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## On the Probabilistic Characterization of Robustness and Resilience

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### Abstract

Over the last decade significant research efforts have been devoted to the probabilistic modeling and analysis of system characteristics. Especially performance characteristics of systems subjected to random disturbances, such as robustness and resilience have been in the focus of these efforts and significant insights have been gained. However, as much of the undertaken research and developments aim to fulfill the particular needs of specific application areas and/or societal sectors somewhat diverging perspectives and approaches have emerged. In the present paper we take basis in recent developments in the modeling of robustness and resilience in the research areas of natural disaster risk management, socio-ecological systems and social systems and we propose a generic decision analysis framework for the modeling and analysis of systems across application areas. The proposed framework extends the concept of direct and indirect consequences and associated risks in probabilistic systems modeling formulated by the Joint Committee on Structural Safety (JCSS) to facilitate the modeling and analysis of resilience in addition to robustness and vulnerability. Moreover, based on recent insights in the modeling of robustness, a quantification of resilience is formulated utilizing a scenario based systems benefit modeling where resilience failure is associated with exhaustion of the capital accumulated by the system of time. The proposed framework and modeling concepts are illustrated with basis in a simple interlinked system model comprised by an infrastructure system, a governance system, a regulatory system and a geo-hazards system. It is shown how the robustness and the resilience of the interlinked system may be modeled and quantified, how robustness and resilience are influenced by the stochastic dependency structure of the disturbance events and corresponding resistances, how robustness and resilience depends on the capacity of the social system to plan for and respond to disturbances over time and how robustness and resilience interrelate.

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## 1. Introduction

During the last 4-5 decades the understanding and modeling of system characteristics in general and systems robustness and resilience in particular have attracted significant interest and efforts in the international research community, see [1]-[7], [10]-[17]. More recently, these topics have also attracted the attention on the global political agenda as one of the concerns in the context of climate change and sustainable societal developments ([1]). Societal infrastructures including energy production and distribution systems, the built environment, transport systems, communications systems, food production and distribution systems, waste disposal and treatment systems play very significant roles for the success of society in the short, medium and long run. Societal infrastructures provide indispensable functionalities to society and fundamentally comprise the basis for economic growth, health and welfare. Societal infrastructures however also constitute one of the major consumers of raw materials, space, energy and water and thereby severely impose stresses to the environment and at the same time represent significant economic investments and substantial expenditures in terms of maintenance and renewals. In [11] serious gaps are highlighted in the general body of knowledge concerning the performance of complex interlinked systems and it is emphasized that even smaller and localized disturbances of interlinked systems have the potential to trigger scenarios of cascading failures with widespread and disastrous consequences and calls for increased research efforts and focus to close these gaps.

Traditionally in the field of civil and infrastructure engineering, robustness of systems has been understood as a systems ability to limit the consequences of damages, failures and other disturbances within an order of magnitude of the cause of the disturbances ([8]). Whereas the intention of this interpretation of robustness might be intuitively clear, it is not immediately obvious how to apply this qualitative interpretation in support of design and management of systems. First attempts to model and quantify robustness of systems in civil and infrastructure engineering are reported in [3]-[18] taking basis in risk concepts. Robustness of systems in this sense is understood as a systems ability to limit the expected value of total consequences (the risk) originating from damages and failures of individual system constituents to the same order of magnitude as the risks associated with damages and failure of these constituents in isolation.

Similarly as for systems robustness, resilience interpreted qualitatively as a systems ability to plan for, recover from and adapt to adverse events over time ([10], [15], [16], [17]) provides a strong concept and relevant objectives for the design, operation and management of infrastructure systems however, does not give much practical guidance on how to achieve these and to assess whether the achieved level of resilience is sufficient and acceptable. In [14] the many challenges associated with assessing and ensuring the resilience of systems are addressed and it is suggested that present practices of risk based approaches for ensuring resilience are not adequate why there is a need for the development of a new paradigm.

In the present paper we briefly introduce a decision analysis framework for design and management of systems and with this setting the probabilistic representation of system characteristics is addressed with special emphasis on the modeling and quantification of robustness and resilience. Robustness and resilience of systems are introduced as random and causally dependent system characteristics with significant impact on both the short and long term performance of a system. A principal example considering an interlinked system comprised of an infrastructure system, a regulatory system, a governance system and a geo-hazard system is then provided for the purpose of illustrating the proposed framework and modeling concepts. Finally, the results from the principal example are discussed and directions for further research are suggested.

## 2. Decision analysis for systems

### 2.1. Basis for decision support

It is fundamental, that decision alternatives which are considered for the purpose of optimizing the design and/or management of systems subject to uncertainty and incomplete knowledge in a normative decision context shall be ranked in accordance with their expected value of utility (or benefit) in accordance with the Bayesian decision analysis and the axioms of utility theory, see e.g. [12]-[19]. To benefit fully from this theoretical and methodical basis for decision optimization it is necessary to formulate probabilistic models for the performances of the systems

as well as the preferences of the decision maker with respect to the possible outcomes of the decisions. Crucial issues obviously concern the probabilistic modeling of the considered systems and also the identification of strategies and options for their design, operation and management.

## 2.2. Probabilistic system modeling

Here it is proposed to utilize and extend the framework for risk informed decision making for systems suggested by the Joint Committee on Structural Safety ([13]) as basis for the modeling of interlinked systems. Necessary modifications must however be introduced in order to account for the representation of benefits and accumulated benefits of the interlinked systems over time, see Figure 1.

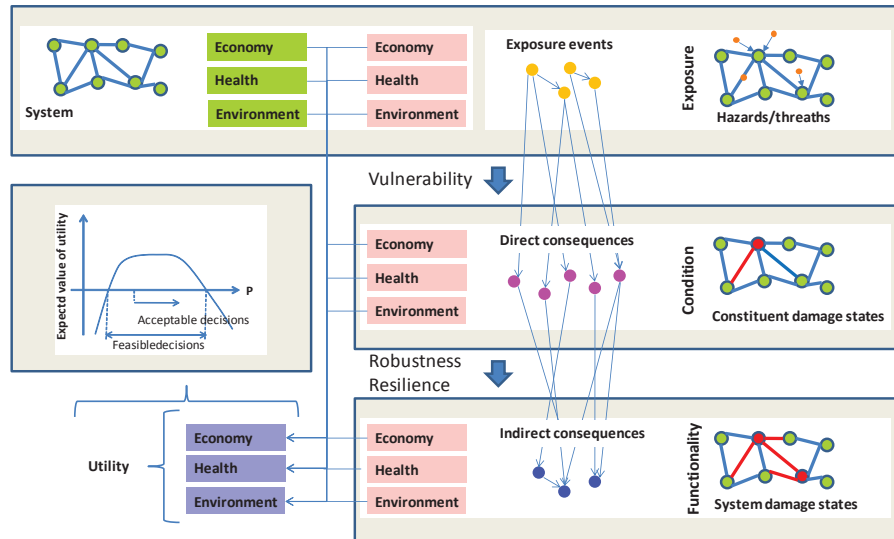


Figure 1 Generic framework for decision analysis of systems ([9])

In Figure 1 it is shown that the interlinked system is represented in terms of its undisturbed configuration with associated benefits together with the ensemble of possible scenarios of system failure events imposing losses to the system over time. In the following it is assumed that a probabilistic system model represents all relevant physical processes, environmental systems, geo-hazard systems, engineered objects and facilities, organizational processes, human activities as well as all decision alternatives envisaged for designing and managing the performance of the system. The system model is comprised by an ensemble of  $n_c$  constituents interacting jointly to provide the desired functionalities of the system. Some of the constituents may be designed while others are given by nature. The system modelling approach suggested by the JCSS ([13]) is utilized to subdivide the scenarios of events leading to consequences for the system into two parts, namely the direct consequences and the indirect consequences. The direct consequences comprise all losses caused by failure states of the constituents of the system except functionality related losses. On the other hand the indirect consequences are assumed caused by functionality losses alone. Besides the differentiated consequence modelling, two phases are introduced in the modelling of the progression of failure of the system; the initiation phase and the propagation phase, see also Figure 2.

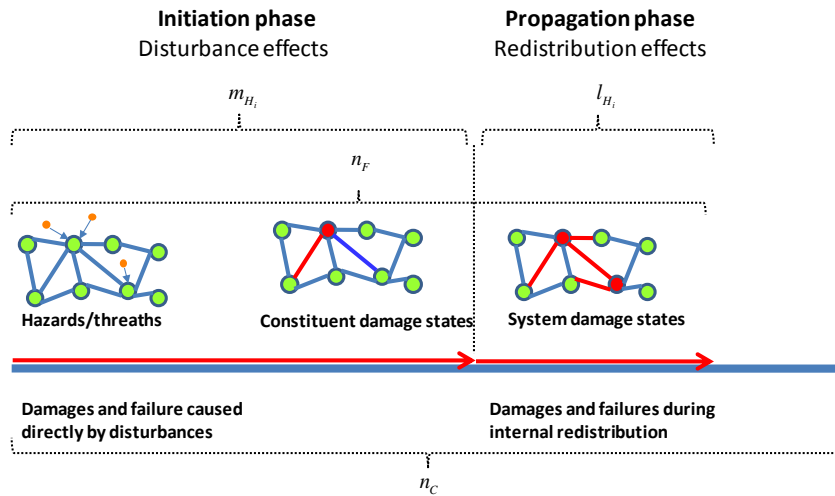


Figure 2 Illustration of the two phase scenario based failure propagation model.

In the initiation phase  $m_{H_i}$  constituent failures are assumed generated by the hazard event  $H_i$ . In the propagation phase further  $l_{H_i}$  constituent failures are generated by the joint effect of internal redistribution of system demands and hazard events. The two-phase failure propagation model facilitates the representation of cascading failure scenarios.

For the purpose of simplification of notation and without loss of generality it is assumed that all possible  $i = 1, 2, \dots, n_s$  different scenarios of hazard events with their occurrence probabilities  $p(i)$ , direct consequences associated with constituent failure events during the initiation phase  $c_{D,I}(i)$  and propagation phases, respectively  $c_{D,P}(i)$  and the indirect consequences  $c_{ID}(i)$  have been identified and assessed. The probabilistic system representation  $\mathbf{S}$  can then be written as:

$$\mathbf{S} = (i, p(i), c_{D,I}(i), c_{D,P}(i), c_{ID}(i)) \text{ with } i = 1, 2, \dots, n_s \quad (3)$$

A probabilistic system representation in this form is generally less than trivial and computationally expensive to establish for most systems of practical relevance. However interesting, the challenges associated with the efficient probabilistic analysis of systems are treated elsewhere and in the analyses presented in Chapter 3 of the present paper crude Monte Carlo simulation is utilized to establish the information contained in Equation (3).

### 2.3. Robustness modeling and quantification

Based on the system representation provided by Equation (3) it is possible to assess the performances of the system with respect to possible hazard events which may occur over time. The robustness of systems is one of the system characteristics that have attracted the most attention in this respect. The objective being to establish a means for assessing the degree to which a system is able to contain or limit the immediate consequences of hazards and thereby ensure that systems are designed and managed with an appropriate degree of robustness. Risk based formulations for the quantification of systems robustness are first provided in Baker et al. ([3]) and JCSS ([13]). In Faber ([8]) these formulations are revisited and a more general and consistent scenario based approach to the quantification of robustness is proposed. The underlying idea is, along the same lines of reasoning proposed in Baker et al. ([3]), to relate the robustness of a system to the ratio between direct consequences and total consequences. Fundamentally this ratio is random and in Baker et al. ([3]) it is suggested to assess it through the expected values of the two terms individually (or equivalently through the direct and total risks). In Faber ([8]) this

ratio is taken scenario wise and in this manner an index of the robustness of a system with respect to a given scenario  $i$ , i.e.  $I_R(i)$  may be assessed as:

$$I_R(i) = \frac{c_D(i)}{c_T(i)} \quad (4)$$

The direct and total consequences  $c_D(i)$  and  $c_T(i)$  entering Equation (4) may be interpreted with some flexibility depending on the focus of the system assessment. If the focus of the system assessment is directed on the representation and analysis of cascading failure event scenarios Equation (4) may be rewritten as:

$$I_R(i) = \frac{c_{D,I}(i)}{c_{D,I}(i) + c_{D,P}(i)} \quad (5)$$

where  $c_{D,I}(i)$ ,  $c_{D,P}(i)$ , represent the direct consequences associated with the initiation phase and the propagation phase of the failure scenario of the system, respectively.

If on the other hand the emphasis is directed on the ability of the system to contain the development of consequences Equation (4) may be written as:

$$I_R(i) = \frac{c_{D,I}(i) + c_{D,P}(i)}{c_{D,I}(i) + c_{D,P}(i) + c_{ID}(i)} \quad (6)$$

As the scenarios  $i$  are random in nature, as reflected by their occurrence probabilities  $p(i)$ , it is realized that the robustness index  $I_R(i)$  itself is a random variable which may be analysed further by categorization and ordering of the different scenarios in accordance with the hazard, damage, failure and consequence events they are composed of. In this manner robustness indexes for a given system can, as illustrated in Chapter 3 be assessed probabilistically conditional on e.g. the type and/or intensity of the hazard event as well as the magnitude of direct, indirect or total consequences. Moreover, the scenario based approach allows for tracking which constituent damages and failures contribute the most to e.g. poor robustness performance as well as to the total consequences.

It should be noted that robustness is not desirable per se. As already underlined in Chapter 2 systems shall fundamentally be designed and managed based on a holistic modelling and assessment of service life benefits. The robustness of a system can often be increased but generally only in a trade-off with efficiency. However, as will also be apparent in Chapter 3, robustness and resilience are strongly interdependent and the optimal design and management of systems depends on a thorough understanding of this dependency.

#### 2.4. Resilience modeling and quantification

A relatively large variety of propositions for the modelling and quantification of systems resilience are available in the literature, see e.g. [5]- [14] Most often the suggested models are directed on the short term representation of the ability of the system to sustain and recover from disturbances, fast, without substantial loss of functionality and without the support from the outside. Hazard and disturbance events are generally specified in terms of type and intensity and the ability to sustain and recover from disturbances is modelled through the social, organisational and adaptive capacities together with traditional characteristics of technical systems such as strength, ductility, brittleness, redundancy, segmentation and diversity, see e.g. [7] and [18].

Following the life-cycle benefit considerations in the resilience model presented Faber ([8]), however, systems resilience models and assessments should ultimately account for not only the loss of functionality, but also for the generation of the capacity which is critically important for the fast and successful reorganisation, adaptation and rehabilitation following disturbances and hazard events. Therefore a life-cycle model of systems resilience is proposed here in which scenarios of benefit generation and losses are modelled and analysed and where insufficient resilience or systems resilience failure is defined as exhaustion of system capacity (social, financial and/or environmental). Resilience, in the same manner as robustness is thereby a system characteristic of a random nature

and requirements to resilience may only meaningfully be specified probabilistically; e.g. in terms of an acceptable annual probability of resilience failure.

In Figure 3 this idea is illustrated for the simple case of a system for which the only explicitly considered capacity is a financial reserve collected as a fixed percentage of the annual benefit generated by the system over time. The general shape of the benefit loss curves in the aftermath of disturbances reflects that in general a certain time is required before the functionality can be re-established, and in the first instance only up to a certain level, reflecting that interim solutions are foreseen, implemented and operated while waiting for the preparation and implementation of full and possibly even improved system rehabilitation. In Figure 3 two pairs of time histories of benefit generation and accumulated economic reserves are illustrated. It is seen how disturbance events both reduce the benefit generation as well as the reserves. In the time history illustrated with a green line it is seen that a disturbance event exhausts the accumulated reserves and causes a resilience failure.

As for the case of robustness, conditional resilience may be modelled and assessed utilizing the scenario based life-cycle oriented approach. Conditioning on hazard events of given characteristics, the resilience can be defined as recovery within a given time horizon without exceeding available reserves.

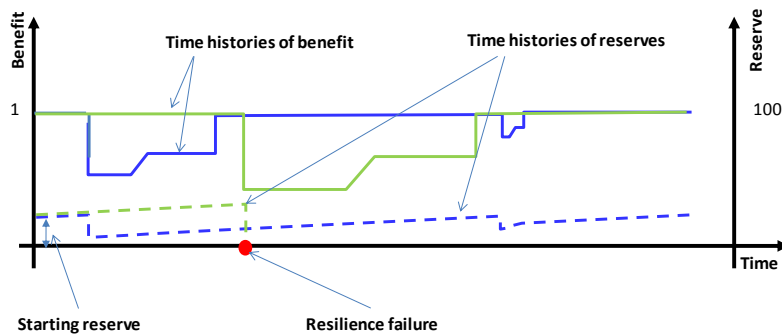


Figure 3 Illustration of the resilience model in terms of time histories of benefit generation and corresponding time histories of accumulated economic reserves.

Examination of Figure 3 reveals that the first immediate drop in the benefit rate (or functionality) after a disturbance event relates directly to the systems robustness. Even with moderate assumptions concerning the contribution of indirect consequences to total consequences it is apparent that cascading failures and loss of functionality plays a significant role for the resilience of the system. Moreover, it is seen in Figure 3 that a starting capital or reserve is assumed available at time  $t = 0$ . In a normative perspective such a reserve is indeed possible, provided that the portfolio of assets in the considered system is sufficiently large. In the design and management of systems, however, sufficient resilience critically depends on the maintenance of this reserve as illustrated in the example presented in Chapter 3.

### 3. Principal example – interlinked infrastructure system

In the following example the probabilistic performance of an interlinked system over time is addressed and analyzed in accordance with the framework and approaches outlined in Chapter 2. The considered interlinked system is assumed to be comprised by the following subsystems:

- Infrastructure system
- Governance system
- Geo-hazards system
- Regulatory system

The temporal performance of the interlinked system is represented by the time-slice model illustrated in Figure 4:

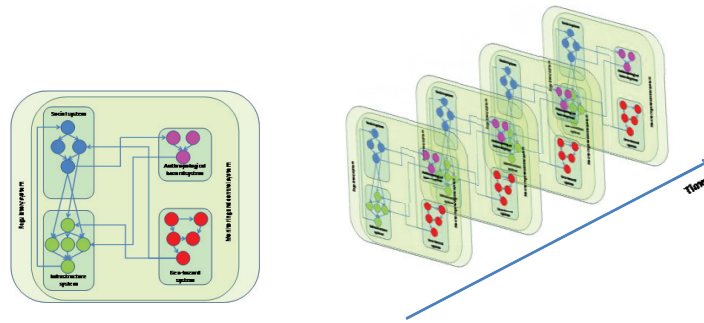


Figure 4 Illustration of interlinked system with time slicing over time ([9])

It is assumed that decision optimization is performed from a regulatory normative perspective – why the available decision alternatives are identified and ranked by the regulatory system. In the present example human health and safety as well as local and global effects to the environment are excluded for purposes of simplification – however, these can be included by constraints to the decision optimization. Finally, the systems performance is assessed for a life-cycle equal to 100 years. In the following an outline is provided on the modeling of each of the sub-systems illustrated in Figure 4.

### 3.1. Infrastructure system

For simplicity the infrastructure system is represented through a Daniels system comprised of  $n_c$  constituents, see Figure 5. Each constituent has a resistance with respect to operational loading  $L$ , i.e.  $R$  and a resistance with respect to geo-hazard disturbances  $H$ , i.e.  $\eta$  (see also Figure 6).

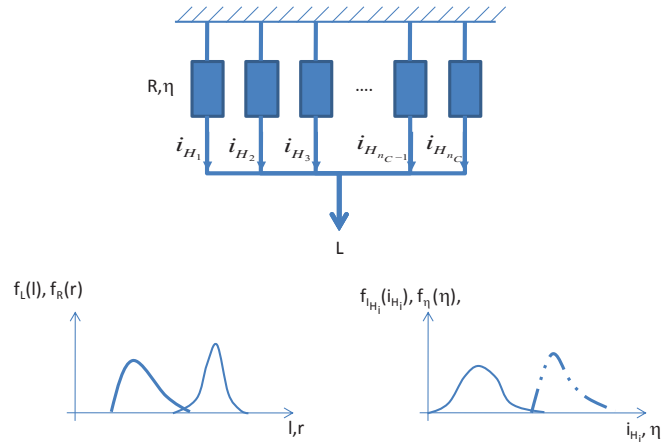


Figure 5 Illustration of probabilistic model for the operational loads and resistances of the infrastructure system with respect to operational loads as well as geo-hazard disturbances.

The resistances of the infrastructure system  $R$  and  $\eta$  are modeled by Log-normal random variables and the operational annual maximum loading  $L$  is modeled by a Gumbel distributed random variable. The daily maximum operational load is modeled by a Weibull distribution (to ensure non-negative realizations) which is fitted such that it provides the same 98% upper fractile value as the Gumbel distribution for the annual maximum. The daily

maximum operational load is relevant in the case where a natural hazard disturbance has damaged the system and the further progression of system damage is assessed subject to redistribution of internal system demands. The natural hazard disturbances are defined in the description of the geo-hazard system.

The random variables representing the resistances and operational loading are assigned the following moments:

Table 1 Probabilistic model for component resistances and operational load disturbances

| Variable | Expected value | Coefficient of Variation |
|----------|----------------|--------------------------|
| $R$      | $E[R] = 1$     | $COV[R] = 0.2$           |
| $\eta$   | $E[\eta] = 1$  | $COV[\eta] = 0.3$        |
| $L$      | $E[L] = 1$     | $COV[L] = 0.3$           |

The resistances with respect to operational failures and natural hazard disturbance failures,  $R$  and  $\eta$  are assumed correlated with correlation coefficient  $\rho[R, \eta] = 0.8$ .

The limit state functions representing failure of the individual constituents of the infrastructure system with respect to operational annual maximum operational loads and natural hazard disturbances are given as:

$$\begin{aligned} g_O(\mathbf{x}) &= z_1 r - l \\ g_H(\mathbf{x}) &= z_2 \eta - i_H \end{aligned} \quad (7)$$

where  $z_1$  and  $z_2$  are design parameters which may be chosen to comply with the requirements of the regulatory system with respect to target probabilities of constituent failure (see Section 3.4).

Failure of the system takes place either due to annual maximum operational loads exceeding the capacities of the constituents with possible subsequent cascading failure scenarios, or by constituent failures due to natural hazard events which then due daily maximum operational loads may lead to cascading constituent failure scenarios for the system.

The infrastructure system is assumed to provide benefit (here for simplicity of notation but without loss of generality assumed net of ordinary maintenance costs) to society and thereby support the governance system. In case the infrastructure is not disrupted by failure events caused by disturbances the annual rate of benefit is assumed to be constant in time and equal to  $b(t) = 1(y^{-1})$ . The benefit generation may, however, be disrupted and reduced by disturbance events as outlined later in the description of the governance system (see Section 3.3).

In case of failure of constituents of the infrastructure system the constituents are replaced. The costs of the replacement of all constituents of the infrastructure i.e. to build a new infrastructure system is assumed to correspond to the benefit generated over a period of 10 years (return of investment period). Thus it is assumed that the replacement costs in a given event scenario of infrastructure system constituent failures is assumed directly proportional to the number of failed constituents in that event scenario, i.e.  $C_F = \frac{10n_f}{n_c}$ .

### 3.2. Geo-hazard system

The natural hazard disturbance events are assumed to follow a Poisson counting process with annual occurrence rate  $\lambda_H = 0.01$ . The intensity of disturbance events acting on each of the  $n_c$  constituents of the infrastructure system is modelled by a random vector  $\mathbf{I}_H$  with components assumed to be log-normal distributed. The realizations of  $\mathbf{I}_H$  are assumed independent from time to time but the disturbances acting on the constituents at a given time are assumed correlated with correlation coefficient  $\rho_{i_H}$ . In Figure 6 Probabilistic modeling of geo-hazard disturbance events acting on one constituent of the infrastructure system and their inter arrival times  $T$ . the probabilistic modelling of the geo-hazard disturbances acting on one constituent of the infrastructure system is illustrated.



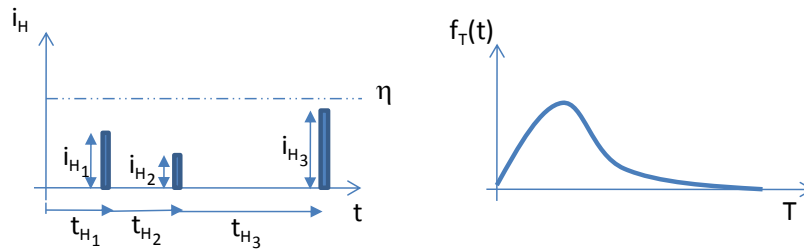


Figure 6 Probabilistic modeling of geo-hazard disturbance events acting on one constituent of the infrastructure system and their inter arrival times  $T$ .

The expected value and the coefficient of variation of the intensity  $I_H$  i.e.  $E[I_H]$  and  $COV[I_H]$  are equal to 1 and 0.4 respectively.

### 3.3. Governance system

The main function of the governance system is simplified to be the response to failures of the infrastructure caused by disturbances and operational loads. Strongly simplified it is assumed that the governance system can be represented by the functionality disturbance and recovery curve illustrated in Figure 7.

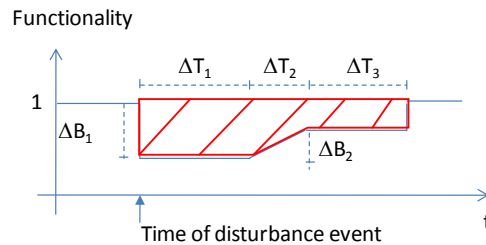


Figure 7 Illustration of the representation of the performance of the governance system with respect to reorganization and recovery of infrastructure functionality after disturbances.

The time variation of functionality illustrated in Figure 7 shows how the functionality is reduced by  $\Delta B_1$  at the time of disturbance.  $\Delta T_1$  represents the time till the governance system has established an overview of the situation and initiates commissioning of temporary measures to re-establish functionality. The temporary measures are assumed to be fully functional after a period  $\Delta T_2$  with a resulting functionality gain equal to  $\Delta B_2$ . In parallel to and after commissioning temporary measures it is assumed that permanent measures for re-establishing functionality are being planned and prepared. These are assumed commissioned after a period  $\Delta T_3$ .

The loss of functionality of the system  $\Delta B_1$  in a given scenario of failed constituents is for simplicity assumed to be proportional to the number of failed constituents  $n_F$ , i.e.  $\Delta B_1 = \frac{n_F}{n_C}$ . The periods  $\Delta T_i, i = 1, 2, 3$  describing the principal functionality loss and recovery curve are modeled by log-normal distributed random variables. In order to model the preparedness performance of the governance system two levels of preparedness are considered namely

low and high. The expected values  $E[\cdot]$  and coefficients of variation  $COV[\cdot]$  for the random variables are given in Table 2. To account for the dependency between the loss of functionality and the preparedness and capacity to rehabilitate the expected values of the duration  $\Delta T_1$ ,  $\Delta T_2$  and  $\Delta T_3$  are assumed proportional to  $\Delta B_1$ . The integral off the losses (shown as the red striped area in Figure 7) constitutes the loss function  $A$ . Based on this assumption, both total recovery time  $T$  and losses  $A$  are log-normal distributed with parameters proportional to  $\Delta B_1$  and level of preparedness.

Table 2 Definition of the probabilistic model of the governance system with respect to preparedness and capacity.

| Variable     | Distribution model | Low preparedness               |     | High preparedness                      |     |
|--------------|--------------------|--------------------------------|-----|--|-----|
|              |                    | Expected value (y)             | COV | Expected value (y)                     | COV |
| $\Delta T_1$ | Log-normal         | $E[\Delta T_1] = \Delta B_1$   | 0.2 | $E[\Delta T_1] = \frac{\Delta B_1}{2}$ | 0.1 |
| $\Delta T_2$ | Log-normal         | $E[\Delta T_2] = 5\Delta B_1$  | 0.2 | $E[\Delta T_2] = \Delta B_1$           | 0.1 |
| $\Delta T_3$ | Log-normal         | $E[\Delta T_3] = 20\Delta B_1$ | 0.2 | $E[\Delta T_3] = 10\Delta B_1$         | 0.1 |
| $\Delta B_1$ | Deterministic      | $\frac{n_F}{n_C}$              |     | $\frac{n_F}{n_C}$                      |     |
| $\Delta B_2$ | Deterministic      | $0.5 \times \Delta B_1$        |     | $0.8 \times \Delta B_1$                |     |

It is assumed that the governance system maintains a reserve capital to be available over the life-cycle of the infrastructure system for covering the cost of replacement of system constituents which may fail over time due to disturbance events. At time  $t = 0$  the starting capital reserve is modelled as a percentage  $\chi$  % of the expected value of the accumulated benefits over the life-cycle of the infrastructure system.

### 3.4. Regulatory system

The regulatory system is assumed to manage the performance of the infrastructure system on behalf of the governance system in the context of the operational hazards and the geo-hazard system. It is thus assumed that the management is undertaken by setting requirements to the following parameters of the model:

- The annual probability of individual constituent failure with respect to operational load disturbances is set to  $p_{f,o} = 10^{-3}$ , and incorporated into the model through calibration of  $z_1$  (see Equation (7))
- The annual probability of individual constituent failure conditional on the event of a geo-hazard disturbance is set to  $p_{f|H} = 10^{-2}$  and incorporated into the model through calibration of  $z_2$ .
- The percentage of annual benefit  $\chi$  which is saved for financing of repair and replacement of infrastructure constituents after future disturbance events and thereby to ensure a certain level of resilience.

### 3.5. Analysis results

Following the introduction of the principal example together with the proposed framework (see also [8], [9]) and models, the robustness and resilience of the interlinked system are investigated. The constituents of the infrastructure system are assumed to behave brittle at failure, implying that they lose their carrying capacity completely after their capacity limit is reached.

The robustness is studied with the robustness index conditional on the scenarios disturbance from the geo-hazard and operational hazard load in dependency of the number of infrastructure constituents. The direct consequences are

calculated as the replacement costs associated with constituents failed due to the disturbance (before internal load redistribution) and the indirect consequences are associated with replacement due to failures caused by internal load redistribution (see Equation 5). The CDF (cumulative distribution function) representing the non-exceedance probability of the conditional robustness index is depicted in Figure 8 for the number  $n_c = 2, 4, 6, 8$  and 10 of infrastructure system constituents. It is observed that the probability of a robustness index lower than  $x$  is larger for operational hazard events than for geo-hazard events. The reason for this is that when constituents fail due to extreme (annual) operational loads, then the internal loads which must be redistributed subsequently also correspond to annual maximum operational loads. When on the other hand constituents of the system fail due to geo-hazard events then the internal loads which must be redistributed subsequently correspond only to daily maximum operational loads. It is thus more likely that cascading failure scenarios develop in the former case. Already for systems with six or more constituents, the probability of a robustness index smaller than 1, conditional on a geo-hazard event tends to be very close to zero why cascading failure scenarios in these cases are very unlikely.

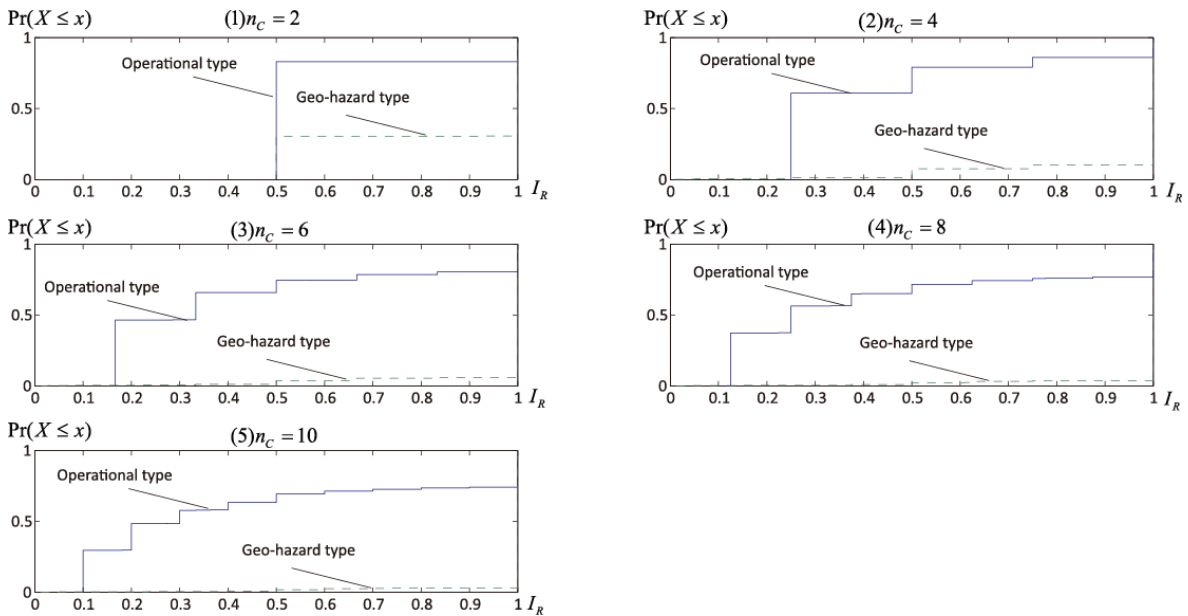


Figure 8 Illustration of conditional probability distribution functions for the robustness index of the interlinked system in the cases of operational and geo-hazard type disturbances and for different numbers of infrastructure system constituents  $n_c$

The resilience of the interlinked system as described in Section 2.4 depends on a number of factors such as the frequency and types of disturbances, the capacity and robustness of the infrastructure system and the capacity of the governance system. Important characteristics of the governance system are comprised by the total recovery time  $T$  and the total loss of benefit  $A$  given a disturbance. For  $n_f / n_c = 0.5$  the probability distribution function of  $T$  conditional on different quantiles of loss of benefit  $Q_A$ , for the cases of governance systems with low and high preparedness, are shown in Figure 9. Not surprisingly shorter recovery times are significantly more probable for governance systems with a high level of preparedness. This is expected as the expected value of the recovery time is directly proportional to the factors by which the individual recovery times are defined, in dependency of the benefit (see Table 2).

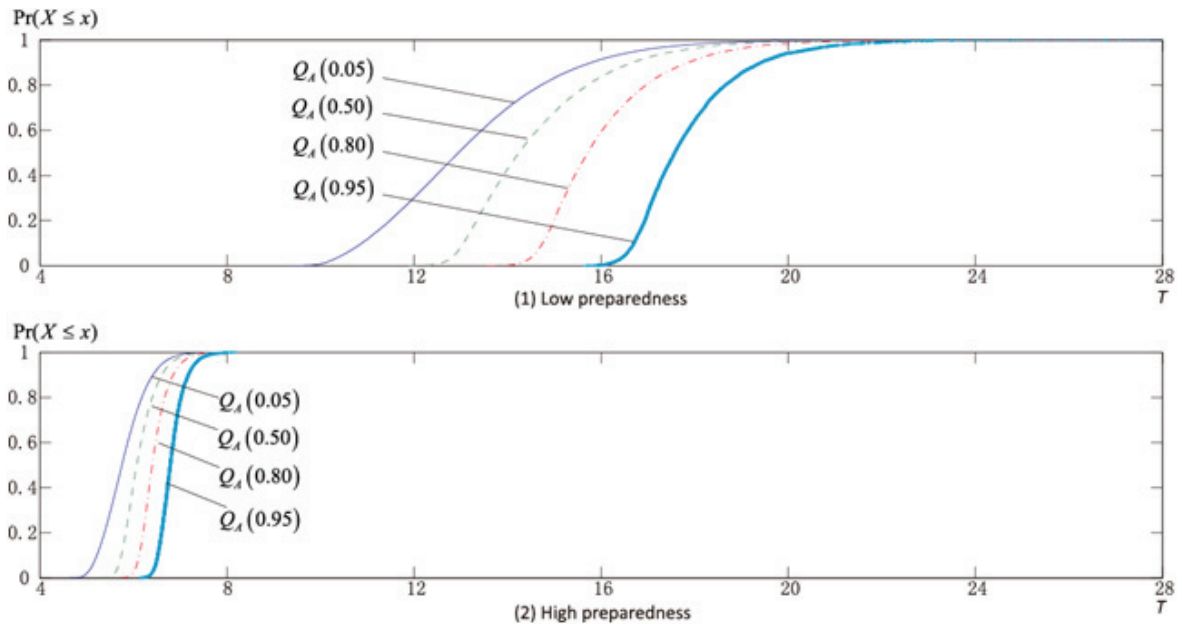


Figure 9 The CDFs of the total recovery time  $T$  with different quantiles of the area  $A$  for the system with low or high preparedness given that  $n_f / n_c = 0.50$

The probability of resilience failure, i.e. the annual probability of exhaustion of the accumulated financial reserve by disturbance events as a function of the decision parameter  $\chi$  are illustrated for the cases of low and high preparedness of the governance system, respectively (note that for the study of the resilience the infrastructure system is comprised by 5, 10, 15 and 20 constituents respectively). By comparison of Figure 10 and Figure 11 it may be observed that the difference between the curves representing different numbers of constituents are more pronounced for the case of high preparedness. This might indicate that in Figure 10 the (lack of) resilience is dominated by a poor capacity of the governance system rather than the performance of the infrastructure system, whereas in Figure 11 the performance of the infrastructure system plays a stronger role. Moreover it is seen that for the case of low preparedness (Figure 10) the curves are relatively flat up until  $\chi = 16\%$  where after it decreases rather rapidly. For the case of high preparedness (Figure 11) a similar behavior is exhibited already for  $\chi = 8\%$ .

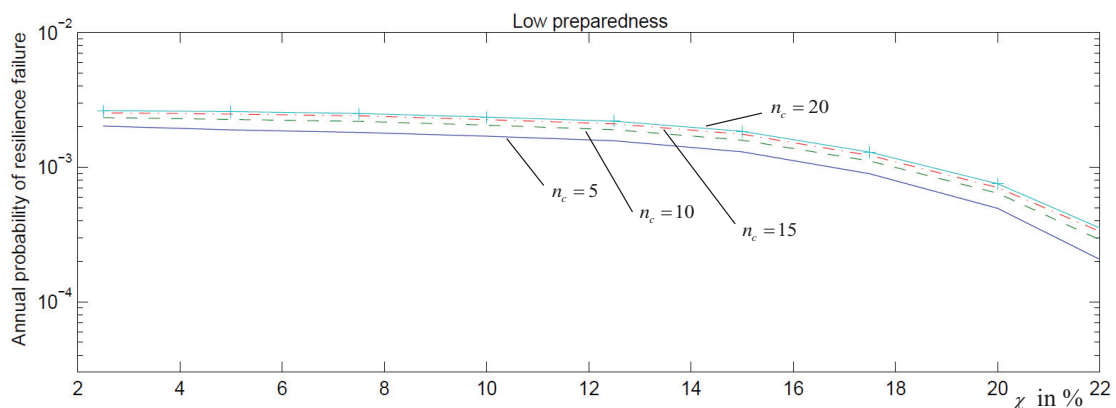


Figure 10 Annual probability of resilience failure for low preparedness as a function of the decision parameter  $\chi$  with  $n_c = 5$  (blue/continuous),  $n_c = 10$  (green/dashed),  $n_c = 15$  (red/dash-dot),  $n_c = 20$  (cyan/crosses)

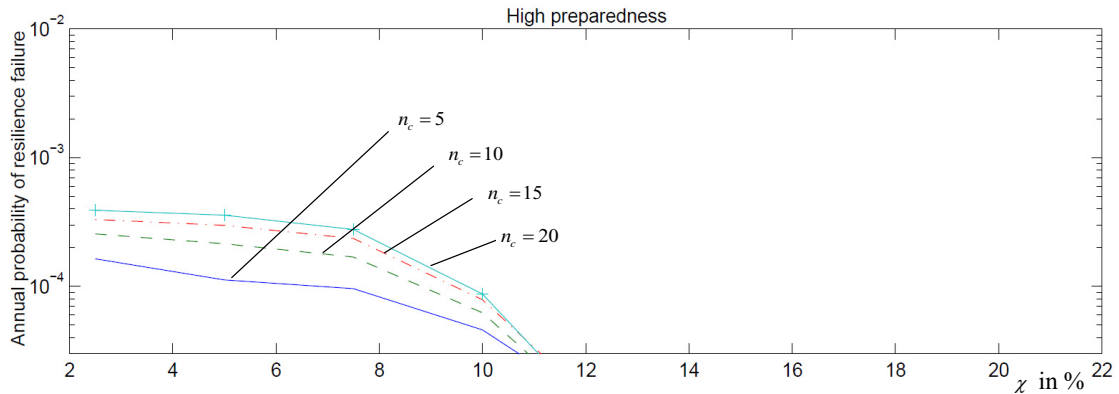


Figure 11 Annual probability of resilience failure for high preparedness as a function of the decision parameter  $\chi$  with  $n_c=5$  (blue/continuous),  $n_c=10$  (green/dashed),  $n_c=15$  (red/dash-dot),  $n_c=20$  (cyan/crosses)

#### 4. Discussion and conclusions

In the present paper a novel probabilistic framework for the representation and assessment of interlinked systems is presented. The framework aims to address optimal design and management of infrastructure systems and the built environment in general in the context of societal governance, regulation as well as operational and natural hazards and to facilitate that the interactions of these subsystems are accounted for. Based on the presented framework a principal example is presented in which the robustness and the resilience of system is quantified and analyzed as a function of the characteristics of the infrastructure system which may be influenced by decisions regarding the design (number of constituents acting in parallel), the governance system through its preparedness with respect to disturbances and the regulatory system for what concerns the decision on how much of the generated benefit from the infrastructure systems shall be kept in as a reserve for reconstructions and repairs following future major disturbances. The proposed framework facilitates that the robustness of the considered interlinked system may be analyzed in detail with respect to which scenarios contribute the most to potential inappropriate robustness and thereby also points to efficient means of improvements. Finally it is shown how the proposed framework allows for assessing the probability of resilience failure as a function of the parameters describing the various subsystems of which the interlinked system is comprised in addition to the more traditional assessments of functionality losses and time till recovery.

The presented framework and corresponding approaches for the modeling and assessment of the robustness and resilience of interlinked systems is still rather new and several features need to be explored in more detail. However, the framework appears to be relatively flexible and accommodates for a number of more detailed assessments that usually considered and thereby could add significant value as a platform for decision support on the design and management of interlinked systems.

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