines espoused, and takes into account the years that elapsed between Socrates' lifetime and the writing of the dialogues.

In the next section I adopt a similar approach as I examine the use of the geometrical examples in the dialogues. Throughout my work I have referred to Plato's use of geometry but if he is recording Socrates' conversations, even if not verbatim, a more correct designation may be 'Socrates' use of geometry'. I examine Socrates' likely knowledge of geometry, and the dating of geometrical discoveries, to show that the references to the geometrical examples which I have examined can be seen as anachronistic, if they are assumed to originate with the historical Socrates. This implies that they are Plato's own ideas, which offers some support to a 'mouthpiece' interpretation.

Socrates and the geometrical content

I examine here the limited amount of geometry available to Socrates. I base my analysis, in part, on Plato's comments about geometry in the *Republic* (536d). It is written there that 'calculation, geometry, and all the preliminary education required for dialectic must be offered to the future rulers in childhood'. As I explained earlier, Plato's insistence on the teaching of geometry indicates that it was not commonly taught in fourth century Athens. This implies that neither he nor Socrates received formal training in geometry in their youth. There is further support for this in Isocrates (*Panathenaicus*, 26). There Isocrates declares: 'I even commend *that which has been set up in our own day* – I mean geometry, astronomy, and the so-called eristic dialogues, which our young men delight in more than they should, although among the older men not one would not declare them insufferable' (my italics). The italicised phrase suggests that in earlier times, when Socrates was young, the teaching of the listed subjects, including geometry, had not been introduced. However, Plato's establishment of the Academy meant that he had access to the great geometers of his time, as I suggested earlier. This would enable him to become familiar with the geometry of his

day even if he had not had formal tuition in it in his youth. Such an opportunity was not available to Socrates.

Regardless of whether or not Socrates knew any geometry, there is an additional problem, which arises from the content of Plato's geometrical references. To take the case of *diagramma* first, there is only one example, given in the *TLG,* of its use in texts written up to the time of Socrates' death. That is in fragment 3 of Hippocrates and Radice (1977, p.134) suggests that it is 'unlikely that any of the extant writings of the Hippocratic school are written by him [Hippocrates]'. If this is the case it would throw some doubt on the original date of writing of the fragment. While this is not proof that the word was unknown to Socrates it casts some doubt on Socrates choosing to use it, especially as it is used so rarely in any extant ancient work. There is also a problem with the notion of the distant object, since finding the true height would involve geometry, in particular the use of proportion which Eudoxus developed. Before his time the subject was not well understood and since he lived from about 390 this means that Socrates could not have benefited from his research, although Plato could. Plato would be able to refer to work on proportion, but only after the discovery was made. This detail throws some further light on the passages which I have considered.

An examination of the texts which contain the distant object analogy demonstrate how the work of Eudoxus is incorporated into them. One of my criticisms of the use of the apparently diminished distant object analogy in the *Protagoras* was that there is no mention, in that text, of how the true size of the object can be found by calculation. Socrates speaks only of measurement. As I explained earlier, the *Protagoras* was probably written in the late 380s. Although great mathematicians through the ages have tended to produce innovative work while still very young it is unlikely that Eudoxus would have finished producing such ground-breaking work while still a boy. This is the situation if we accept the dates of 390 for his birth and the 380s for the writing of the *Protagoras* and, although the dates are not certain, some slight inaccuracies do not affect my argument. This means that when Plato wrote the *Protagoras* the work on proportion, which enabled the size of a distant object to be calculated, would not have been completed. However, in the two other texts where the analogy is found calculation is mentioned. These are both accepted as being written at later dates and Eudoxus is likely, therefore, to have made his discoveries by the time Plato planned his work on them. It is difficult for us, accustomed to algebraic manipulation of lengths, to grasp how innovative the work on proportion was. As I explained in Chapter One, it revolutionised the work on similar triangles and these are needed to calculate the size of a

distant object. It would not be surprising for Plato to incorporate it into his references to distant objects, but it is impossible for Socrates, several decades earlier, to have any idea that such calculations would be possible. The geometrical examples cannot indicate whose doctrines are being presented in the dialogues but they do suggest that some, at least, of the supporting discussions are Plato's own.

All my work up to this point has been based on only two geometrical examples, and it could be argued that this is too small a number to allow any valid conclusions to be drawn. However, the dialogues contain many other geometrical references which could be used as further support for my claims. Although it is impossible to list all of them, or give an in-depth study of the philosophical implications of each, I examine next the geometrical content of a few passages, with comments on their possible philosophical significance.

Plato's other geometrical references

There are several different types of geometry, and when I considered the distant object analogy I called this aspect of geometry 'practical', because the estimation of the size of such an object would be a practical exercise. The word *diagramma* falls within the area of theoretical plane geometry since it applies to geometrical diagrams being used to demonstrate constructions and prove theorems. I now look at various areas of geometry which existed in Plato's time and show that Plato used many examples from different areas. I suggest some philosophical insights which could come from noting such references. Since both of the examples which I have examined were repeated, and I found significance in the repetition, the value of some of my work would be undermined if, apart from these, there are only isolated references to geometrical terms and concepts. However, single examples are difficult to find. The fact that there is an almost universal repetition of any geometrical references supports my claim that Plato used geometry, which stands out in a literary composition, to link together several discussions. There is no space here to cite all the occurrences of each topic and the purpose of this section is to demonstrate that there are numerous examples which could be the basis of further research. I mention a representative sample, drawn from different areas of geometry, but only cite some of the repetitions which occur.

I first consider a geometrical topic which was known to Plato, but which he complains was under-researched. We know from the *Republic* that solid geometry was not a popular topic for study in Plato's time. In a discussion of geometry, Socrates suggests that they should now look at 'the third dimension' but it is noted that 'this subject hasn't been developed yet'

(528b). Plato's Socrates was speaking in the fifth century but it is possible that Plato is describing a state of affairs which existed in his own time. However, he makes several references to solid bodies and I examine some which could be the basis for investigation into any associated philosophy.

One example of a solid object which Plato uses several times is that of a sphere or globe, *sphaira*. In the Sophist (244e) the visitor is discussing with Theaetetus whether the whole is different from the one being, and quotes from Parmenides, saying:

> All around like the bulk of a well-formed sphere,
> Equal-balanced all ways from the middle, since neither anything more
> Must it be, this way or that way, nor anything less.

These are the words we know as coming from near the end of fragment B8 and they lead to a discussion on parts and the whole. Another use in the *Philebus* is also promising for future work on philosophical insights, since it is connected with a discussion on understanding justice. Socrates draws a comparison with someone who 'knows the definition of the circle and of the divine sphere [*sphairas*] itself but cannot recognize the human sphere [*sphairan*]' (62a). There are other passages in the dialogues where the same word is used to denote a ball used in play. At *Euthydemus,* 277b, Dionysodorus 'picked up the argument as though it were a ball [*sphairan*] and aimed it at the boy again'. This could be the basis for some analysis of philosophical arguments, as could the passage in *Laws* (898a-b), where the 'cyclical movement of reason' is compared to 'a sphere [*sphairas*] being turned on a lathe'.

Perhaps the most famous reference to solid bodies is the example of the so called Platonic solids. However, there is considerable doubt about Plato's role in their discovery. Thomas (1967, p.379) records a scholium to Euclid's *Elements* 13, which states that of the five Platonic figures, three are due to the Pythagoreans, the cube, pyramid and dodecahedron, with the last two, the octahedron and the icosahedron, being credited to Theaetetus. Plato lists these solids in the *Timaeus,* 54d-55c, and it is possible that he was the first to record that there are five regular solids. He describes how four of the shapes can be made, using triangles and squares, and then assigns them to fire, air, water and earth. However, he glosses over the construction of the fifth solid, the dodecahedron, which requires pentagons, saying only that 'this one the god used for the whole universe, embroidering figures on it' (55c). I suspect that much could be written on this application of geometry, with an analysis of why each solid was related to a particular element. Black (2000), for example, has attempted this, although he concentrates on determining how the four bodies representing fire, air, water and earth could be related by proportion. I suggest that a more geometrical approach could

be worthwhile. There are also other references to solid bodies just as 'solids', without specifying which type, which appear to support the statement that little research had been done because there was little interest. Examples include the *Meno* (76a), the *Phaedrus* (246c), and the *Philebus* (51c).

In Chapter One I gave an account of Plato's references to diagrams, indicating the different words he used, before selecting *diagramma* as my topic for investigation. Here, I will refer to some of the constituents of diagrams, namely lines, triangles and circles, and give a brief account of a few, taken from a cross section of topics and dialogues. The basic constituent of most geometrical diagrams is a line, with *grammē* being commonly used. In the *Meno* it is used to describe the side of a square, for example at 82e and 84a. Since this is connected to Socrates' exploration of the theory of *anamnesis* it offers much scope for philosophical enquiry and the geometry in the *Meno* has been the subject of several investigations, unlike that in the other dialogues. Greek mathematicians also used lines to obtain various 'mean' numbers. They drew a line with a point somewhere along it and the position of the point in relation to the length of the line in different circumstances gave a particular mean value. Plato does this in *Timaeus* 31c-32a, in a complicated passage where he derives a 'middle term'. There is also the reference in *Republic* 509d where sections of a line are related to different levels of knowledge.

A triangle, *trigōnon*, formed from lines, is also often mentioned in the dialogues and the properties of triangles had been investigated many years before the time of Plato. Their use to form some of the five regular solids, at *Timaeus* 54d-55c, has already been given but a different reference can be found in the *Meno*, where Socrates is discussing how geometers carry out their investigations:

> ...if they are asked whether a specific area can be inscribed in the form of a triangle [*trigōnon*] within a given circle, one of them might say: "I do not yet know whether that area has that property, but I think I have, as it were, a hypothesis that is of use for the problem... 86e-87a

He goes on to compare this state of mind with those who speak about virtue since 'we do not know either what it is or what qualities it possesses' (87b). An investigation of this connection between an unproven geometrical hypothesis and knowledge of virtue could provide a greater understanding of Plato's philosophy.

Some passages which contain a reference to triangles are puzzling. Near the end of the *Timaeus* Plato turns to diseases, and relates these to triangles. He particularly mentions mathematicians, 'or the ardent devotee of any other intellectual discipline' (88c) because

such people should take part in gymnastics, to ward off disease. However, each species has an allotted life span because 'its triangles [*trigōna]* are so made up, right from the beginning, as to have the capacity to hold up for a limited time beyond which life cannot be prolonged any further'(89c). Correlating such a passage with Plato's philosophical thoughts could prove challenging.

When describing triangles, Plato often refers to an angle, *gōnia*. At *Republic* 510c, it is said that students of geometry hypothesize about 'the three kinds of angles [*gōniōn*]' among other things. He criticizes the approach where they make 'their hypotheses and don't think it necessary to give any account of them, either to themselves or to others, as if they were clear to everyone. And going from these first principles through the remaining steps, they arrive in full agreement' (510c). Plato compares the situation in geometry with that of reason, using 'the power of dialectic' and there is scope for much investigation on this comparison.

A circle, *kuklos*, features frequently in the dialogues. One geometrical use of the circle in the *Meno* has already been mentioned, since the construction of a triangle in a circle is discussed at 86e-87a. In some dialogues where *kuklos* is used translators give alternative words although the meaning of a circle is implicit. One example is in the *Theaetetus* (160e) where the version given in Cooper (1997), translated by Levett, reads 'we must make it in good earnest go the round of discussion'. Fowler (1967, p.75) translates the passage as 'we must in very truth perform the rite of running round with it in a circle [*kuklō*] – the circle of our argument'. An associated word, *periphora*, revolution, implies movement in a circle, as at *Republic* 616c: 'the spindle of Necessity, by means of which all the revolutions [*periphoran*] are turned'. The circular nature of arguments could provide much discussion and show how our understanding of Plato's philosophy can be enhanced through his geometry.

It is only possible to hint at the wealth of geometrical content in the dialogues by giving these few examples, but they demonstrate that there are many repetitions of geometrical references. There are some obvious philosophical connections with many of these references, indicating that there is much worthwhile work left to be done, in the future, on interpreting Plato's philosophy through his geometry. This reinforces my claim that Plato's geometry should not be ignored.

Summary of Chapter Six

There are many writers who believe that Platonic interpretation is impossible and perhaps Dillon (1999, p.222) is correct when he gives as his opinion 'the task of commenting

adequately on a Platonic dialogue ... is probably beyond the capacity of any mere mortal'. He then suggests that 'of course, we should never give up trying'. The application of my findings to theories of Platonic interpretation demonstrates the problem of 'commenting adequately' but shows that if we 'never give up trying' we can obtain some worthwhile insights. In this chapter I have shown that Plato's use of geometrical examples can throw some light on several theories about the dialogues.

I first considered the notion that the dialogues are separate entities, which can be fully understood in isolation. I showed that my work on the repetitions of geometrical examples supports those who disagree with this theory since I have demonstrated the presence of cross-dialogue connections and shown that they enhance our understanding of the associated discussions. I next considered some questions about the way the dialogues were initially received. I argued that there was sufficient evidence to support the possibility that they were read, perhaps in addition to sometimes being heard as they were read out. I then applied my findings on the geometrical repetitions to show that they supported Plato's views on writing, since they could act as a reminder of the presence of previous discussions when the ability to remember things diminished as reading became more popular.

If the dialogues were initially read privately or read to large groups of listeners they could have been available to people outside the Academy. I suggested that this wider audience could have been envisaged by Plato and I proposed that his choice of geometrical examples demonstrates his attitude to such an audience. He is able to use geometry to discriminate between readers who would be able to understand the subject and those who would not and I suggested that it is philosophers who would gain most from the associated discussions. However, limited understanding is available to non-philosophers when the topic is related to an example which can be appreciated by a non-geometer, such as the apparent diminution in the size of a distant object.

I then looked at some theories about the intended purpose of Plato's dialogues for which my research offered support. I first considered the suggestion that the dialogues were intended to teach. I explained how the geometrical examples which I have covered could teach both Academy members and outside readers how to approach philosophy. I also considered the dialogues as advertisements for the Academy. Here, the use of *diagramma* would indicate that higher knowledge was available to the members and both examples would demonstrate the geometrical expertise available in the Academy, in addition to the method of philosophical discussion which is demonstrated.

Having looked at the reception of the dialogues I concentrated on the form of the dialogues. One major problem in modern interpretation theories relates to Socrates and whose doctrines are presented in the dialogues. I gave support to a moderate mouthpiece interpretation, by suggesting that the geometrical references could indicate that Plato was using his own examples to clarify his discussions, rather than recording examples which Socrates might have used.

Although some authors have declared that the problems of Platonic interpretation might be insurmountable, I suggest that my work on Plato's use of geometry has allowed me to clarify some key issues. The possibility of enhanced clarification reinforces the importance of geometry in the dialogues. I finished the chapter by noting some of the many geometrical references that can be found in Plato's texts, from all aspects of the subject. There were a few obvious philosophical connections but I suggest that a study of any of the examples given could yield further worthwhile insights and also, possibly, add to our understanding of how the dialogues can be interpreted.

I leave the last word on interpretation to a writer who recognised, in the sixth century CE, the same problems that we now face. In the *Anonymous prolegomena to Platonic philosophy* (1. 29-35) it is said that Plato:

> ...shortly before his death, had a dream of himself as a swan, darting from tree to tree and causing great trouble to the fowlers, who were unable to catch him. When Simmias the Socratic heard this dream, he explained that all men would endeavour to grasp Plato's meaning, none, however, would succeed, but each would interpret him according to his own views, whether in a metaphysical or a physical or any other sense. Westerink, 1962, p.4

The point about this anecdote is not whether the description of the dream is true but that it represents views of scholars in the sixth century, and probably earlier, and such views still persist today. However, the application of my work to theories of interpretation has offered some insights, which strengthens the view that Plato's geometry should not be ignored.

Conclusion

I started from an observation that Plato repeats the distant object analogy in three dialogues and continued, through my investigation into his use of *diagramma*, to give a thorough account of the use Plato makes of geometry within the dialogues. I built on this to show the value of the geometrical references for Platonic interpretation. I set out to answer the question: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how? A positive response to the first part would indicate that Plato's geometry should not be ignored. It is necessary to support this assertion by showing *how* our understanding of his philosophy is improved by considering the geometrical references. I give a summary of my work to demonstrate several ways in which this has occurred.

I chose two of Plato's geometrical examples, which are repeated in several dialogues, in order to track them across the different passages to see how the discussions are connected. I started with the analogy of an apparently diminished distant object which is used in the *Protagoras*, *Republic*, and *Philebus*. In each of these Plato compares the distant object with pleasures and pains. The fact that the analogy appears in three dialogues means that taking note of all the references allows the three discussions associated with it to be combined, so that omissions in one text are considered in another. The analogy provides a recognisable situation and we can agree that it is possible to falsely estimate the size of pleasures and pains. Plato speaks of being able to measure or calculate the true size, in order to dispel the illusion. Although measuring is problematic I showed that calculations are possible for the objects, using geometry available in Plato's time, but there are difficulties with the assumption that the size of pleasures can be calculated. There are also other problems with the analogy and examining these, as Plato discusses them in the different dialogues, gives a greater understanding of the situation with regard to pleasures and pains, as I now demonstrate.

I summarize the points which I found, showing how the omissions which I highlighted in the *Protagoras* are addressed in the *Republic* or *Philebus*, building up a complete analysis of the estimation and choice of pleasures and pains. Measurements are mentioned in the *Protagoras* but neither a measuring device nor the knowledge needed to use one is considered. However, in the other dialogues calculations are proposed and, in the *Republic*, the knowledge needed is said to reside in the rational part of the brain. The time scale over which the estimations should be made is initially ignored, but the myth of Er, in the *Republic*,

shows an awareness of the necessity to consider all eternity. The unit of measurement for pleasures and pains is not given but, in the *Philebus*, it is suggested that only the gods could know of this. Also in this dialogue, a hierarchy of pleasures allows some information to be obtained about the quality of different pleasures. Even if the true size can be established there remains the problem that the illusion of the false size may continue. The division of the soul described in the *Republic* shows how this can be overcome. In addition, moderation in pleasure, advised in the *Republic*, takes care of what I called the multiplication problem, where multiplying pleasures does not necessarily give greater pleasure. Finally, the fact that a pleasure or pain may not materialise as expected is dealt with in the *Philebus*, where it is claimed that good men will not anticipate falsely. Looking at one dialogue alone would only reveal some of these points so I conclude that tracking these geometrical references concerning a distant object enhances our understanding of Plato's work.

Another point came to light as I examined the three texts. The geometry needed to find the size of a distant object was a recent discovery when Plato was writing and I used this to suggest that only geometers would gain fully from the discussions. I explained how some people who knew geometry did not progress to philosophy but all philosophers were expected to know geometry. I, therefore, made a division into two groups. The first comprised non-philosophers, including geometers, and the second contained philosophers, to include those who had just started on the path to philosophy after studying geometry. In the next section of my work I again tracked a geometrical reference, *diagramma*, across several dialogues but I also examined more closely the possibility of Plato discriminating between philosophers and non-philosophers.

The term *diagramma* appears in the *Lesser Hippias*, *Cratylus*, *Theaetetus, Euthydemus*, *Phaedo*, and *Republic* as well as the possibly spurious *Epinomis.* Each time Plato mentions *diagramma*, in the first six of these dialogues, it is associated with advice about how to gain knowledge. He gives warnings about obtaining information from experts, who may lie, and others who may be genuinely mistaken. He also warns about false premises, which could be used by philosophers. Then he gives more positive advice as he suggests that knowledge can be discovered, using *anamnesis*, and that it can only come from applying reason from within, although discussions are helpful. Noting the rarity of the word allowed me to suggest that it was a specialist term which could be aimed at philosophers, with the implication that the knowledge being acquired is philosophical knowledge of the Forms. This provided support for a theory proposed by several writers, such as Silverman (2001, p.32) for the *Cratylus* and Cornford (1964, p.99) for the *Theaetetus,* that the Forms are indicated in some dialogues

where a word for 'Form' is absent. The use of *diagramma* in the *Epinomis* was shown to be anomalous, which I claimed supports the spuriousness of this dialogue. In addition, the rarity of the use of the term could be taken to imply that the writer of the *Epinomis* used a discarded fragment from Plato's work which contained the word *diagramma*. This corroborates the theory that the writer of the dialogue was a scribe of Plato's, and some scholars, such as Sedley (2003, p.14) have named him as Philip [Philippus] of Opus. Taking the references to *diagramma* across all the dialogues in which it appears gives a deeper understanding of several aspects of Plato's philosophy. This gives further support to my conclusion that noting the use of geometrical references enhances our understanding of Plato's work.

Finally, I showed that noting the way that geometry is used by Plato has some impact on theories about how best to interpret his work. I started with the fact that the geometrical references are repeated across several dialogues, which overturns the view that each dialogue is best approached as a separate entity. I also tied the repetitions to Plato's suggestion, expressed in the *Phaedrus*, that writing does not improve memory but it can act as a reminder. I proposed that his repeated use of the geometrical examples might be intentional in order to remind people of previous discussions. Since, with the exception of Socrates, almost no one is present at more than one discussion I suggested that Plato is aiming his philosophical insights at the readers of his texts.

I was also able to throw some light on several other theories. I considered the value of the dialogues as teaching aids and showed how the geometry could have a role to play in this. Since the geometrical examples are found in several dialogues, written some years apart, the associated discussions are also spread across more than one dialogue. This would enable readers to think about, or discuss with others, any problems in each account before the next dialogue became available. They would thus learn how to assess a philosophical text and gradually learn what is written in the various dialogues. For readers outside the Academy, the content of the dialogues could serve as an advertisement for the Academy and the geometry has a part to play here also. It would indicate to prospective entrants the quality of the geometry, in addition to the philosophy, which could be learnt there. When I moved on to the vexed question of whose views are given in the dialogues I was able to clarify several points about the role of Socrates. For instance, I examined the notion that he was unlikely to know all the geometry which is used in the dialogues and I concluded that the geometrical references are Plato's own. Examining some theories of Platonic interpretation, in the light of my findings regarding Plato's use of geometry, has given many insights. I conclude that the

enhancement of our understanding of Plato, gained from noting his geometrical references, has a wider application than the philosophical content of the dialogues. It is also relevant for our interpretation of them.

Taking only two geometrical topics from Plato's dialogues has provided evidence for the enhancement of Plato's philosophy but they form a small sample from which to draw conclusions. However, I indicated in the last section of my work that there are many geometrical references in the dialogues. I suggest that the insights which have emerged from my study can be compared to the tip of an iceberg. The other examples which Plato uses should allow much worthwhile future research, reinforcing the importance of taking note of the geometry found in the dialogues.

The legendary sign outside the Academy could be seen as an entrance requirement for admittance to Plato's school but, even if the sign never existed, philosophers in the Academy were likely to have had knowledge of geometry, as I have explained. Plato could use geometrical examples and assume that members of the Academy would have no trouble in understanding the references. I examined a hierarchy of knowledge which suggests that, for Plato, geometry was the stepping stone between the sensible world and the world only philosophers could access, that of the Forms. Geometry is based on things of this world, such as shapes, lines and circles but reveals things beyond the sensible world, for example perfect circles. The ultimate knowledge is of the Forms, for instance the Form of Circle. Therefore, Plato could use geometry to support his philosophy, to transfer thought from things of this world to things only philosophers could know. Those outside the Academy, which includes contemporary readers and those of the future, are not ignored. Plato includes some references which anyone could understand, and some of the discussions, particularly those on choosing pleasures and pains, associated with the distant object analogy, are relevant to everyone.

I started with the aim of revealing the importance of Plato's geometry. The method I chose was to cite geometrical references which are repeated in several dialogues and examine the associated discussions. In doing this I have uncovered several specific examples which demonstrate how the enhancement to our understanding of Plato's philosophy is given by his geometrical references. For modern readers, Plato's philosophy is shaped by our interpretation of the dialogues and this depends on our understanding of them. Plato included geometry for some purpose which we cannot know but I claim that taking note of his geometrical references increases our understanding of the dialogues, bringing us closer to

an interpretation which he would accept. This means that interpreting Plato's philosophy through his geometry has been a worthwhile enterprise.

Throughout my work I have kept in mind the notion of the 'fusion of horizons' developed by Gadamer. I presented Plato's views as they are found in the dialogues and explored their significance for both the philosophers and non-philosophers of his day. However, I realise that my assessment of Plato must be grounded in the world of the twenty-first century. I therefore demonstrated the importance of the geometrical references both within the dialogues and with regard to modern interpretations of Plato's work. The many worthwhile insights obtained demonstrate, as Gadamer (2007, p.62) declares: 'the interpreter and the text each possess his or her and its own horizon, and each moment of understanding represents a fusion of these horizons'. My final conclusion is that I have demonstrated a positive response and supporting evidence for the question I set: does Plato's use of geometry enhance or even significantly re-shape our understanding of his philosophy and, if so, how?

I started my investigation with a quotation about Plato, which suggested his commitment to geometry. I end with a quotation from Plato. The wording leads me to imagine that he would agree with my final conclusion. I use Allen's translation of *Republic* 527b (2006, p.244):

> Geometry is knowledge of what always is. So it would draw a soul toward truth, my friend, and be productive of philosophical understanding, by directing upward what we now wrongfully direct downward.

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Ancient texts 176
Other references 179
On line sources 195

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Appendix

Mathematical expressions and calculations

Proportions found in Euclid's *Elements* 196
The calculation of the size of a distant object 197
A proof of Pythagoras' theorem 199

Proportions found in Euclid's *Elements*

Elements V, proposition 16

If four magnitudes be proportional, they will also be proportional alternatively.

Using modern algebra: If $a/b = c/d$ then $a/c = b/d$

Using modern algebra, such rearrangements are obvious but I emphasize that we must be aware that without algebra the necessary working is difficult. However, while keeping this in mind, I see no need to use fourth century BCE terminology. Lasserre (1964, p.101) gives other relations which follow from Eudoxus' work, in modern algebraic form.

If $a/b = c/d$ then $(a-b)/b = (c-d)/d$

If $a/b = c/d$ then $(a+b)/b = (c+d)/d$

If $a/b = c/d$ then $(a-c)/(b-d) = a/b$

If $a/b = d/e$ and $b/c = e/f$ then $a/c = d/f$ and then $(a+b)/c = (d+e)/f$

Lasserre (1964, p.102) concludes that the general theory of proportion, which gives these relationships, is 'one of the most remarkable contributions made by ancient mathematics to the progress of human thought'.

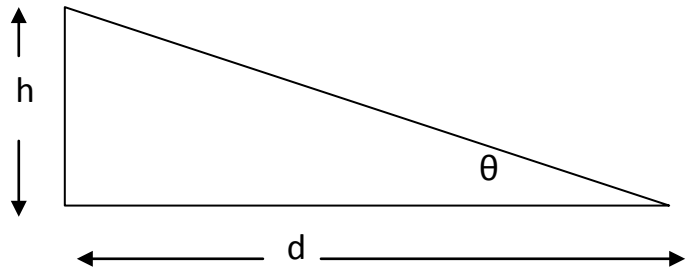
Similar manipulations would allow the size of a distant object to be found, as shown below.

I give, first, modern methods using trigonometry, and, secondly, methods based on geometry which was known in Plato's time. All presentations are my own and are representative of several possible variations.

The calculation of the size of a distant object

(i) An object a known distance away, using trigonometry

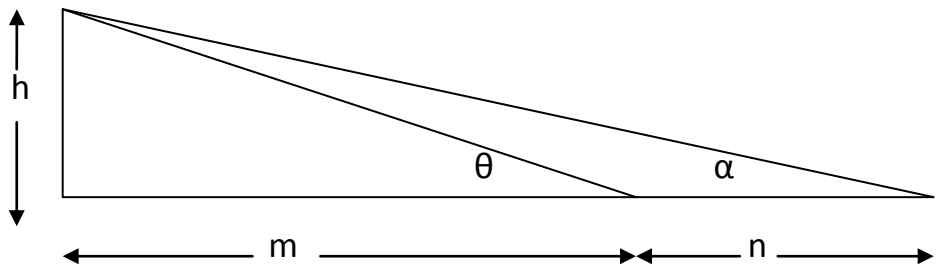
If the distance of the object is known the calculation is simple. If the object is of height h and distance d away then a triangle can be drawn with the angle, θ, which the top of the object makes with the eye of the observer (assumed to be at ground level, a correction for the true position is easy but makes the diagrams look more complicated).



By trigonometry tan θ = h/d so that **h = d tan θ**

(ii) An object an unknown distance away, using trigonometry

If the distance to the object is not known then the calculation is more complicated and requires two measurements of angles. The geometry is shown below.

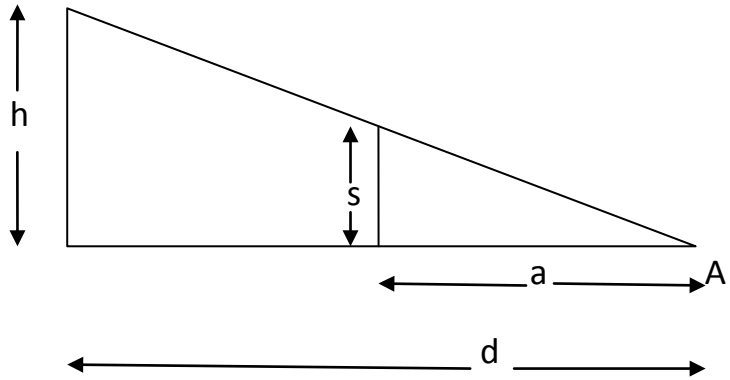


Here, the observer takes the measurement of the angle, as before, with the distance m unknown, but then moves back a known distance n and takes another measurement of the angle to the top of the object. The unknown distance m can now be eliminated mathematically to give

$$\mathbf{h = n \tan\theta \ \tan\alpha / (\tan\theta - \tan\alpha)}$$

(iii) An object a known distance away, using geometry of Plato's time

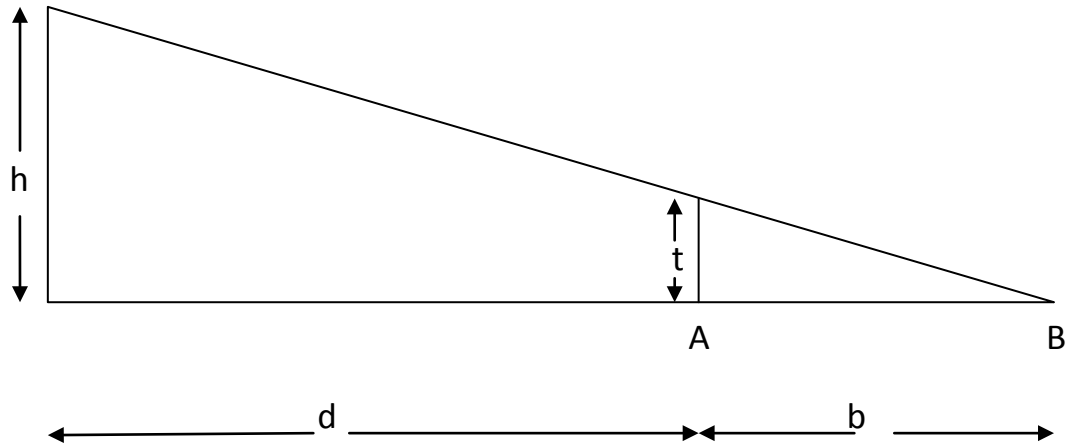
Assume a stick of length s is held so that it just covers the distant object of height h, for an observer at A, where a can be measured.



h/d = s/a gives **h= sd/a**

(iv) An object an unknown distance away, using geometry of Plato's time

In addition to the above, a second stick of size t is used to cover the object for an observer at B, b to be measured. For simplicity I have chosen the second position of the stick to be point A. The calculations are slightly more complicated for other positions.



From the first situation

h/d = s/a which rearranged differently gives d = ha/s

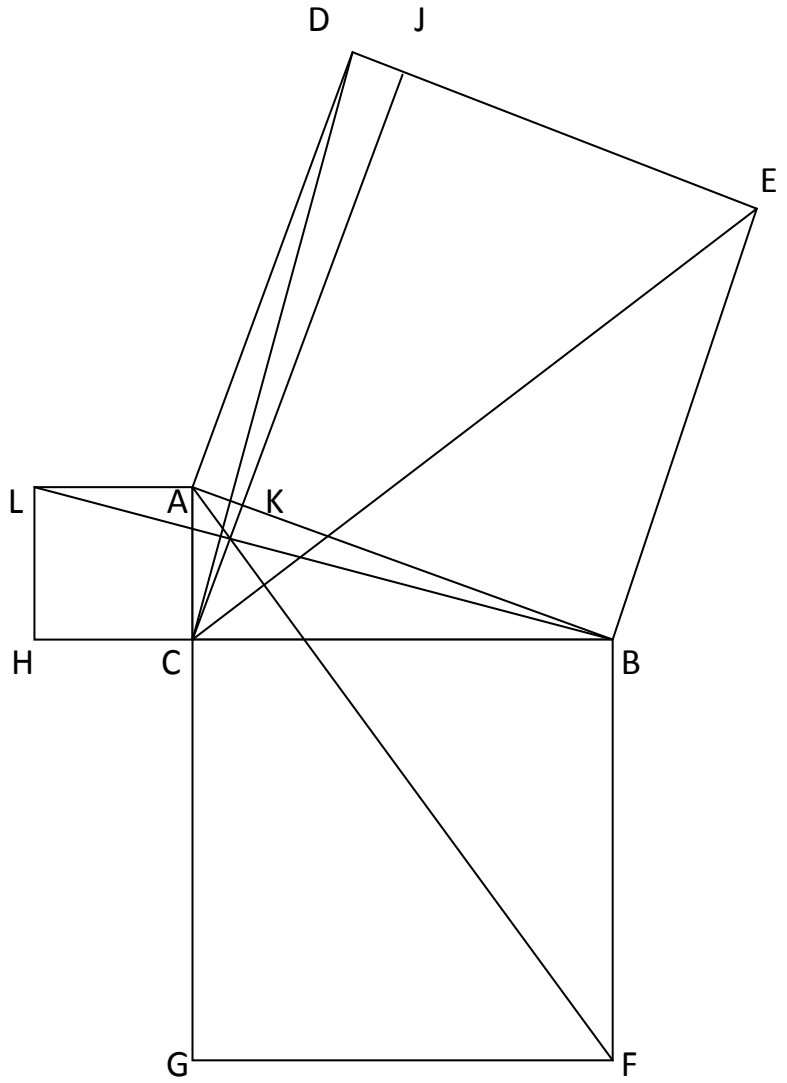
From the second situation

h/(d+b) = t/b which, substituting d from above, gives h/(ha/s + b) = t/b

Rearranging to obtain h gives

h = bts/(sb-ta) All lengths on the right hand side are known.

A proof of Pythagoras' theorem



2Δ LAB = square LACH (same base LA, between same parallel lines LA, HB)

2Δ ABF = square BFGC (same base BF, between same parallel lines BF, AG)

2Δ ADC = rectangle ADJK (same base AD, between same parallel lines AD, CJ)

2Δ EBC = rectangle EBKJ (same base BE, between same parallel lines BE, CJ)

∴ 2(LAB + ABF) = LACH + BFGC and 2(ADC + EBC) = ADJK + EBKJ = square ADEB

But Δ LAB is congruent to Δ ADC (SAS) and Δ ABF is congruent to Δ EBC (SAS)

∴ 2(ADC + EBC) = LACH + BFGC

∴ ADEB = LACH + BFGC

i.e. **For a right-angled triangle ABC, the square on the hypotenuse, AB, equals the sum of the squares on the other 2 sides.**