Global Sustainability Institute

Resources, Financial Risk and Dynamics of Growth

Systems and Global Society

ONLINE APPENDIXES

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Pasqualino R. and Jones A. (2020) Resources, Financial Risk and Dynamics of Growth – Systems and Global Society, Routledge, Oxford.

This document provides a technical appendix to the Part II of the book 'Resources, Financial Risk and Dynamics of Growth – Systems and Global Society' published by Pasqualino R. and Jones A in 2020.

The aim of the book (and this appendix) is both to build clarity around the use the system dynamics approach for the modelling of economic and ecological systems, as well as address an important gap in the literature between the Limits to Growth study (Meadows et al 1972, Meadows et al 1974), Meadows et al 1992, Meadows et al 2003) and today's decision making. In so doing, a novel System Dynamics model named Economic Risk Resources and Environment (ERRE), starting from a basic framework of the System Dynamics National model as proposed in Sterman (1981), and the last version of the Limits to Growth World3-03 model (Meadows et al 2003) is developed, extended with a climate module and analysed.

In particular, Part I of the book provides a review of the Limits to Growth model and compares that with reality. We provide a basic description of system dynamics to learn how the World3 model works. From a top-down perspective, the World3-03 model is then presented. This is done in a way that was not available in the literature, giving emphasis on the fourteen non-linear relationships that generate behaviour, showing how the dynamics of both growth and collapse emerge in the system due to the interconnection among the two. As a response to the work of Turner (2008, 2012, 2013), who compared the recent historical data trend to the behaviour of the scenario 1 of the Limits to growth, we provide a calibration of World3 with real world data in Chapter 2. This demonstrates how different the world evolved in comparison to the Limits to Growth scenarios, which conclusion can also be found also in Pasqualino et al. (2015). In Chapter 3, we provide a description of how the real world has evolved since Limits to Growth was first published, starting from the principles of capitalism, finance, and reasons why productivity and technology growth were considered the engine for prosperity. Such a review provides a metric of comparison between the Limits to Growth forecast and the reason why the world evolved as we it is today. Most important, it reveals climate change as the major treat to today's society, being interlinked with both energy and food systems globally.

Building on this, Part II shows how the science of modelling and policy consultancy evolved. Chapter 4 shows the evolution of economic understanding of systems, including how different schools of thought defend their own values. With the aim of comparing computer modelling schools, highlighting their strengths and complementarities, the system dynamics school is analysed in terms of its relationship with the economic profession since the time it was initially conceived (Forrester 1956). Thus both the Limits to Growth, and further work of the community, are analysed from the economic modelling perspective. Thus Chapter 4 provides further elucidation of those elements that, we believe, a system dynamics modeller should be aware of before engaging in economic modelling activities to influence system policy change. On the other hand, other economic communities should be aware of the potential of system dynamics as contributing to the behavioural, evolutionary, post-Keynesian and institutional economic schools of thoughts while providing disciplined and rigorous formal modelling methods.

Chapter 5 and 6 provide a description of the ERRE model, starting from the framework emerging from the World3-03 model (Meadows et al. 2003), and the System Dynamics National Model as proposed in Sterman (1981) in the Energy Transition and the Economy model. The ERRE model can be considered as a stock and flow consistent impact assessment model to address the financial risks emerging from the interaction between economic growth and environmental limits under the presence of shocks. We frame the resulting ERRE as a system dynamics model, which overlaps with neo-classical, evolutionary, behavioural, Post-Keynesian and ecological schools of economic thought. Finally, statistical validation, analysis of resilience in the presence of short-term shocks, and long term stress tests and scenarios are assessed to analyse the fat tail extreme risks dependent on the interaction between economic growth, financial risk and global resource limits.

While the Chapter 5 of the book provides the structures and data architecture that allows all the subsystems to be interconnected together, the Appendix 1 below shows the detail of the model in line with all system dynamics structures, equations, and assumptions that impact the behaviour of the ERRE. This is followed by an Appendix 2, which provides a variety of behavioural tests to explore the whys the model can be considered a disequilibrium model, and links to, at least, five schools of economic thoughts. Thus, Appendix 2 forms the basis for the Chapter 6 of the book, where a statistical comparison between model output and historical data is performed, and where stress testing scenarios are presented in relation to the analysis of sustainability in today's world. An online version of the model can be found at https://doi.org/10.25411/aru.10110710.

1. System Dynamics modelling of ERRE

Appendix summary

This appendix is supportive of Chapter 5 of Pasqualino and Jones (2020), which provides a top down system perspective that is required to understand the core dynamics of the ERRE model. It describes the System Dynamics structures necessary to capture the complexity of the real world system in greater detail. In particular, a modular approach has been adopted resulting in a fully integrated system theory. The idea is that system structures that are commonly representative of business sectors could be used more times to describe sub-systems within systems. Idiosyncrasies have been applied whenever necessary to capture different dynamics across systems.

ERRE model structures

Table 1.1 describes the sub-sectors that represent the Government, Financial system, Energy market, and Climate Impact sectors. In particular, the Government sector can be seen as an integrating sector for the rest of the economy. For example, GDP is calculated as sum of investments and consumption from the entire economy, whereas the Labour market picks information from the every sector to determine how labour would move across them. The financial sector controls finances across sectors. The climate system accumulates depletion while the economy grows, and acts as negative feedback to the economy when damage overcomes a certain threshold. The energy market allows the collection and distribution of energy demand across the energy providers as well as allocating shortages in case of energy crisis.

Table 1.2 and Table 1.3 shows the subsystems used to model the real sectors and how many times these structures have been reused to describe different sectors. The sub-sectors named ‘System Boundaries’ and ‘Initialization’ captures the rationale described in Chapter 5 of this Pasqualino and Jones (2020), aiming at assuring stock and flow consistency in both financial and real dimensions of the economy. In particular, these are used to determine the flows of cash and output across sectors, the values of key parameters and the initial values of most stocks are calculated based on the relationships with the other sectors.

Table 1.1- Subsectors of the Energy market, Government and Financial Sector

Energy Market	**Government**	**Financial Sector**	**Climate Sector**
Prices	System Boundaries/Tax Revenue	System Boundaries/Money Supply	System Boundaries
Orders fraction	Initialization	Initialization	Initialization
Shortfall Allocation	GDP – National Income Accounts	Interest Rate Nominal	GHG emissions and Impact
Indicators	GDP Deflator - Inflation	Money creation	-
-	Labour Market	Financial Leverage	-
-	Balance Sheet (Gov)	Balance Sheet (Bank)	-
-	Cash Flows and Spending	Cash Flows, Income, Taxes	-
-	Income Tax/Corporate Tax	Financial Decisions	-
-	Subsidies	-	-
-	Energy Transition Policies	-	-

All other sub-sectors describe specific aspects of the model in a highly modular way. For example, although having a unique role in the model as well as being at the heart of the entire financial system, the household sector shares a similar structure with the firm sector. In particular, capital formation structure, balance sheet, debt, financial risk and interest rates, financial assets, labour mobility, and energy requirements sub-sectors present little to no differences between households and firms. However, sub-sectors of firms such as wages, dividends, return of investments, prices, as well as a labour hiring structure, are not present in the households sector. Firms' structures overlap among themselves nevertheless presenting idiosyncrasies giving sectorial differences. In addition, productive sectors compete for labour based on wage in the labour market. Commodity markets is also included between producers and users of their output.

In total, the ERRE model is composed of 47 sub-structures reaching 148 sub-dimensions when accounting for their reusability. This explains the large numbers of variables and elements in the system. The ERRE model accounts for approximately 250 stock variables, 3500 auxiliary variables, 500 parameters, and 120 non-linearities.

The appendix follows by describing the system sub-dimension after sub-dimension grouping them by sectors. It first describes the Government and Banking sectors that lie above the system. Secondly each sub-dimension of the real sector is shown. Each of them will be presented once only, while describing possible idiosyncrasies for each particular sector it has been applied to. The Labour market will be represented while describing wages and labour mobility across sectors, while the energy market after having Prices and Production for each sector.

Table 1.2- Use and Reuse of sub-components of systems to describe all sectors of the economy (1 of 2)

Sub-Sector	**Capital**	**Goods and Services**	**Agriculture**	**Fossil Fuels**	**Renewables**	**Households**
System Boundaries	✓	✓	✓	✓	✓	✓
Initialization	✓	✓	✓	✓	✓	✓
Production (general)	✓	✓			✓	
Production (Agriculture)			✓			
Production (Fossil Fuels)				✓		
Utility						✓
Capital	✓	✓	✓	✓	✓	✓
Agricultural Land			✓			
Good and Services						✓
Food	✓					✓
Energy Requirements of Capital	✓	✓	✓	✓	✓	✓
Labour Productivity	✓	✓	✓	✓	✓	
Labour force	✓	✓	✓	✓	✓	
Labour Supply	✓	✓	✓	✓	✓	✓
Wage	✓	✓	✓	✓	✓	
Price	✓	✓	✓	✓	✓	

Table 1.3 - Use and Reuse of sub-components of systems to describe all sectors of the economy (2 of 2)

Sub-Sector	Capital	Goods and Services	Agriculture	Fossil Fuels	Renewables	Households
Balance sheet (firm)	✓	✓	✓	✓	✓	
Balance sheet (Household)						✓
Financial Decisions	✓	✓	✓	✓	✓	✓
Cash Flows	✓	✓	✓	✓	✓	✓
Savings Propensity						✓
Value of Capital/Assets	✓	✓	✓	✓	✓	✓
Marginal values of Assets						✓
Depreciation	✓	✓	✓	✓	✓	
Income Statement & Taxes (firm)	✓	✓	✓	✓	✓	
Income statement & Taxes						✓
ROI, Adj Returns, Capital Charge Rate	✓	✓	✓	✓	✓	
Dividends	✓	✓	✓	✓	✓	
Interest Rate and Risk	✓	✓	✓	✓	✓	✓
Borrowing	✓	✓	✓	✓	✓	✓

Government

The role of the Government in the ERRE model is to control the public accounting system, collect taxes from each sector of the economy and return expenditure, subsidies and service the economy via tax change. The Government is assumed to not accumulate physical assets, and it should be seen in conjunction to the households sector. In fact, all Government expenditures are assumed to go direct to households as Government Transfers. Households can make purchase decisions and accumulate assets. Thus the physical assets of the household sector should be seen as aggregate between private consumption and public property at the service of the economy.

In addition, the government is allowed to produce debt thus raising money expenditure, and boosting the economy. Options for subsidy, interest rate accumulations and tax changes are also modelled and allow the user to test options of financing those via debt creation or reduction in expenditure.

GDP, NATIONAL INCOME ACCOUNTS AND GDP DEFLATOR

The calculation of nominal GDP in the model, follows the standard formulation. As the model is globally aggregated, import and export have been neglected, thus resulting in the sum between consumption and investment as proposed in Eq. 5.25:

Eq. 1.1 - GDP

$$GDP = \sum_{\Pi} \Psi_i + \sum_{H} C_i$$

Where Ψ_i represents the investments of each firm *F* allocated to the construction of new assets (both capital and agricultural land), and C_k represent the money spent in consumption in the household sector. These include capital construction, purchase of durable goods and services, consumption of food and energy.

Similarly the GDP Deflator is calculated as the average change in inflation for all commodities used to calculate GDP and used to measure inflation in the model. The equation used is:

Eq. 1.2 – GDP Deflator

$$GDP_{DEF} = GDP_{DEF_0} \frac{\sum_{\Pi} P_i p_i + E_H p_E}{\sum_{\Pi} P_i p_{i_0} + E_H p_{E_0}}$$

Where $P_i p_i$ is the multiplication between Production and Prices of output for each producing sector, (thus measuring the revenue for each sector), $E_H p_E$ is the multiplication between Households energy consumptions and Price of energy (thus accounting for households' energy expenditure). The denominator accounts for the same equations keeping price of each commodity constant at the initial time. The result is multiplied by the initial GDP deflator. The GDP deflator has been normalized to 2010 values, thus assuring $GDP_{DEF} = 1$ in the year 2010 of the calibrated simulation.

The GDP Deflator is one of the most important variables in the model in which more than 13000 feedback loops pass through. The GDP deflator trend is used as a reference to calculate yearly inflation, thus input to every investment function and interest rate in the model. GDP deflator is also used to model the national accounting system converting nominal to real values. For example, Real GDP is calculated accordingly.

GOVERNMENT BALANCE SHEET, CASH FLOWS AND REVENUE

Figure 1.1 presents the Balance sheet of the Government sector showing both assets (left hand side) and liabilities (right hand side). While the government receives income as tax revenues from the every sector of the economy, it is allowed to increase debt via increasing borrowing above debt retirement over time. The Debt determines the interest payments that the government owes to creditors and financing via reduction in spending. In a similar manner, government can distribute subsidies in each sector of the economy, and decide to finance it via debt or reducing expenditure. The Government is assumed to spend instantaneously all cash received as input, thus maintaining the stock of cash at zero for the entire simulation time.

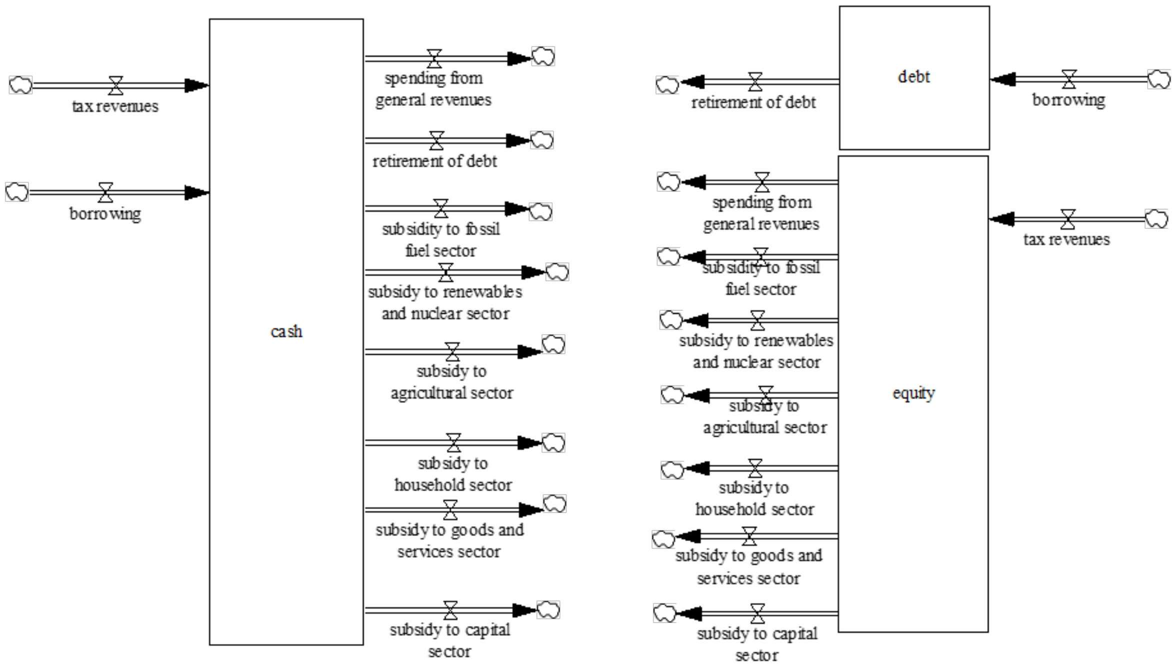
The main input to the balance sheet is the Tax Revenue calculated as:

Eq. 1.3 – Government revenue

$$R_{Gov} = \sum_{\Pi+H+B} T_i + T_E$$

Where $\sum_{F+H+B} T_i$ is the tax payments from each firm, household and bank, and T_E represent the tax income received form the energy sector in case of carbon tax or similar. In the standard run, tax rates are considered constant and carbon tax null. However, other policy scenarios could assume variations on those.

Figure 1.1 – Balance Sheet of the Government sector



The interest rate on Government rate is lower than the interest rates applied to firms, thus a coefficient θ is introduced to account for such a reduction. Interest payments Y_{Gov} is calculated as follows:

Eq. 1.4 – Interest payments from government

$$Y_{Gov} = D_{Gov} \times i_n \times \theta$$

Where D_{Gov} is the debt of the government, i_n is the nominal interest rate applied to the other sectors of the economy, and θ a positive corrective factor lower than 1.

Government spending is a key decision variable in the model. ERRE includes an exogenous growth factor $\Delta(t)$ that allows the government to generate growth in the economy. In particular, Government spending is calculated as:

Eq. 1.5 – Government expenditure

$$Exp_{Gov} = R_{Gov} \times \left(1 + \Gamma(t)\right) - Y_{Gov}$$

Where $\Gamma(t)$ is an exogenous factor that allows for changing spending proportionally to tax revenues R_{Gov}. In so doing, if the user is interested in increasing expenditure above the amount necessary to keep debt constant, the Government is assumed to beable to create all money they need to achieve such a purpose. Thus, the Government keeps the balance of their balance sheet by issuing new debt and controlling the amount of debt returned to lenders. Differently from the other sectors the Government's ability to issue new debt is affected by the ability of Households and Banks to purchase it. As a result, Indicated Return of Debt $IndRoD_{Gov}$ and Indicated Borrowing $Ind\beta_{Gov}$ are formulated as follows:

Eq. 1.6 – Indicated Debt Retirement from government

$$IndRoD_{Gov} = D_{Gov} \times \mu_{Gov}$$

Eq. 1.7 – Indicated Borrowing rate from government

$$Ind\beta_{Gov} = Exp_{Gov} + \Upsilon_{Gov} + IndRoD_{Gov} + \sum_{P} Sub_i$$

Where D_{Gov} is the Debt of the government, μ_{Gov} the average life of debt for the government, Υ_{Gov} is the interest payments from government, and Sub_i the subsidy for each sector. The indicated borrowing becomes a demand for Securities to purchase for banking and households sector, that based on their availability of liquidity can buy less than what is demanded by the government. As a result, borrowing is calculated:

Eq. 1.8 - Government borrowing rate

$$\beta_{Gov} = Sec\beta_H + Sec\beta_B$$

Where $Sec\beta_H$ and $Sec\beta_B$ represent the securities purchase from households and banks respectively.

In turn, Retirement of Debt is corrected from the indicated value based on the differences between actual and desired borrowing. The variable Deficit is endogenously calculated as a difference between borrowing and debt retirement, and the ratio Deficit to GDP allows the level of indebtedness of the economy to be addressed for calibration purposes. Such a cash flow can be used as an input to the banking sectors to create money as newly issued liquidity.

The structure of the government is highly simplified in comparison to the other sectors, but it allows for a high level of control on the rest of the economy. It is worth noting that the current structure allows the creation of subsidies whenever necessary and allocates resources to every desired sector. The choice between financing those subsidies via money creation or reduction of expenditure can be kept under control with the coefficient $\Gamma(t)$.

Bank

Together with the Government sector, the Banking sector is assumed to be detached from physical assets accumulation, and is represented in financial terms only. Banks aim at fulfilling three fundamental purposes for the economy:

- Providing interest bearing loans to the private and public sectors
- Controlling money supply via money creation
- Controlling nominal interest rate

The structure of the financial sector has been highly modified in comparison to Sterman (1981). In particular, the Basel III regulation structure has been introduced. The Bank is now allowed to buy government securities, and a stock of debt money and deposits have been explicitly represented to highlight the ability of the banking sector to create money out of nothing. Most important the stock of Loans has been used in a very different way from Sterman (1981). It is now at the foundation of the stock and flow consistency of the entire economy, assuring a match between the sum of Debt in the private sector, and keeping consistent the relationship between Households' deposits and loans.

BALANCE SHEET, FINANCIAL DECISIONS AND CASH FLOWS

One aspect that is common between banking, firms and household is the control of their Liquidity stock via non-linear financial decisions. Being a system dynamics model which is affected by time delays, feedback systems and oscillations, decision makers that have control on finances have to counterbalance those instabilities with non-linear effects that aim at assuring that the Liquidity stock Λ always remains positive. This is a typical behaviour in the banking system that caused instability in the economy in the past. For example, the bank-run during a financial crisis would require the bank to stop savings withdrawal for households despite their demand.

Figure 1.2 shows the balance sheet of the banking sector. The double entry rule has been explicitly used to control flows in and out of the system. Important to note that Banks uses the stocks of Households' Deposit and Debt Money (representing banks ownership) as a source for loans and securities purchase in the economy. In addition, in line with Basel III regulation a certain fraction of those stocks are stored as liquidity and reserves in the assets side of the balance sheet. The bank is allowed to create money, and use those for providing interest bearing loans to support growth and meet its legal targets. The return on loans (interest payments) is distributed as income to households and banks, based on the ratio between deposits and debt money.

One aspect that is common between banking, firms and household is the control of their Liquidity stock via non-linear financial decisions. Being a system dynamics model which is affected by time delays, feedback systems and oscillations, decision makers that have control on finances have to counterbalance those instabilities with non-linear effects that aim at assuring that the Liquidity stock Λ always remains positive. This is a typical behaviour in the banking system that caused instability in the economy in the past. For example, the bank-run during a financial crisis would require the bank to stop savings withdrawal for households despite their demand.

Figure 1.2 – Financial Sector Balance Sheet

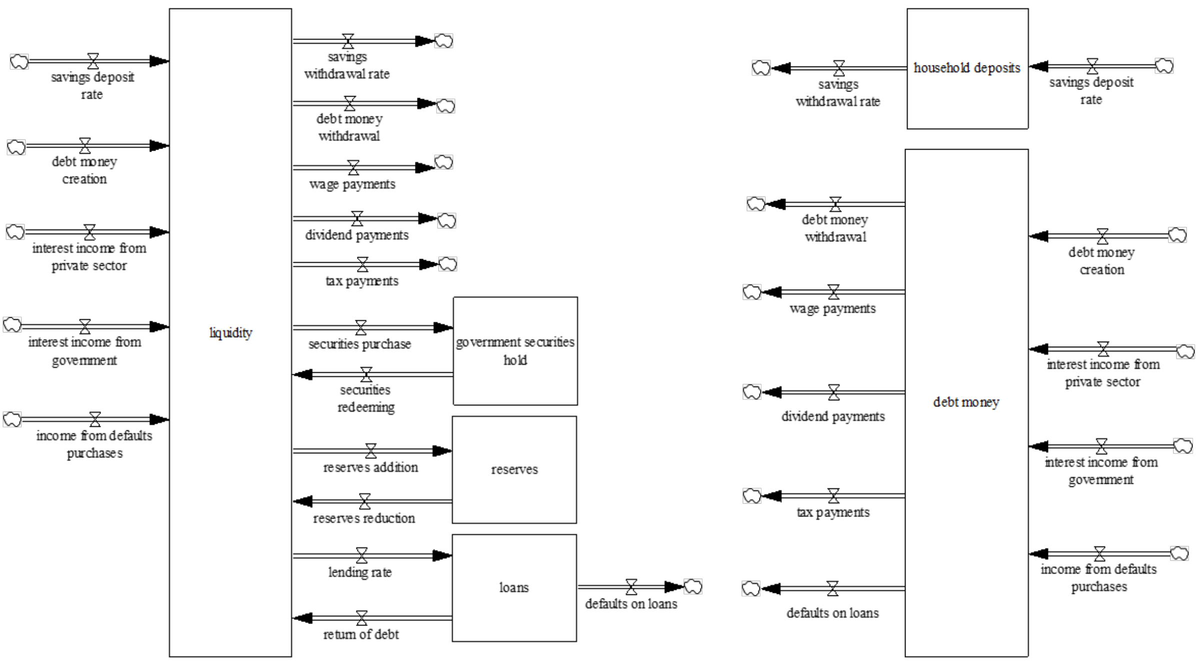
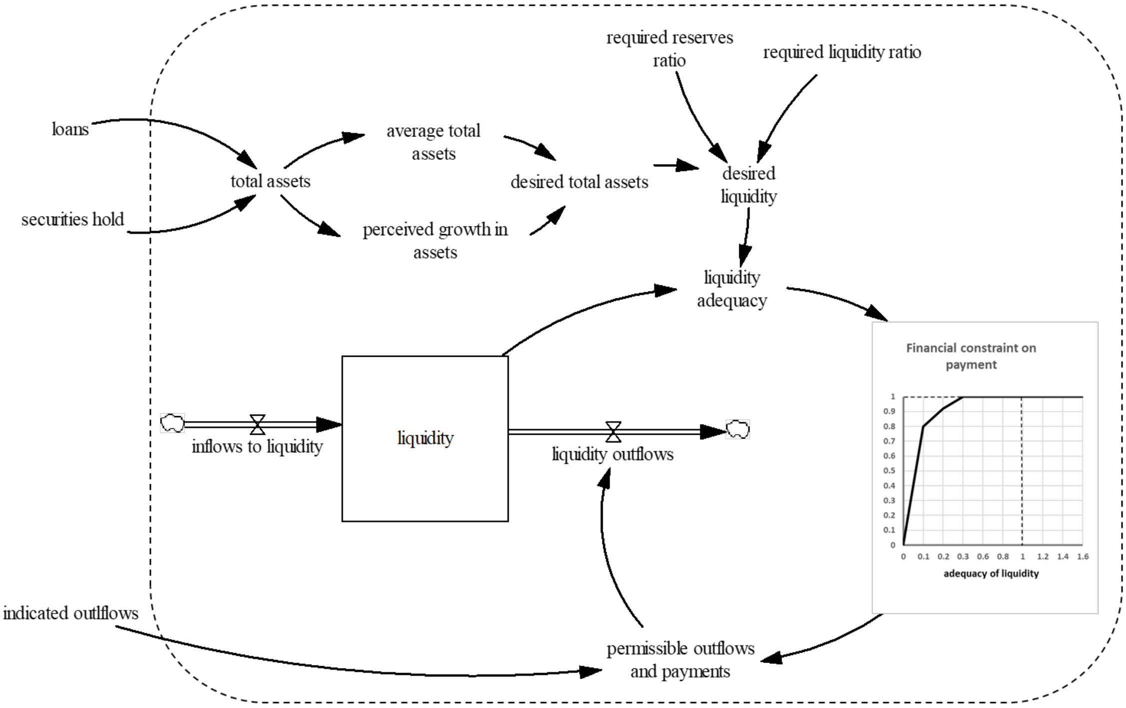


Figure 1.3 shows the decision feedback loop used to control liquidity to desired levels. Based on current total assets, bankers are assumed to form adaptive expectations based on the trend of their assets decision, and use such expectation as input to liquidity control decisions. Desired Liquidity Λ^* is calculated meeting the Basel III requirement on required liquidity l and reserves ratios r. Bankers measure the Liquidity Adequacy $\frac{\Lambda}{\Lambda^*}$ as a ratio between current Liquidity Λ and Desired Liquidity Λ^*. Every time Liquidity is less than half of Desired, banks

constraint their cash-flows non-linearly, till the outflow is reduced to zero when Liquidity is zero.

Figure 1.3 – Effect of Financial Decisions in Banks



In equation form this is:

Eq. 1.9 – Desired liquidity in banks

$$\Lambda^* = A(t) \times (1 + TRNDJ(A(t)) \times AdjT_A) \times \frac{l}{(1-r)(1-l)}$$

Where Λ^* is the desired liquidity, $A(t)$ is banks assets represented by both Loans and Securities, $(1 + TRNDJ(A(t)) \times AdjT_A)$ represents a correction factor describing the extrapolative expectations of future assets based on their trend $TRNDJ(A(t))$ and anchor bias measured over the time $AdjT_A$, r is the required reserves ratio, and l is the required liquidity ratio. It is worth noting that r and l are fractions representing the required amount of Liquidity and Reserves stocks in relation to total banks liabilities (i.e. Deposits and Debt Money). Thus, Eq. 1.9 assures the simultaneous match of both reserves and liquidity to the desired values in relation to Loans and Securities stored on their financial side.

The financial decisions on cash flows CF_i used to control the liquidity stock take the following form:

Eq. 1.10 – Bank cash out flows

$$cf_i = \mathfrak{f}_i\left(\frac{\Lambda}{\Lambda^*}\right) \times cf^*{}_i$$

Where cf_i is the cash flows out of liquidity stock, $cf^*{}_i$ the desired out flow from Liquidity, and $\mathfrak{f}_i\left(\frac{\Lambda}{\Lambda^*}\right)$ the non-linear decision behaviour for each outflow based on the adequacy of liquidity $\left(\frac{\Lambda}{\Lambda^*}\right)$. In the case of the banking sector, all non-linear curves have the same shape as indicated in Figure 1.3 and Eq. 1.11.

Eq. 1.11 – Non-linear effect of liquidity adequacy on cash out flows

$$\mathfrak{f}_i\left(\frac{\Lambda}{\Lambda^*}\right) = \begin{cases} 0 < \mathfrak{f} < 1, & \dfrac{\Lambda}{\Lambda^*} < 0.5 \\ 1, & \dfrac{\Lambda}{\Lambda^*} \geq 0.5 \end{cases}$$

It is worth noting that the current structure allows the maintenance of the Stock Liquidity Λ in proximity to the desired liquidity Λ^* for the full duration of the simulation, maintaining the adequacy of liquidity ratio $\frac{\Lambda}{\Lambda^*}$ in the proximity of 1.

<u>STOCK AND FLOW CONSISTENCY OF THE BANKING SECTOR</u>

The Banking sector remains at the core of the Stock and Flow Consistency condition of the ERRE model. In order to achieve so, the revenue equation presents important characteristics as follows:

Eq. 1.12 – Banks revenue

$$R_B = \sum_{\Pi+H} \Upsilon_i\left(\frac{DM_B}{S_H + DM_B}\right) + \Upsilon_{Gov}\left(\frac{Sec_B}{Sec_H + Sec_B}\right) + \left(\sum\nolimits_{\Pi+H} \partial_{D\,i} - \sum\nolimits_{\Pi+H} \nabla_{D\,i}\right)$$

Where R_B is the revenue of the banks, $\sum_{\Pi+H} \Upsilon_i$ is the interest payments from each firm and Households, $\left(\frac{DM_B}{S_H+DM_B}\right)$ is the ratio of interest payments due as revenue for bank, where DM_B is the cash owned by the bank and S_H households savings deposits in banks, Υ_{Gov} is the interest payment from Government, $\left(\frac{Sec_B}{Sec_H+Sec_B}\right)$ is the ratio of securities held from banks on total securities, $\sum_{\Pi+H} \partial_{D_i}$ is the liquidity gained from the sales of impounded assets, and $\sum_{\Pi+H} \nabla_{D_i}$ is the sum of defaults on debt from both firms and households.

The element $\left(\sum_{\Pi+H} \partial_{D_i} - \sum_{\Pi+H} \nabla_{D_i}\right)$ of Eq. 1.12 represent the way banks protect themselves from financial risk and defaults on debt in the ERRE model. When a company defaults, their assets are impounded by the bank and kept idle from production until they are reinjected in the market via purchase by another firm (this dynamic is described in detail in the firm section). The impounded asset is assumed to have a certain market value, which remains the property of both bank and firm depending on a parameter ν between 0 and 1. At the time the corresponding asset is sold back to market, it is assumed that the relative income ∂ gets distributed between private sector as equity $\partial_{\in}$ and bank as firm assets from debt ∂_D depending on such parameter ν. On the other hand, every default corresponds to a loss of Loans value in terms income for the banking sector, and needs to be accounted as a default outflow from loans stock $\sum_{\Pi+H} \nabla_{D_i}$. Assuming the conservation of value of the asset, the bank gains the same amount of cash lost via the selling of impound assets with a delay necessary to find an acquirer $\sum_{F+H} \partial_{D_i}$. Therefore, banks protect themselves for any discrepancy between defaults and sold assets as reduction in the Revenue R_B reducing the amount of revenue to be re-distributed between dividends, wages, and taxes by the difference between expected defaults.

The Stock and Flow Consistency condition is assured via the assumption that banks profit is redistributed completely back to the economy. In equations:

Eq. 1.13 – Banks wage payments

$$W_B = w_B \times R_B \times \mathfrak{f}_W$$

Eq. 1.14 – Banks Tax payments

$$T_B = \tau_B \times (1 - w_B) \times R_B \times \mathfrak{f}_T$$

Eq. 1.15 – Banks Dividends payments

$$\Phi_B = (1 - \tau_B) \times (1 - w_B) \times R_B \times \mathfrak{f}_\Phi$$

Where W_B is the wage payments, w_B is the fraction of revenue spent for payments of labour, T_B the tax payments, τ_B the fractional bank tax rate, Φ_B the dividends, and $\mathfrak{f}_W$, $\mathfrak{f}_T$, $\mathfrak{f}_\Phi$ the non-linear effects of financial decisions to protect liquidity from becoming negative. Similarly to the role of Government, this correspond to the assumption that banks behave as an auxiliary sector to the economy, performing consumption and investment decisions through Households. Further work can be focused on the detailed modelling of the banking sector.

MONEY CREATION AND LEVERAGE FOR GROWTH

Figure 1.4 shows an abstraction on the modelling of Money creation composed of both endogenous and exogenous elements for testing policies. The endogenous structure assumes that the financial sector would aim at keeping constant the ratio between Real GDP and the money supply in the economy. After applying a correction factor dependent on the expected growth in GDP, an indicative money creation is determined. The result of this structure assumes that while the economy grows, money can be created accordingly, whereas if the economy stagnates or degrowth occurs, money would gradually be withdrawn from it. In other words money creation can both assume the form of positive values (debt money creation) and negative values (debt money withdrawal). Liquidity control is assumed to constrain the money withdrawal to assure the liquidity stock remains positive.

The exogenous structure for testing policies allows both Government and Central banks to print money and inject them directly in the system to boost growth as desired. Whereas Banks can apply any exogenous growth rate to money supply, the Government can decide how much debt growth can be used as money creation depending on a parameter ϑ between 0 and 1. This structure remains particularly useful to increase the realism of the entire model during calibration phase.

Demand for money is represented by the desired borrowing in both firms and household sectors. Based on Basel III regulation, the bank can apply non-linear constraints on desired demand for lending in time of liquidity shortage (permissible debt from available fund). However, the additional cash created via money creation in Banks is assumed to be injected to the economy directly to increase the amount of borrowed cash beyond demand, generating a pressure from the financial system to grow via increasing debt. The so called 'financial sector leverage for growth' FL, represents the ratio between actual lending and the demand for lending. This structure allows the financial sector to make it easier for the private sector to leverage money, thus boosting cash availability, investment, consumption and growth.

Such a rationale can be translated in equations as follows. The variables Money Supply $MS(t)$ is calculated as:

Eq. 1.16 – Money supply

$$MS(t) = \sum_{\Pi+H+B} \Lambda_i + Res(t)_B$$

Where $\sum_{\Pi+H+B} \Lambda_i$ is the sum of all liquidity stock in the entire economy, and $Res(t)_B$ the Reserves in the banking sector.

On the other hand the variable Desired Money Supply MS^* is determined as follows and is used to determine money creation MC. In particular:

Eq. 1.17 – Desired Money Supply

$$MS^*(t) = \frac{MS_0}{RealGDP_0} \times RealGDP(t) + \int_t [ExoQE(z) \times MS(z) + (\beta_{Gov}(z) - DR_{Gov}(z)) \times \vartheta]dz$$

Where $\frac{MS_0}{RealGDP_0} RealGDP$ represents the endogenous element aiming at keeping constant the ratio between Money supply and Real GDP, $\int [ExoQE(z)MS(z) + (\beta_{Gov}(z) - DR_{Gov}(z)) \times \vartheta]dz$ accounts for the accumulation between both Central Banks and Government policies over time. In particular, $ExoQE(t)$ is a set of policies that can be defined by the user as multiplicative fractions on current $MS(t)$, and $\beta_{Gov}(t) - DR_{Gov}(t)$ is the deficit of the government calculated as difference between borrowing $\beta_{Gov}(t)$ and debt retirement $DR_{Gov}(t)$, and ϑ a parameter allowing the amount of government deficit actually used for issuing new money to be addressed.

Indicative Money creation $IndQE$ is a combination between the adjustment generated from the difference between desired and actual money supply over an adjustment time $\frac{MS^*-MS}{AdjT_{MS}}$, a component dependent on the growth rate of the Real GDP $TRNDJ(RealGDP) \times MS$, and the exogenous element determining the ability of Central Banks to issue money when desired $ExoQE(t) \times MS(t)$.

Eq. 1.18 – Indicative Money Creation

$$IndMC = \frac{MS^* - MS}{AdjT_{MS}} + TRNDJ(RealGDP) \times MS(t) + ExoQE(t) \times MS(t)$$

Actual Money Creation MC differentiate between money creation $Max(0, IndMC)$ when the Indicative Money Creation is positive, and money withdrawal $\mathfrak{f}_{QE}\left(\frac{\Lambda}{\Lambda^*}\right) \times Max(0, -IndMC)$ when Indicative Money Creation is negative, assuming a correction factor dependent on the availability of liquidity in banks.

Eq. 1.19 – Money Creation

$$MC = Max(0, IndQE) + \mathfrak{f}_{QE}\left(\frac{\Lambda}{\Lambda^*}\right) \times Max(0, -IndMC)$$

The Money Creation is injected in the economy as a boost to borrowing beyond demand, simulating a higher propensity of banks to borrow in the following way. Demand for money is calculated as the sum of indicated borrowing from each economic sector $\sum_{F+H} Ind\beta$. The standard behaviour of banks is to provide all money required corrected with a Liquidity correction factor $\mathfrak{f}_{FL}\left(\frac{\Lambda}{\Lambda^*}\right)$ (commercial banks) and add the Money Creation coming from the Central Banks. Thus, lending is calculated as $\sum_{F+H} Ind\beta \times \mathfrak{f}_{FL}\left(\frac{\Lambda}{\Lambda^*}\right) + MC$ and a financial leverage for growth FL as ratio between total lending and total demand for money.

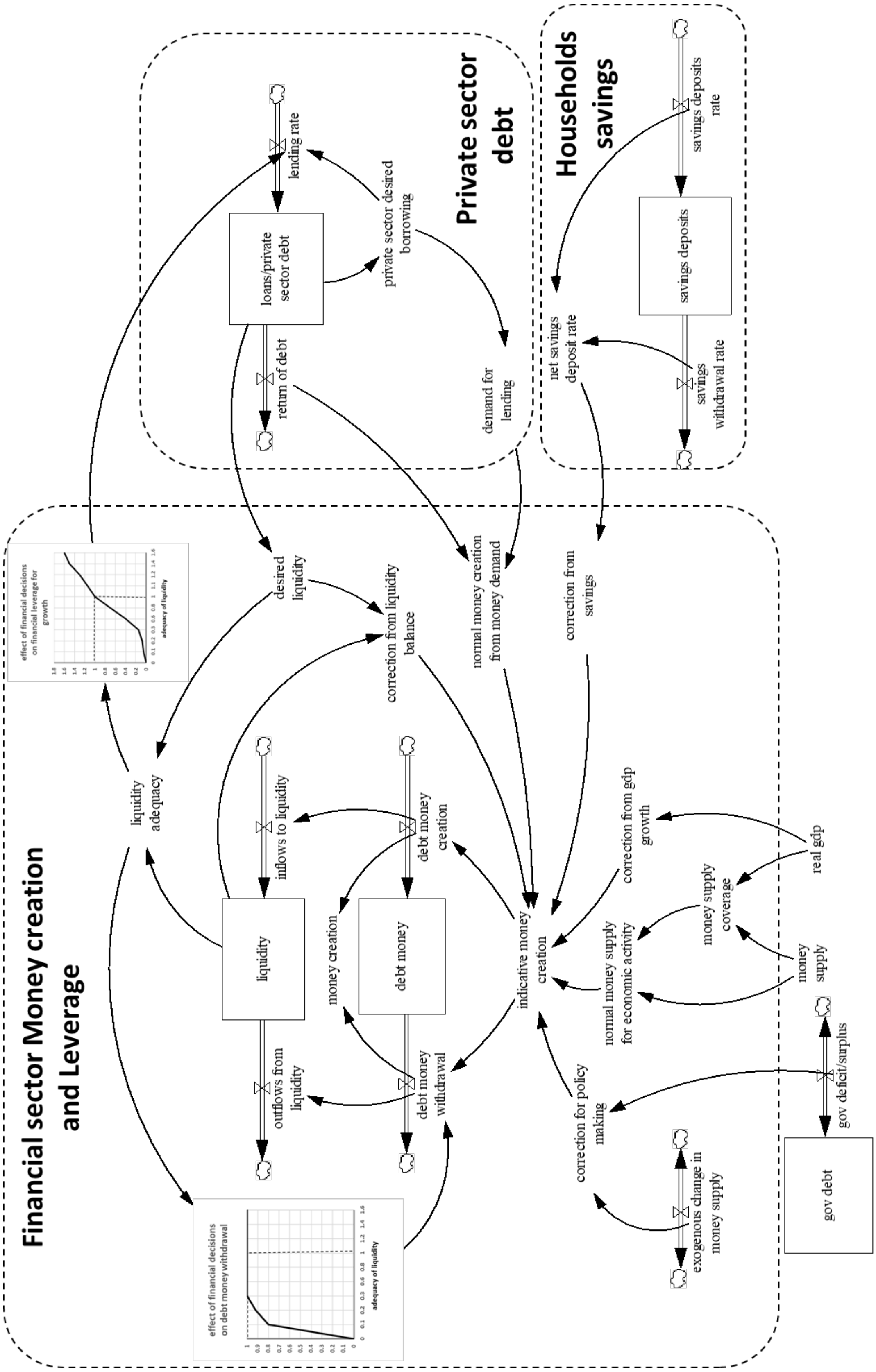
Eq. 1.20 – Financial Leverage for Growth

$$FL = \frac{\sum_{\Pi+H} Ind\beta \times \mathfrak{f}_{FL}\left(\frac{\Lambda}{\Lambda^*}\right) + MC}{\sum_{\Pi+H} Ind\beta}$$

Every sector of the economy determines their actual borrowing β by correcting their demand for money $Ind\beta_i$ of the resulting financial leverage for growth FL.

This entire rationale assumes that all sectors in the economy are treated as equal, all benefitting equally from monetary policies depending on their fraction of demand for money on total demand. A more sophisticated version of the model, could be expanded to target specific sectors and generating money specific to them, while neglecting others.

Figure 1.4 – Money Creation and Financial Leverage for Growth in the Banking system

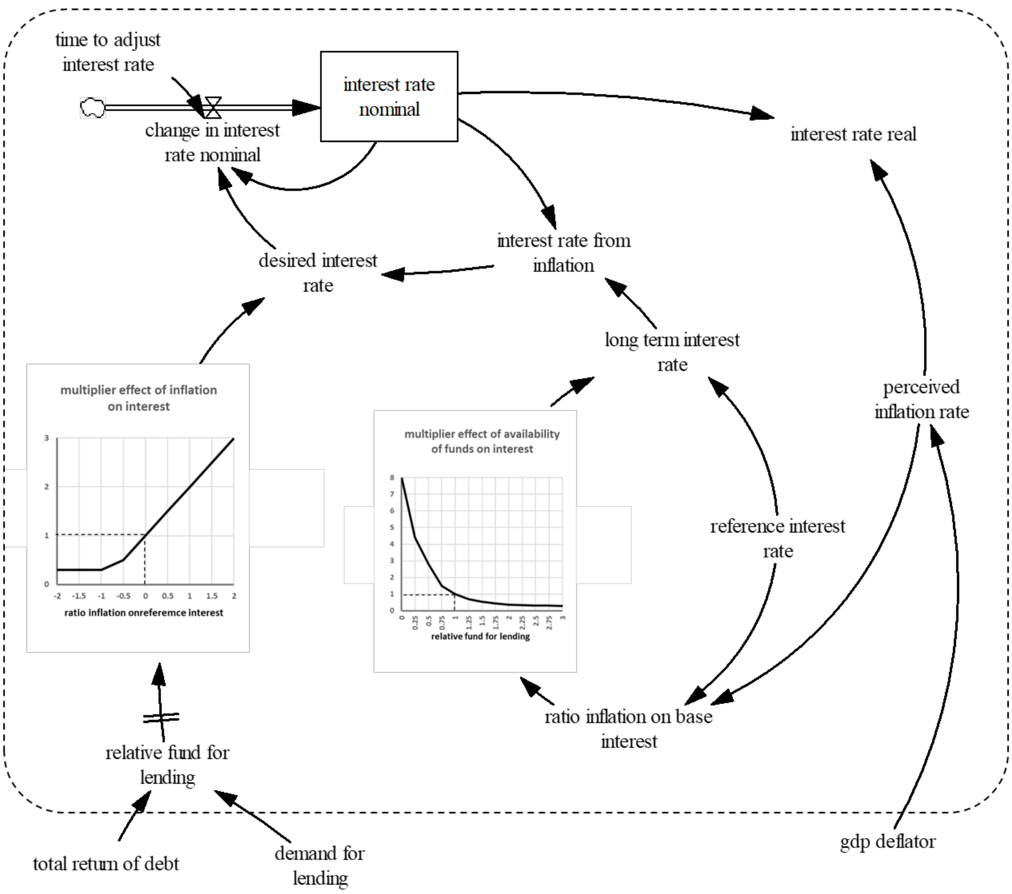


INTEREST RATE NOMINAL

In line with the principles of monetary policies, exogenous creation of money is a driver for increased inflation in the model. However, without a balancing dynamic feedback to keep inflation under control, the inflation could increase to generate important instability throughout the economy. The modelling of nominal interest rate in the banking sector represents such a balancing feedback. The relationships between growing inflation and growing interest rate leads firms to reduce investments, and stimulates households to increase savings. The sum of those behaviours generates the forces necessary to balance inflation back to normal as well as stimulating business cycles. In fact, the presence of delays in perceiving information and taking decisions can still generate volatility.

Figure 1.5 shows the structure of the two non-linear relationships used to model nominal interest rate in the ERRE. These are on the long-term effect of inflation on interest (non-linearity on the right of Figure 1.5) and short-term effect on money availability on lending (left hand side of figure).

Figure 1.5 – Interest Rates Nominal at the base for the entire economy



In the model it is assumed that for positive values of inflation γ_i, long term interest rate i_{LT} would grow proportionally and linearly. However, for negative values of inflation, the interest rate would decrease non-linearly, thus assuring that interest rate will not go negative. The non-linear relationship $\lambda_{i_{LT}}\left(\frac{\gamma_i}{i_b}\right)$ relies on the relative ratio between inflation γ and the base interest rate for policy i_b and uses it as correction factor to the base interest itself as described in Eq. 1.21. This is a necessary condition to assure the banking system makes a profit over time. It is worth noting that such a table function can be easily altered to test alternative theories such as the Taylor rule in linking inflation and interest.

Eq. 1.21 – Long Term Interest Rate

$$i_{LT} = \lambda_{i_{LT}}\left(\frac{\gamma_i}{i_b}\right) \times i_b$$

In addition, to Sterman (1981) an interest rate goal seeking path dependent structure has been added. Such a structure is a system dynamics archetype well exploited in Hynes (1987) for the modelling of interest rates in the System Dynamics National Model. In fact, goals are often affected both by past performance and external pressures, where traditional performance forms slowly, adapting to actual ones. Such a rationale is consistent with the common judgemental heuristic well known in Cognitive and Behavioural Economics as *anchoring and adjustment* (Mainelli and Harris 2011, Kahneman 2011, Thaler 2015, Sterman 2000). Decision makers normally determine a quantity or make a judgement by anchoring, and adjust their judgement to account for factors specific to the case at hand. In the real non perfectly rational world, adjustment tends to be not sufficient, leading to bias toward the anchor, and moving away from the rational model used in mainstream economic theory.

In ERRE, it is assumed that past values of interest rate generate larger friction on the future interest rate while affected by external pressures. A sensitivity parameter σ_i between 0 and 1 measures the weighted average between current interest rate $i_n(t)$ and inflationary pressures i_{LT}, simulating the behaviour of bankers sticking to past decisions when stimulated with external system pressures.

Eq. 5.47 shows the effect of inflation on interest rate i_γ based on such parameter σ_i. It is worth noting that in the extreme case in which $\sigma_i = 0$, i_γ completely ignores the inflation effect, whereas if $\sigma_i = 1$, i_γ correspond entirely to the inflationary long term interest i_{LT}. Exploring variations on such parameter σ_i during the sensitivity and calibration phase would allow an exploration of how sticky to past interest rate bankers can be, despite the most rational decision being to simply keep using the i_{LT} as suggested in the neo-classical academic literature.

Eq. 1.22 – Interest Rate from Inflation

$$i_{\gamma} = i_n(t) \times \left(\frac{i_{LT}}{i_n} \times \sigma_i + (1 - \sigma_i) \right)$$

On the left hand side of Figure 1.5, the banks rises apply non-linear control on interests based on the growing pressure of demand for cash $\sum_{F+H} Ind\beta_i$ in relation to the ability of borrowers to return their debt $\sum_{F+H} IndRoD_i$. The average relative fund for lending ϱ_B is calculated as an exponential smooth on such a ratio measured over the time $AdjT_{\varrho_B}$.

Eq. 1.23 – Average relative fund for lending

$$\varrho_B = Smooth\left(\frac{\sum_{F+H} Ind\beta_i}{\sum_{F+H} IndRoD_i}, AdjT_{\varrho_B} \right)$$

Based on the non-linear adaptation $\lambda_{i_{MD}}(\varrho_B)$, it is assumed that in case the demand for money equals the amount of debt returned by the private sector (relative fund for lending=1), the banking sector would not apply any change in long term interest rate from inflation i_{γ}. Alternatively, if demand for money increases much beyond debt return, the desired interest rate $i_n{}^*$ can increase non-linearly eight fold, whereas in the case of degrowth, it can be reduced until it is the 70% less.

Eq. 1.24 – Desired Interest Rate

$$i_n{}^* = \lambda_{i_{MD}}(\varrho_B) \times i_{\gamma}$$

The anchor and adjustment concludes with Nominal Interest rate accumulating the differences between desired interest rate and actual nominal interest rate as captured in Eq. 1.25. This corresponds to the smooth average on desired interest rate nominal $i_n{}^*(t)$.

Eq. 1.25 – Nominal Interest Rate

$$i_n(t) = i_{n_0} + \int_{t_0}^{t} \frac{i_n{}^*(z) - i_n(z)}{AdjT_{i_n}} dz = smooth(i_n{}^*(t), AdjT_{i_n})$$

Finally, the real Interest Rate is calculated as the difference between nominal interest rate and inflation rate.

Firms and Households

Firms and Households are represented in both financial, operational, and physical levels. Each sector includes financial variables and decisions, the accumulation of capital, labour, orders and demand, and non-linear rationally bounded decision behaviours. In addition, the agricultural and fossil fuel sectors include the modelling of natural resources, accounting for non-linear cost curves the more resources are depleted.

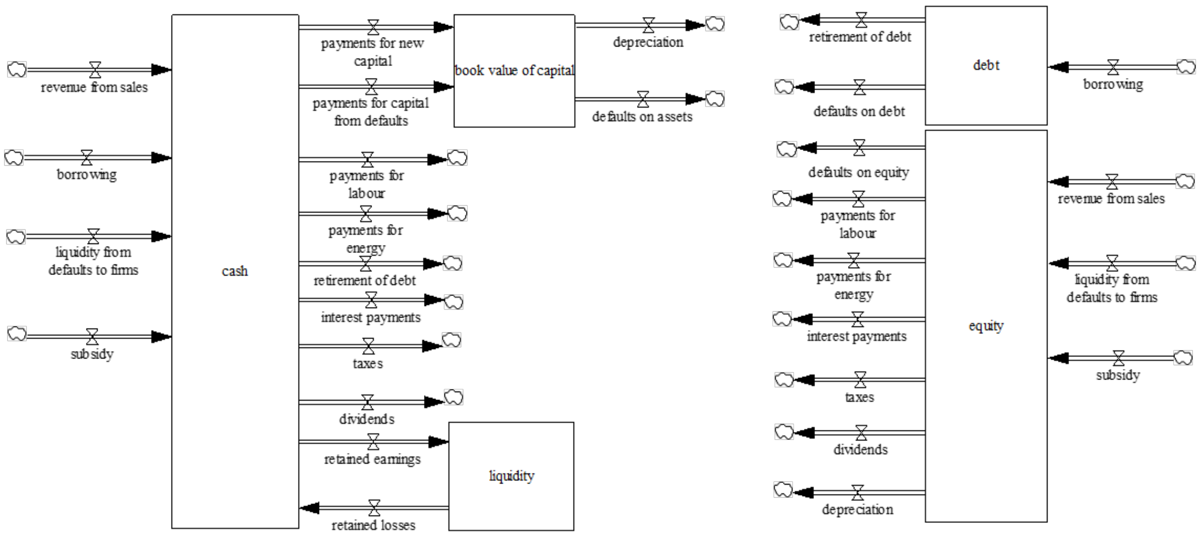
As the model is complex, with many variables and decisions being intertwined across systems, it would be possible to start this treatment from different angles. However, Balance Sheet and Price sub-systems, being the ones with highest degree of interconnectedness within sectors, represent the ideal position to start this section, allowing the reader to maintain their big picture on the functioning of ERRE. The treatment starts with the financial perspective, including Balance Sheet, Income Statement, Financial Decisions Making, Interest Rate, Borrowing and Savings. Then the Price structure is presented, followed by Production an Utility subsectors, Energy Market, Capital accumulation, Energy requirements, Assets value, Depreciation, Labour market, Wages and Dividend payments. Idiosyncrasies for each sector are presented under the relative section.

It is worth noting that the sub-sectors of the Balance Sheet and Price could be used as system maps when looking at every other subsystem described in here.

BALANCE SHEET AND FINANCIAL DECISIONS

Figure 1.6 shows the balance sheet of a generic firm sector, as composed of five main stocks (Cash, Liquidity, and Value of Capital on the assets side; Debt and Equity on the liabilities side) and all their in and out flows represented based on the double entry rule typical for accounting systems. It is worth noting that the flows Retained Earnings and Losses, Payments for New Capital and Defaulted Capital do not need to be represented on the liabilities side since they are still part of Debt or Equity. In addition, the 'Defaults on Assets' outflow is split between 'Defaults on Debt' and 'Defaults on Equity' on the liabilities side. The balance sheet is a powerful tool to present the ERRE model because it allows to keep track of all flows in each sector, and support their top-level view.

Figure 1.6 - Balance sheet of a generic firm sector



Based on the ERRE structure, the Liquidity stock represent by far the most important financial decision variable in both Households and Firms allowing the control of payments they make over time , and assuring their Liquidity level remains in proximity of desired levels. Debt responds to the accumulation of assets via borrowing, controls investments, and determines the interest payments. Borrowing responds to liquidity deficiencies as well as supporting the further consumption, investments and payments in general. These two stocks and relative importance are described in the following section.

The stock Book Value of Capital on the assets side is used here as a mean for accounting and communication, whereas the structure underpinning the modelling of Assets Value used for decision making, as well as their Depreciation is explained in the chapter.

Differently from Sterman (1981), the stocks of Cash and Liquidity have been separated as two elements to demonstrate the stock and flow viability of the ERRE model. In particular, the variables Retained Earnings and Retained Losses are calculated as difference between all inflows and all outflows from the stock Cash, generating the accumulation of money in the stock Liquidity. The outcome of this allows the stock of Cash to be kept at zero for the entire time in the simulation. The stock Equity is not used to determine any specific decision, but is fundamental to maintain the equality between assets and liabilities in the model, thus assuring the necessary condition of Stock and Flow Consistency in the ERRE model.

Based on the balance sheet, it is possible to calculate the Net Income Before Taxes $NIBT_i$ and Net Income NI_i as follows:

Eq. 1.26 – Net Income Before Taxes

$$NIBT_i = R_i - W_i - \mathrm{\epsilon}_i - \Upsilon_i - \Delta_i$$

Eq. 1.27 – Net Income

$$NI_i = NIBT_i - T_i$$

Where R_i is the revenue of firms, W_i the wage payments for labour, $\mathrm{\epsilon}_i$ the energy payments, Υ_i the interest payments, Δ_i the depreciation of assets, and T_i the tax payments. Both variables are used in various sub-systems in the firm sector of the ERRE model.

Every other element of the Balance Sheet is described in detail through the rest of this appendix.

FINANCIAL DECISIONS AND EFFECTS

In the ERRE model, the Liquidity stock should be seen as the fundamental control variable determining how rationally bounded non-linear financial decision making would spread, impacting every part of both Firms and Households sectors. Consistently with the Anchor and Adjustment heuristic, well known in Cognitive economics, the financial decisions act to maintain the Liquidity stock in proximity of a desired value based on expected payments to assure the firm remains solvent and stable over time. Figure 1.7 shows the five categories of effects that any discrepancy between the Liquidity stock Λ and Desired Liquidity Λ^* would trigger across the firm sector and towards its boundaries.

The Liquidity stock Λ represents the accumulation of cash in the firm sector as dynamic difference between all cash in and outflows. The Desired Liquidity Λ^* variable is calculated as a metric of reference for Liquidity in the following way. First, the desired payments CF^* is calculated as the exponential average ($smooth$) on total expected cashflows $\sum_{CF} {cf^*}_i$ measured over the time $AdjT_\Lambda$ as:

Eq. 1.28 – Desired cash flows in firms and households

$$CF^* = smooth\left(\sum\nolimits_{CF} {cf^*}_i\,,\, AdjT_\Lambda\right)$$

Secondly, Desired Liquidity Λ^* is calculated as:

Eq. 1.29 – Desired liquidity in Firms and Households

$$\Lambda^* = CF^* \times \ l_{cov} \times \left[1 + TRNDJ\big(adjA(t)\big) \times AdjT_\Lambda\right]$$

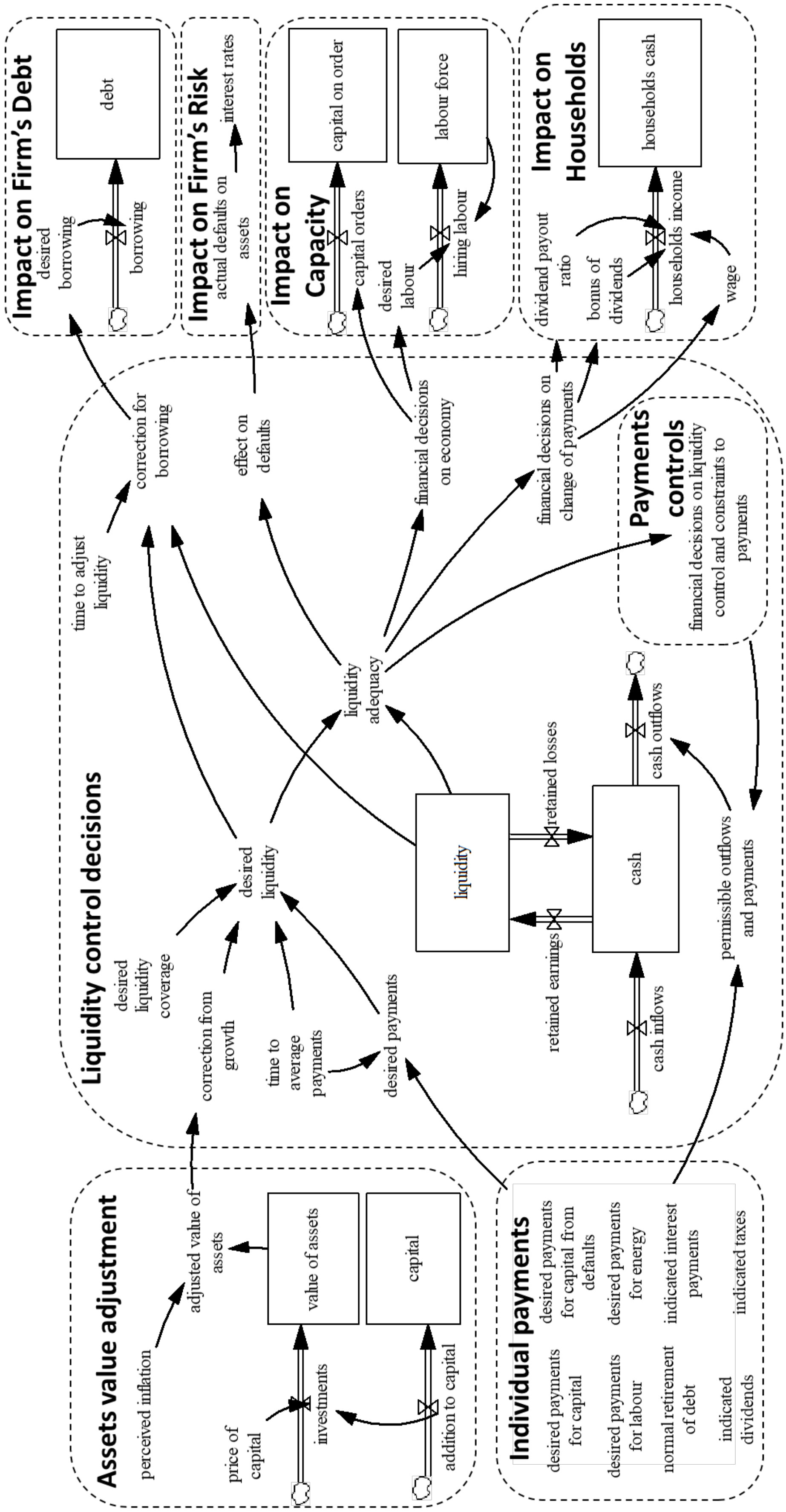
Where l_{cov} represents the desired liquidity coverage based on their cash flows, and $TRND\big(adjA(t)\big) \times AdjT_\Lambda$ represents a correction factor based on the adaptive expectations of the firm in measuring their assets growth $TRNDJ\big(adjA(t)\big)$ over the adjustment time period $AdjT_\Lambda$. The top-left hand corner of Figure 1.7 shows an abstraction of the modelling of capital asset in ERRE. It is important to know that real capital determines the accumulation of value in the balance sheet of firms, whose value is corrected with an inflation trend to obtain an adjusted value of assets $adjA(t)$, assumed general practice to evaluate companies assets in their markets.

The five areas where liquidity adequacy is used within Firms and Households are:

1. Measure of control to each outflow in the system similarly to the Banking sector.
2. Influence on decisions at the operational level supporting both increases and decreases in capacity.
3. Effect on payments adaptation including both wages and dividends
4. Change in default rates and cascading impact on interest rates
5. Demand for additional cash via Borrowing.

These effects are treated in detail in the following.

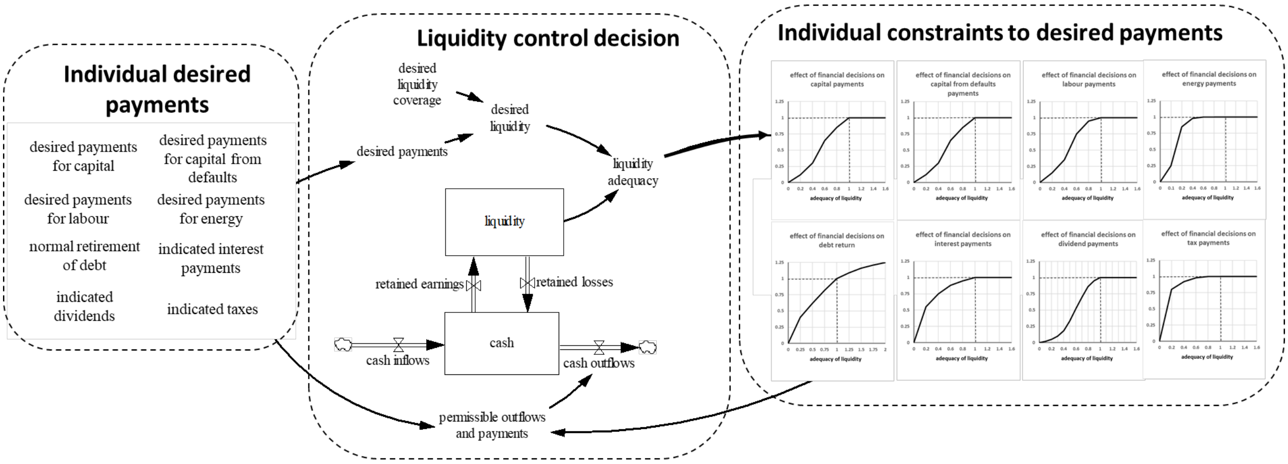
Figure 1.7 - Effects of Financial decisions based on the Liquidity stock control variable



Financial decisions to control liquidity

Figure 1.8 and Eq. 1.30 show how financial decisions are applied to constraint cash flows in time of liquidity shortage.

Figure 1.8 – Balancing feedback loop to control liquidity outflows



Eq. 1.30 – Cash flows in firms and households

$$cf_i = cf^*{}_i \times \mathfrak{f}_i\left(\frac{\Lambda}{\Lambda^*}\right)$$

Where cf_i is each actual cash flow from Liquidity Λ, $cf^*{}_i$ is the indicated cash flow assuming no money constraint, $\frac{\Lambda}{\Lambda^*}$ is the Adequacy of Liquidity, and $\mathfrak{f}_i\left(\frac{\Lambda}{\Lambda^*}\right)$ is a non-linear relationship in the form depicted in Figure 5.23. Every $\mathfrak{f}_i$ in the Firm and Household sectors has a specific meaning, and can have different shape. However, their structure implies that if actual liquidity Λ is above or equal to desired liquidity Λ^* the multiplier effect would be 1 (i.e. no effect), whereas if liquidity Λ is lower than desired, the actual cash flows would be diminished by the non-linear multiplier effect till reaching zero when Liquidity approaches zero. For example, Eq. 1.31 shows the application of this rationale to the Payments of Tax.

Eq. 1.31 – Tax payments

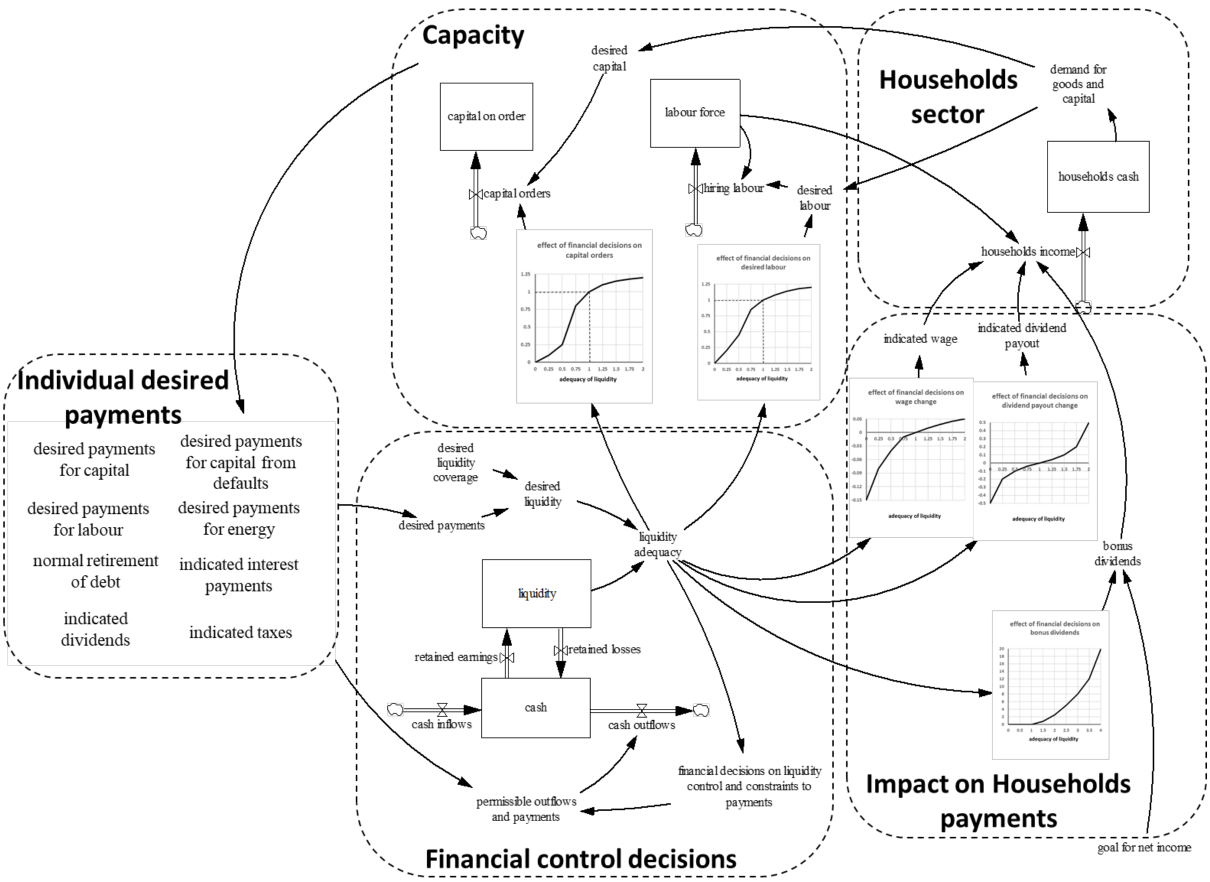
$$T_i = \mathfrak{f}_i\left(\frac{\Lambda}{\Lambda^*}\right) \times (\tau_i \times NIBT_i)$$

Where $(\tau_i \times NIBT_i)$ is the indicated tax payment calculated as multiplication between Tax Rate τ_i and Net Income Before Taxes $NIBT_i$, and $\mathfrak{f}_i\left(\frac{\Lambda}{\Lambda^*}\right)$ the financial decision implying that if Liquidity Λ is beyond 60% of Desired Liquidity Λ^*, all taxes would be paid as they should be, whereas if Liquidity Λ is below 60% of desired levels, taxes would be negotiated or probably evaded till reaching zero an absence of liquidity. It is worth noting that the ERRE model aggregates each sector to the level of the global economy, assuming that the more liquidity decreases below certain levels, the more some companies would start losing value and generate behaviour of failure thus paying less taxes. By editing such a non-linear relationship, it would be possible to test options for tax evasion to much higher levels of liquidity adequacy than what assumed in ERRE.

Financial decisions on capacity adaptation

Figure 1.9 shows how financial decisions are assumed to affect decisions in hiring labour force and ordering capital assets. In both cases, capital owners are assumed to respond to an abundancy of cash by increasing both their levels of labour and operative capacity beyond optimal capacity to increase their competitive advantage and firm stability. Both equations are presented in the Capital and Labour sections of this appendix. In a more disaggregated solution of this model, financial decisions could be made more granular, assuring that different type of capital and labour could be employed towards specific purposes.

Figure 1.9– Effects of financial decisions on the other parts of the firm



Financial decisions on inequality

The right hand side of Figure 1.9 shows the non-linear behaviour of capital owners in distributing finances across wages, and dividends depending on their adequacy of liquidity. These relationships aim at capturing the asymmetry in the control (as well as in risk) of firms towards the inequality between capital owners and workers. When companies do not perform well, it is assumed that Wages can be decreased till they are a 15% below the current level, whereas Dividend Pay-Out Ratio can be reduced till 50% below current levels of dividend. This behaviour, captures the inability of capital owners to perform their operations without workers, while keeping dividends low (even zero) in time of liquidity shortage.

However, when companies are successful and liquidity rises beyond their desired levels, Wages are assumed to increase to a maximum of +3% on current levels to stimulate workers commitment, whereas dividend pay-out ratio can increase non-linearly with increasing marginal returns till +50% on current levels. It is worth noting that both effects are multiplicative factors on current levels of Wages and Dividend Pay-Out Ratio, which means that maintaining liquidity to high levels for longer time than required would result in exponential growth of 3%/year growth rate in the case of wages and exponential growth of 50%/year growth rate in the case of dividends.

In addition, capital owners are willing to pay a bonus on their dividends when recording successful performance. The effect is non-linear and applicable only when Liquidity is beyond desired levels. In this particular case they would be distributing their Goal for Return on Investment, and multiply it up to twenty times in the case of very large abundancy of liquidity. All equations are described in the Dividend and Wage sections of this appendix.

The result of these effects taken together is additional cash available to the Household sector that, based on the same principle (and explained later in the appendix) is meant to increase their consumption levels thus boosting demand even further. In a more detailed version of this model, a disaggregation between capital owners and workers in terms of their financial decisions and behaviour would benefit the study of inequality dynamics in ERRE.

Financial effects on defaults and interest rate

Figure 1.10 shows the dynamic structure of defaults and interest rate as driven by availability of liquidity. Given that this structure is representative of the entire global firm sector, it is assumed there will always be a fraction of companies that declare default over time (constant Normal Defaults on Debt ξ_i). Thus the level of available liquidity in the sector would allow to increase or decrease non-linearly the amount of defaults in comparison to normal default levels.

The cascade effect of liquidity shortage on defaults implies, a loss in real capital Ω_i that is accounted for in assets value terms ∇_{A_i}. Such a default is distributed between equity $\nabla_{\in_i}$ and debt ∇_{D_i}, assuming that the average ownership of assets at the time of default ν would remain constant, and split the distribution of loss between firm and creditor of that particular loan.

The equations capturing these dynamics are provided elsewhere in this appendix (see Eq. 1.138, Eq. 1.188, Eq. 1.189). In this section, it is assumed the case of modelling capital as a first order delay, which allows to describe the defaults on real assets Ω_i as:

Eq. 1.32 – Defaults on Real Assets (simplified structure)

$$\Omega_i = K_i(t) \times \xi_i \times \mathfrak{f}_\nabla\left(\frac{\Lambda}{\Lambda^*}\right)$$

Where $K_i(t)$ is the capital stock, ξ_i is the normal default on debt, and $\mathfrak{f}_\nabla\left(\frac{\Lambda}{\Lambda^*}\right)$ is the non-linear effect of liquidity adequacy on defaults. Such value is translated in financial terms as defaults recorded to the assets side of the balance sheet ∇_{A_i} as:

Eq. 1.33 – Defaults on Financial Assets (simplified structure)

$$\nabla_{A_i} = \Omega_i \times mAdjp_{k_i}$$

Where $mAdjp_{k_i}$ is the market adjusted price of those assets, that is described in formulation and structure in the Value of Assets section of this appendix at Eq. 1.188. The value of defaults ∇_{A_i} with details for a generic capital vintage is shown in Eq. 1.189.

The corresponding value is divided between defaults from debt ∇_{D_i} and defaults on equity $\nabla_{\in_i}$ based on the average ownership ratio between borrower and creditors ν, with $0 < \nu_i < 1$.

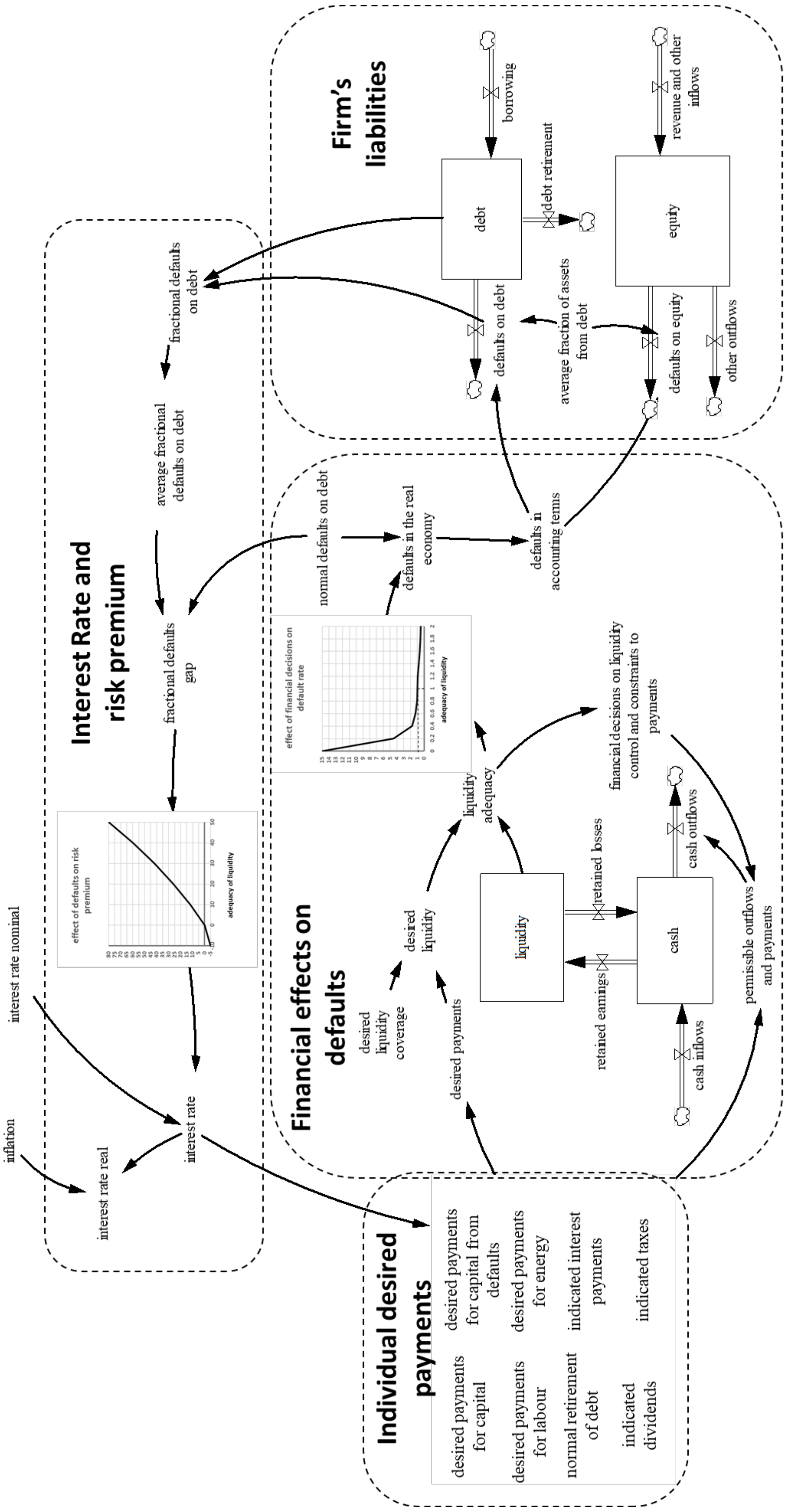
Eq. 1.34 – Defaults on Debt

$$\nabla_{D_i} = \nabla_{A_i} \times \nu_i$$

Eq. 1.35 – Defaults on Equity

$$\nabla_{\in_i} = \nabla_{A_i} \times (1 - \nu_i)$$

Figure 1.10 – Effect of liquidity on interest rates per each firm



Given that the Defaults on Debt in the firm sector ∇_{D_i} corresponds to a loss of value for loans in the banking sector, it is assumed that banks would increase their Risk Premium i_{rp_i} for lending money to that specific sector as follows:

Eq. 1.36 – Interest with Risk premium

$$i_{rp_i} = i_n + \xi_i \times \lambda_{rp}(\overline{\xi_\iota} - \xi_i)$$

Where i_n is the nominal interest rate from the banking sector, ξ_i is the normal default on assets, $\lambda_{rp}(\overline{\xi_\iota} - \xi_i)$ is the non-linear adjustment behaviour of banks in increasing risk premium, and $\overline{\xi_\iota}$ is the average default rate measured as:

Eq. 1.37 – Average default rate

$$\overline{\xi_\iota} = smooth\left(\frac{\nabla_{D_i}}{D(t)_i}, AdjT_\xi\right)$$

Where ∇_{D_i} is the default rate on debt, $D(t)_i$ is the debt stock, and $AdjT_\xi$ the adjustment time to measure average defaults. As depicted in Figure 1.10 the increase in interest is higher than the linear proportion on defaults rate. This aspect is meant to capture the behaviour of the banking industry to quickly overreact to the defaults in each particular sector to protect their interest and profitability. On the side of firms, this would imply lowering investments, with larger companies maintaining their ability to act in those markets and small companies leaving them.

The resulting interest rate i_{rp_i} impact the interest payments Y_i, that is also dependent on Debt D_i and a financial decision $\mathfrak{f}_Y\left(\frac{\Lambda}{\Lambda^*}\right)$ as follows:

Eq. 1.38 –Interest payments

$$Y_i = i_{rp_i} \times D(t)_i \times \mathfrak{f}_Y\left(\frac{\Lambda}{\Lambda^*}\right)$$

Borrowing and Debt

Figure 1.11 and Figure 1.12 show Borrowing and Debt as a decision dependent on both the assessment by the bank (see Permissible Debt PD_i), and what is demanded by the borrower (see Desired Borrowing β^*).

In particular, the lender requires four variables for taking their decision: adjusted value of assets $adjA(t)_i$, interest rate real i_{r_i}, net income before tax $NIBT_i$, and the average fractional default on debt $\overline{\xi_i}$. The bank uses the borrower's assets value $adjA(t)_i$ as a standard metric to determine the amount of debt permissible to them PD_i, and measures their income $NIBT_i$ to assess their viability to service their debt at current interest rate i_{r_i}. Both measures are compared to the initial value of these variables in the simulation, thus assuming that their reference metric system would not change over time. Additional adjustment is dependent on their market average default rate $\overline{\xi_i}$ over time.

In equations, Permissible Debt from Assets PDA_i is calculated as:

Eq. 1.39 – Permissible Debt from Assets

$$PDA_i = adjA(t)_i \times \omega_i$$

Where $adjA(t)_i$ is the adjusted value of assets of every borrower, and ω_i is the constant normal debt to assets ratio.

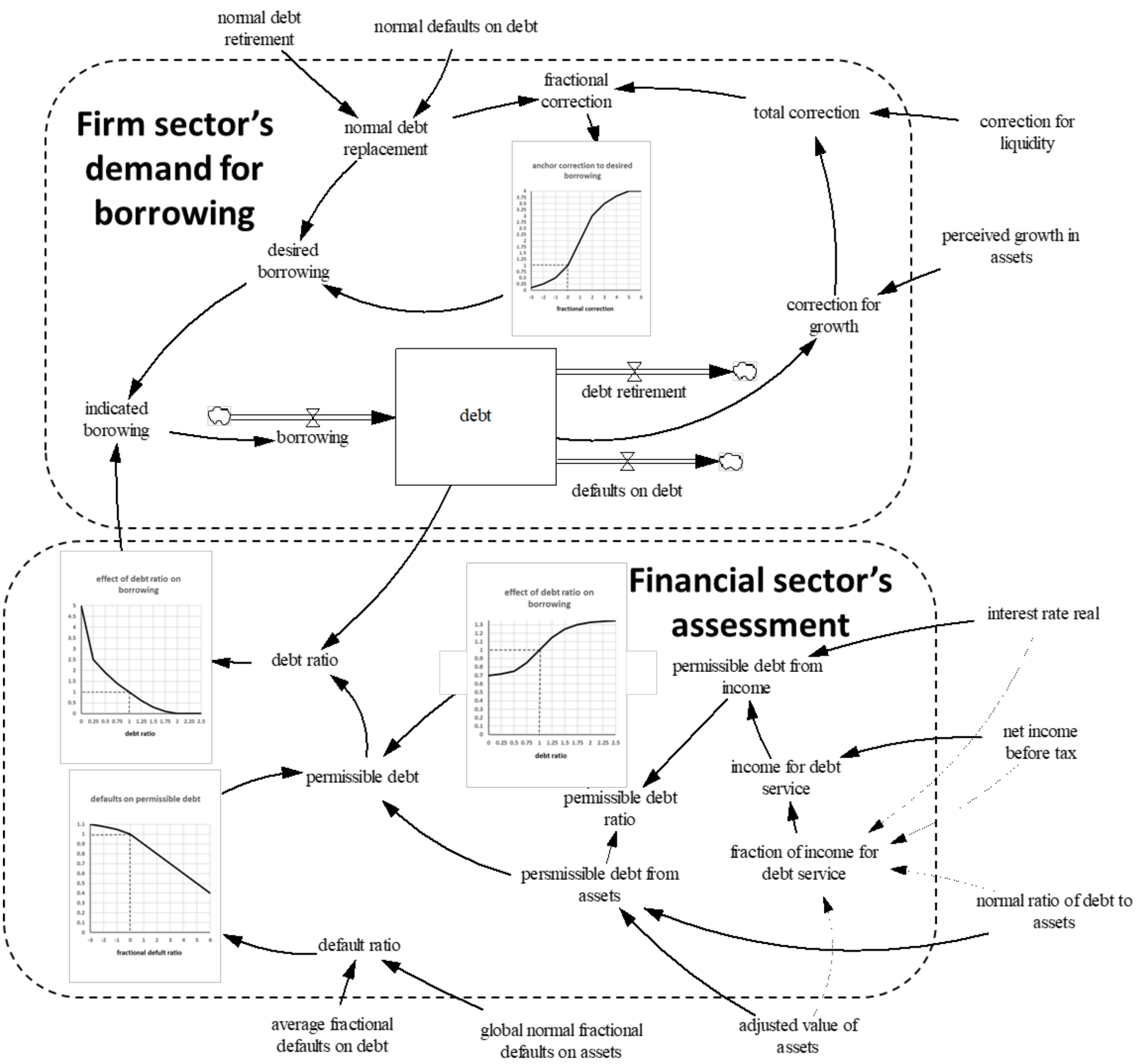
Permissible Debt from Income PDI_i is calculated as:

Eq. 1.40 – Permissible Debt from Income

$$PDI_i = \left(PDA_{i_0} \times \frac{i_{r_{i_0}}}{NIBT_{i_0}}\right) \times \frac{NIBT_i}{i_{r_i}}$$

Where PDA_{i_0} is the Permissible Debt from Assets at the initial time in the simulation, the factor $\left(PDA_{i_0} \times \frac{i_{r_{i_0}}}{NIBT_{i_0}}\right)$ is a constant value named the '*fraction of income to debt service*', and the ratio $\frac{NIBT_i}{i_{r_i}}$ accounts for every variation in income and interest to determine the change in lending. It is worth noting that Eq. 1.40corresponds to PDA_i at the initial time in the simulation, therefore allowing to initialize the system to equilibrium.

Figure 1.11 – Borrowing and Debt and their relationship with interest rate and liquidity



Permissible Debt is calculated as:

Eq. 1.41 – Permissible debt

$$PD_i = PDA_i \times \lambda_{PD}\left(\frac{PDI_i}{PDA_i}\right) \times \lambda_{\xi}\left(\frac{\overline{\xi_i}}{\xi_i}\right)$$

Where $\lambda_{PD}\left(\frac{PDI_i}{PDA_i}\right)$ is a non-linear relationship addressing the correction that the bank makes on Permissible Debt from Assets PDA_i in relation to the ratio $\left(\frac{PDI_i}{PDA_i}\right)$, and $\lambda_{\xi}\left(\frac{\bar{\xi}}{\xi}\right)$ is non-linear effects of changes in average defaults $\overline{\xi_i}$ in comparison to normal defaults on debt ξ_i. This relationship assumes that those who own assets will find it easier to receive lending from banks, with income and defaults having a more marginal effects.

On the other hand, each borrower anchors their desired borrowing ${\beta^*}_i$ to the expected Return on Debt $IndRoD_i$, performing adjustments $\mathrm{J}corr_\beta$ based on both liquidity gap $\frac{\Lambda^*-\Lambda}{AdjT_\beta}$ and perceived growth in adjusted value of assets $D_i \times TRND\mathrm{J}(adjA(t))$. Figures 5.28 show the non-linear effects determining desired borrowing ${\beta^*}_i$. In particular, in periods in which both demand for cash and assets grow, it is assumed that firms would aim to borrow a maximum of three times their indicated debt replacement $IndDR_i$. However, in time of abundancy of liquidity and degrowth in assets, borrowing would need to decrease, assuming a correction reaching a minimum of 30% on indicated debt replacement $IndDR_i$.

The equations describing the calculation of desired borrowing ${\beta^*}_i$ follow as:

Eq. 1.42 – Desired borrowing

$${\beta^*}_i = IndRoD_i \times \lambda_\beta \left(\frac{\mathrm{J}corr_\beta}{IndRoD_i}\right)$$

Where $IndRoD_i$ is the indicated Return on Debt, $\mathrm{J}corr_\beta$ the anchor correction on indicated Return on Debt, and λ_β the non-linear behaviour of borrowers in determining their change in borrowing. In turn:

Eq. 1.43 – Correction for borrowing

$$\mathrm{J}corr_\beta = \frac{\Lambda^* - \Lambda}{AdjT_\beta} + D_i \times TRND\mathrm{J}(adjA(t))$$

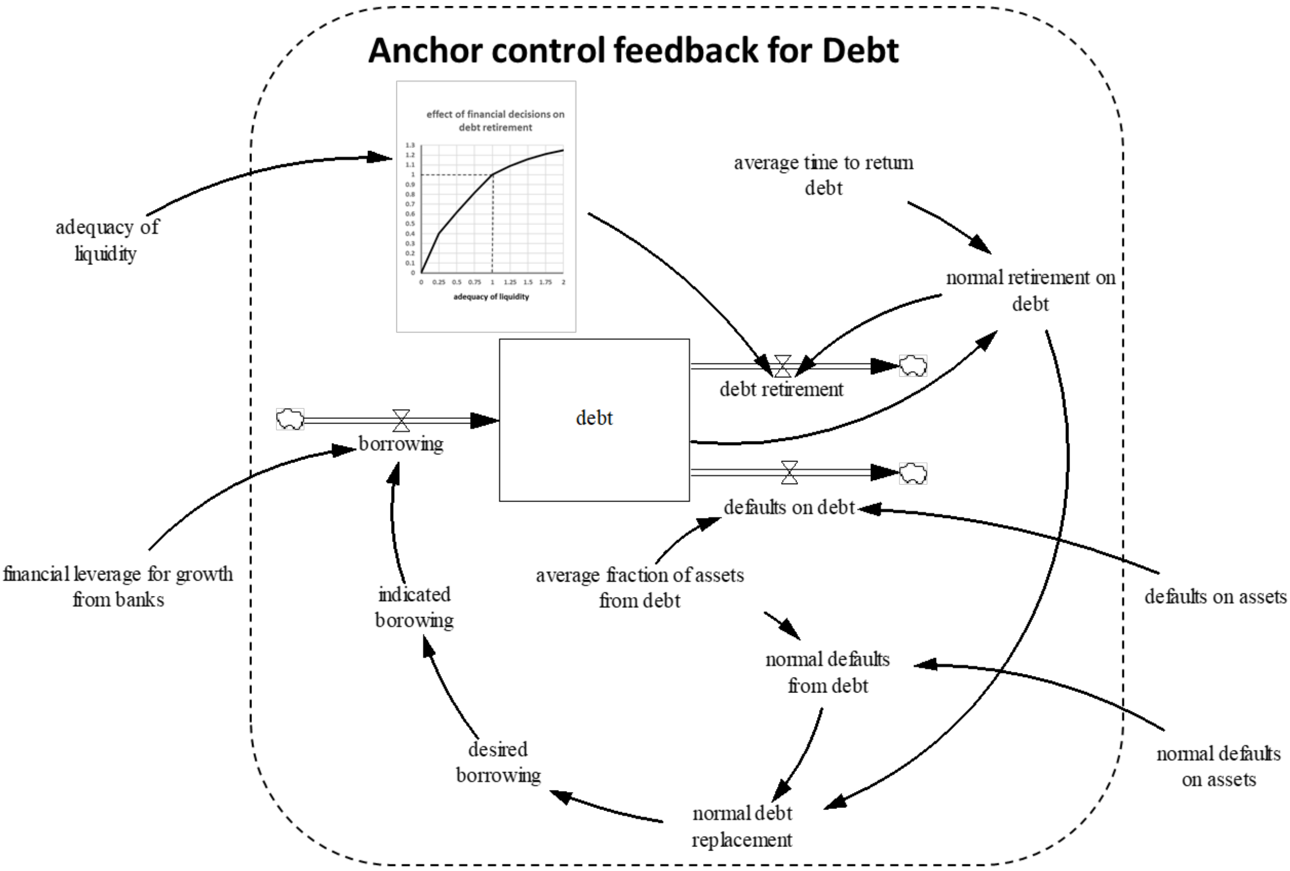
Eq. 1.44 – Indicated Return on Debt

$$IndRoD_i = D_i \times \mu_i$$

Where $\frac{\Lambda^*-\Lambda}{AdjT_\beta}$ is the liquidity adjustment gap based on the adjustment time to rise money via debt $AdjT_\beta$, D_i is the Debt, $TRND\mathrm{J}(adjA(t))$ represent the trend with anchor on adjusted value of assets $adjA(t)$, and μ_i the average time to return debt.

The Indicated Borrowing Rate $Ind\beta_i$ should be seen as the result of the negotiation process between borrower and banks, given the ratio between actual debt D_i and what banks would consider permissible PD_i based on previous rationale. The non-linear effect would be two-fold based on the discrepancy among the two. It is in the interest of the bank to lend money and keep every sector of the economy at a healthy debt ratio, but when debt becomes too large to be managed, banks would stop borrowing controlling their risk of defaults. The non-linear decision variable $\lambda_{Ind\beta}$ (see Figure 1.11) would imply the bank increases the desired borrowing $\beta^*{}_i$ till five times what is desired in case the debt of a firm would approach zero, and push any borrowing demand to zero when D_i would be more than double of what banks would consider permissible.

Figure 1.12 – Anchor control feedback for debt



Eq. 1.45 – Indicated Borrowing Rate

$$Ind\beta_i = \beta^*{}_i \times \lambda_{Ind\beta}\left(\frac{D_i}{PD_i}\right)$$

It is worth noting that Indicative Borrowing $Ind\beta_i$ represents the negotiated demand for money for each sector, and is transferred as input to the bank to determine lending and money creation decisions (see Eq. 1.20). In ERRE it is assumed that based on their objective for growth, the bank can interfere with the borrower decision making it easier for them to leverage money in addition to what is indicated by rational calculation, determining their financial leverage for growth FL. The actual borrowing β_i is then calculated as follows.

Eq. 1.46 – Borrowing rate

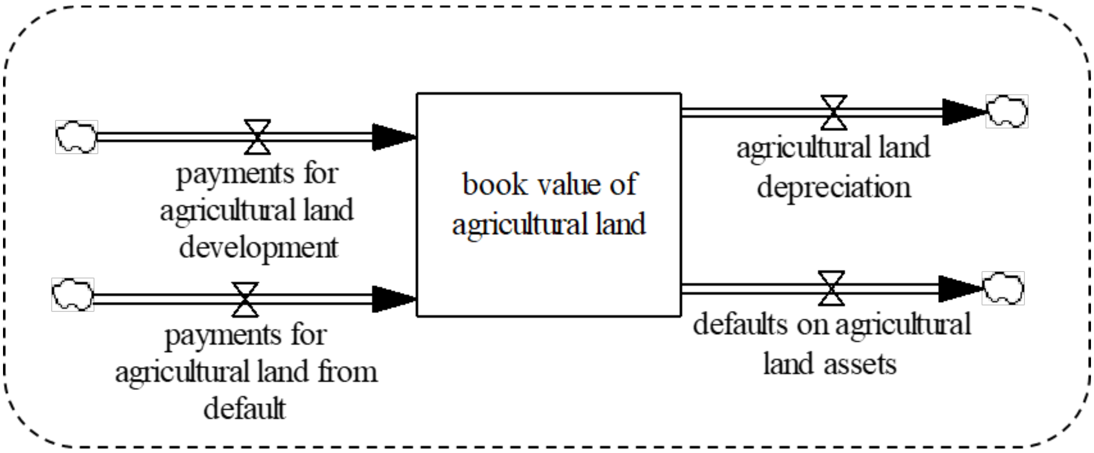
$$\beta_i = Ind\beta_i \times FL$$

AGRICULTURE AND FOSSIL FUEL SECTORS IDIOSYNCRACIES

The structures proposed above are common across all firm sectors. However, given the idiosyncrasies of agriculture and fossil fuel sectors, amends had to be taken into account for their specific cases.

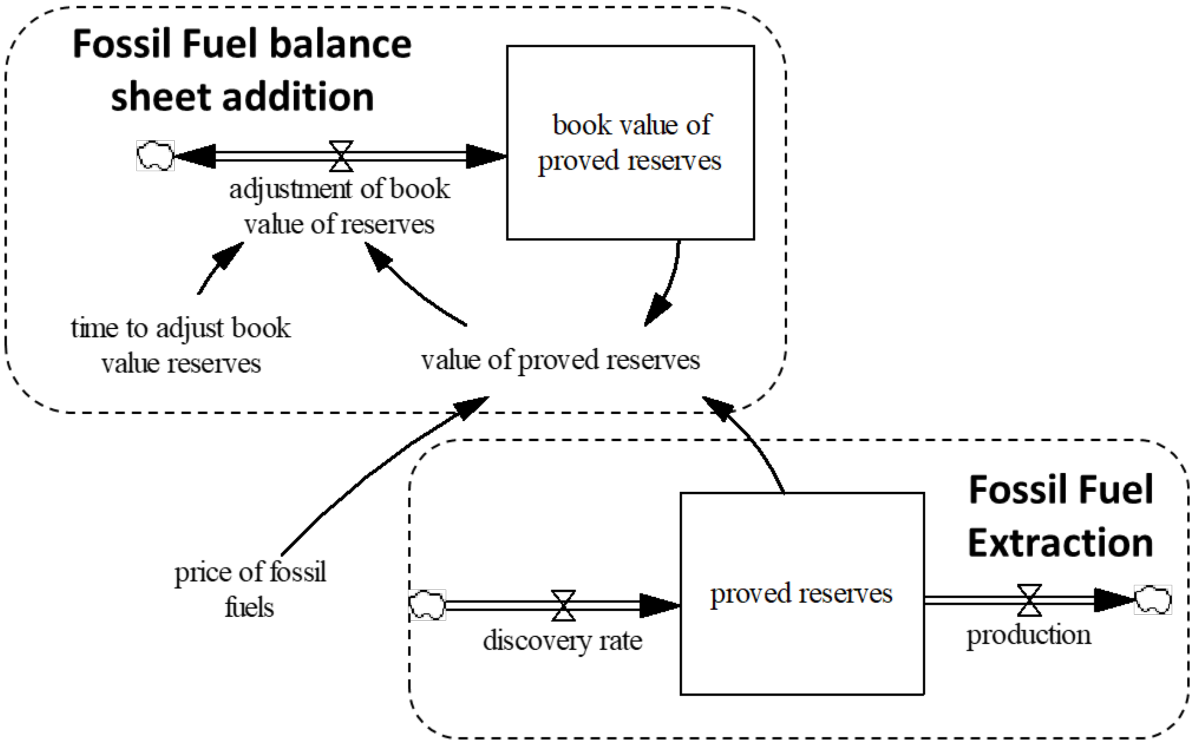
Figure 1.13 shows the book value of agricultural land as addition to the assets side of the Balance Sheet as shown in Figure 1.6 for the case of the agriculture sector. Agricultural Land is an additional input to production, and as such is the result of accumulation of investment in land development and discard due to land erosion, defaults, and development of urban lands. As a result, all of the financial structure seen in this appendix so far has been updated accordingly, including liquidity, interest rate, and borrowing decisions without altering their core philosophy. Details have been made explicit in the relevant section of this appendix.

Figure 1.13 – Idiosyncrasies in the agricultural sector for balance sheet



In a similar way, the fossil fuel sector must account for the proved reserves value in their balance sheet. Despite Real Proved Reserves are dependent on production and discovery rate variables, the Proved Reserves Value For Balance Sheet is a stock that adjusts based on the volatility of the fossil fuels' price as shown in Figure 1.14.

Figure 1.14 – Idiosyncrasies in the Fossil fuel sector for balance sheet

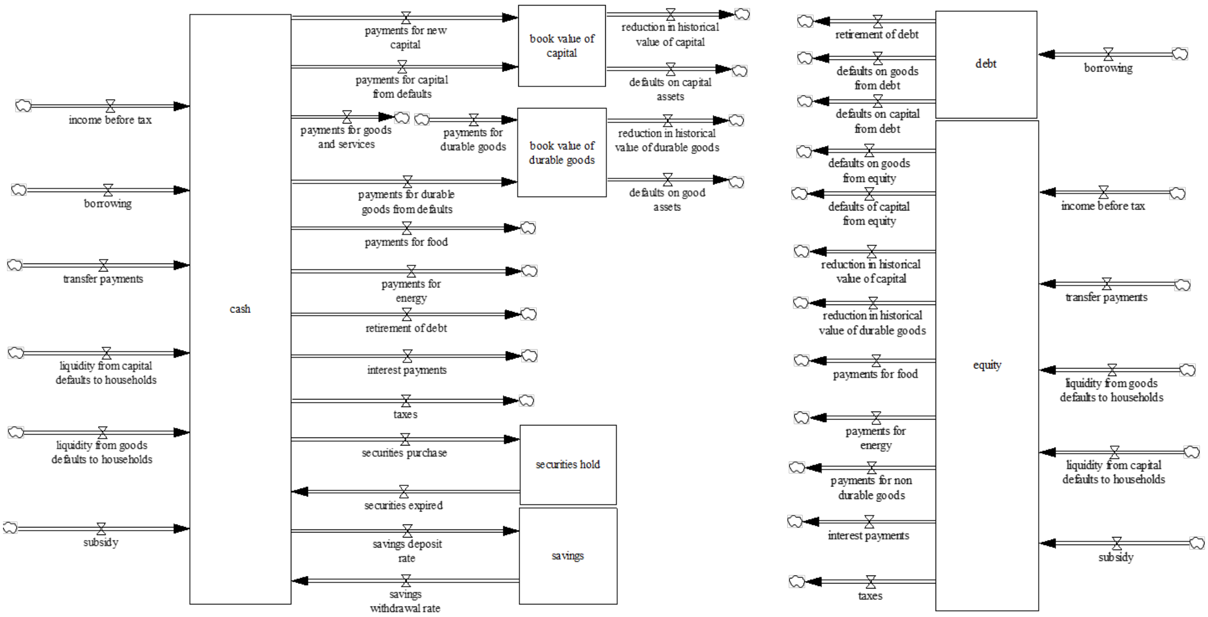


HOUSEHOLDS IDIOSYNCRACIES

The Household sector presents a particular case of the Balance Sheet (Figure 1.15). In addition to being particularly rich in elements, it presents the additional stock of Securities, and the Liquidity stock is substituted with Savings which is controlled slightly differently from the firm sector. All cash inflows differ from the ones in the firm sector, since households income mainly consists of firms payments of both dividends and wages, government transfers and interest income from Savings. In addition, their consumption decisions affect demand for the entire economy.

Households represent the core of the stock and flow consistency of the ERRE model since banks, government and firms converge together to provide the cash necessary to generate household consumption and savings. The rationale behind the determination of the inflows ‘income before tax’ IBT_H and ‘transfer payments’ TP_H demonstrate the relevance of such an argument. In particular, income before tax IBT_H is calculated as follows:

Figure 1.15 – Balance sheet in the Household sector



Eq. 1.47 – Income before tax

$$IBT_H = \sum\nolimits_{\Pi+B} \Phi_i + \sum\nolimits_{\Pi+B} W_i + \sum_{\Pi+H} \Upsilon_i \times \left(\frac{S_H}{S_H + DM_B}\right) + \Upsilon_{Gov} \times \left(\frac{Sec_H}{Sec_H + Sec_B}\right)$$

Where $\sum_{\Pi+B} \Phi_i$ is the sum of dividends from the firms and banking sectors, $\sum_{\Pi+B} W_i$ represent the wage payments from both firms and banks, $\sum_{\Pi+H} \Upsilon_i$ is the total interest payments of the private sector, $\left(\frac{S_H}{S_H+DM_B}\right)$ the fraction of households savings on total banks liabilities, Υ_{Gov} the interest payments from government, and $\left(\frac{Sec_H}{Sec_H+Sec_B}\right)$ the fraction of securities owned by households.

The transfer payments TP_H correspond to:

Eq. 1.48 – Transfer payments

$$TP_H = Exp_{Gov}$$

Where Exp_{Gov} is the government expenditure calculated as in Eq. 1.5. Putting together Eq. 1.48 (government transfers to households), Eq. 1.5 (government expenditure), and Eq. 1.3 (government revenue) it is possible to calculate:

Eq. 1.49 – Transfer payments calculation

$$TP_H = (\sum_{\Pi+H+B} T_i + T_E)^*\big(1 + \Gamma(t)\big) - \Upsilon_{Gov}$$

Finally, putting together the Eq. 1.12 (bank revenue), Eq. 1.13 (bank wage payments), Eq. 1.14 (banks taxes), and Eq. 1.15 (banks dividends) with Eq. 1.47 (income before tax), and summing the result to Eq. 1.49 (Government transfers) it is possible to show that:

Eq. 1.50 - Total income for households

$$IBT_H + TP_H = \sum_{\Pi} \Phi_i + \sum_{\Pi} W_i + \sum_{\Pi+H} \Upsilon_i + (\sum_{\Pi+H} T_i + T_E)^*\big(1 + \Gamma(t)\big)$$

Eq. 1.50 shows the high level of aggregation in the household sector ERRE and most importantly demonstrates how tax policies and bank behaviour remain absorbed into the household decisions. In addition, households depicts both capital owners and workers as if they had the same behaviour. In fact, $\sum_{\Pi} \Phi_i$ represents the income of capital owners from firms in terms of dividends, $\sum_{\Pi} W_i$ represents the income from wages from the firm sector, $\sum_{\Pi+H} \Upsilon_i$ is the total interest payments from the private sector thus inclusive of all interest income of banks secondly distributed in terms of taxes, wages and dividends, and $\sum_{\Pi+H} T_i$ represents the entire revenue of the government that in the base run is accounting for both expenditures and interest payments from government.

In the base run, energy policy taxes T_E and government deficit $\Gamma(t)$ are kept to zero, thus determining the equilibrium condition of households in the model. However, as Eq. 1.50 points out, every government policy determines an increase in household ability to spend that is treated in detail in the following section.

Financial decisions and Savings

The treatment of Savings and Financial Decisions in the household sector is analogous to the treatment of Liquidity in the Firm sector (See Figure 1.7) for generating a source for cash outflows control, their impact on investments and consumption, interest rates and borrowing decisions. Their differences can be listed as follows:

1. The Savings stock is initialized taking into account the requirements of the Banking Sector as Deposits
2. Financial decisions on investment and consumption for Households affect the growth rate of all sectors in the economy
3. The Desired Liquidity coverage in the firm sector has been substituted with a decision variable named Propensity for Savings Ratio $\mathrm{s(t)}$ to simulate the volatile behaviour in households in savings depending on economic condition.

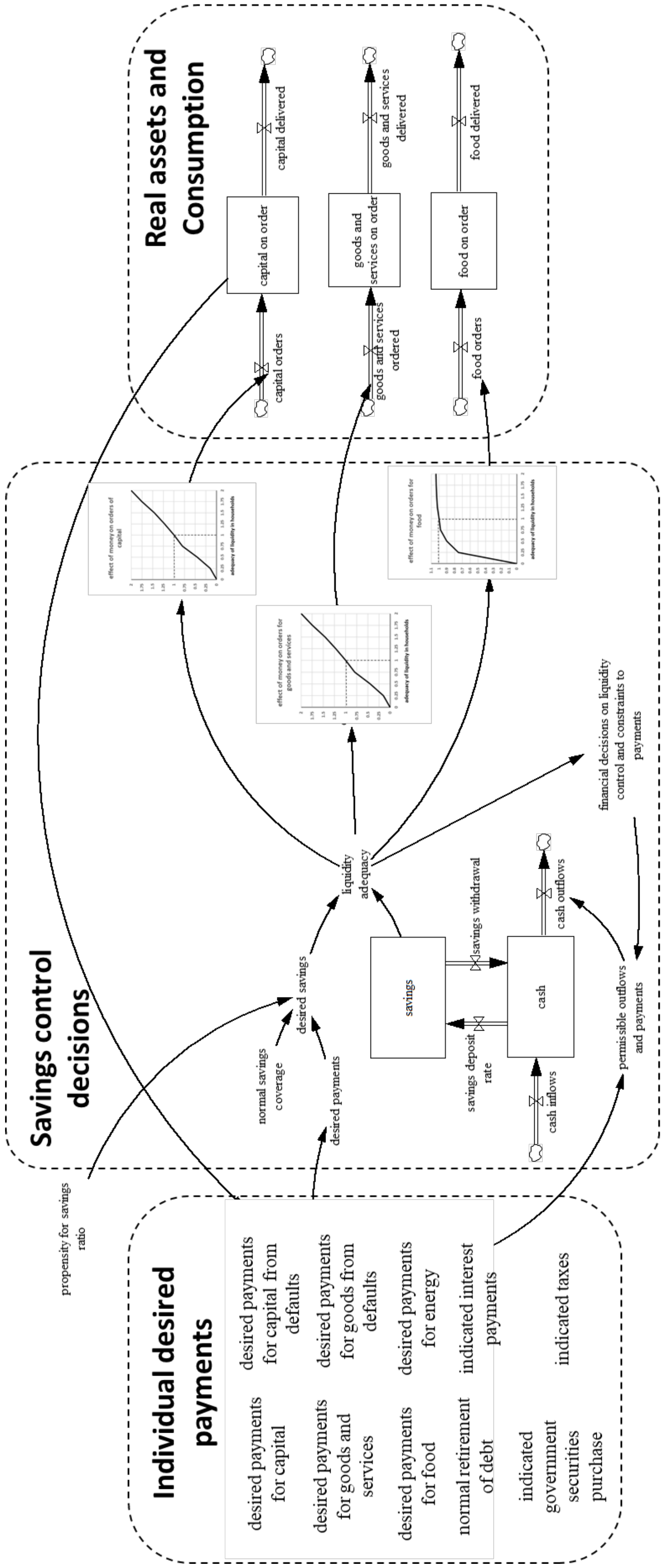
Whereas the first point above has been treated in detail in the previous section of this appendix, we shall focus here on the latter two.

Figure 1.16 shows the effect of Savings Adequacy $\frac{S}{S^*}$ on the consumption and investment decisions of Households, that are the ordering decisions of food, capital, and goods and services. These three non-linear relationships have important effects on the entire economy, given in particular, that every increase in government expenditure generate an increase in the available cash to the households, and thus implying the dynamics of growth and price change for every commodity in the model.

In ERRE capital and energy are consumed by each sector of the economy, whereas Goods and Services and Food are assumed to have only Households as customer. It is worth noting that the capital sector represents most investments in the economy, accounting for about 20% of total GDP, with agriculture remaining lower than 15% of total GDP, and the rest accounted for in the Goods and Capital sector. Most energy output is embedded in the production and price of other commodities. Thus the Household demand for Food, Goods and Services generates cascade demand for Capital as well.

Given the non-linear relationships in Figure 1.16 Household's spend any surplus money in Goods and Services and Capital almost linearly, and when Savings are below desired, they reduce consumption non-linearly assuming an inertia in consumers towards changing their consumption habits until it reaches zero when liquidity is zero. Such a relationship shows the importance of government to assure the liquidity level in the household sector remains above desired (for example using subsidies or tax breaks), so stimulating their consumption behaviour and boosting economic growth. Such hypothesis can be tested and explored in more detail.

Figure 1.16 - Effect of household financial decisions on the economy



On the other hand, their consumption behaviour for food has a very different shape. Food is assumed to be a commodity that is necessary for human life, where average short-term liquidity surplus would have little impact on consumption. Such an effect would remain true in case of cash deficit as well until the point in which liquidity approaches zero, in turn abruptly reducing food consumption to zero. This would reduce the impact of money availability to agriculture volatility. The accounting for long term income change on agriculture commodities consumption is described instead in the following section.

Savings Propensity Ratio

Figure 1.16 show that, differently form the firm sector, the Desired Savings Coverage is dynamic based on a Propensity for Savings Ratio $s(t)$. Such a structure is proposed in Figure 1.17 below.

The Propensity for Savings Ratio $s(t)$ is a multiplicative variable for the constant savings coverage s_{cov} assumed to have a neutral effect at the beginning of the simulation ($s(0) = 1$). The objective is to capture the behaviour of most households in savings more money over time when they can profit more from it, and spend more in the opposite case.

In so doing the variable Return on Savings $\mathrm{RoS_H}$ is introduced as:

Eq. 1.51 – Return on Savings

$$\mathrm{RoS_H} = \frac{\mathrm{NIBT}(\Upsilon)_\mathrm{H} \times (1 - \tau_\mathrm{H})}{\mathrm{S_H}} - \gamma$$

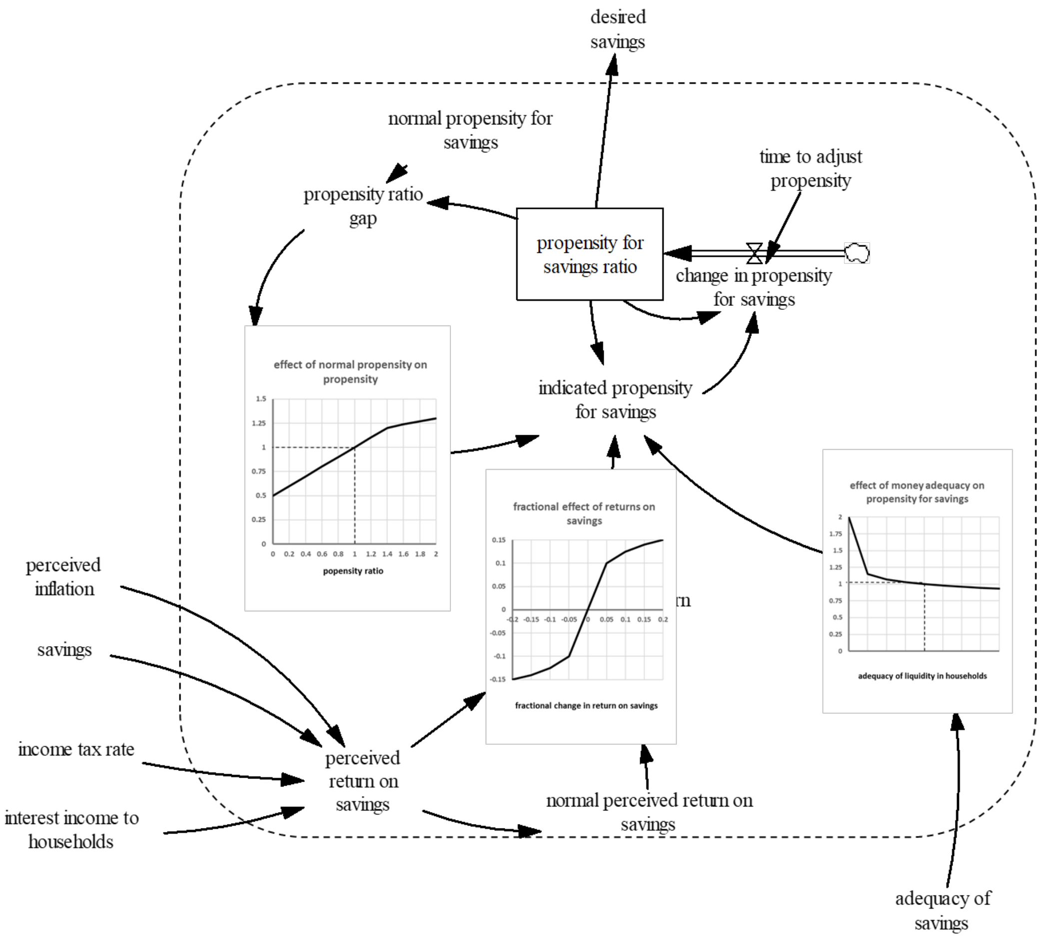
Where $NIBT(\Upsilon)_H$ is the income from interest payments directed to the household sector, τ_H is the income tax rate, S_H is the current savings deposits, and γ the overall inflation in the economy.

The effect of savings return on change in propensity $\lambda_{RoS_H}\left(\frac{RoS_H}{RoS_{H_0}}\right)$ is assumed to be symmetric and non-linear, varying between -0.15 and +0.15 fractional change in savings correction as described in Figure 1.17. The Indicated savings propensity is thus calculated as:

Eq. 1.52 – Indicated savings propensity

$$Inds(t) = s(t) \times \lambda_{RoS_H}\left(\frac{RoS_H(z)}{RoS_{H_0}}\right)$$

Figure 1.17 – Propensity for Savings Ratio



Thus the Propensity of Savings $s(t)$ is determined as the adaptive expectations on past behaviour measured over the time $AdjT_s$ as follows:

Eq. 1.53 – Propensity of Savings Ratio

$$s(t) = smooth(Inds(t), AdjT_s)$$

and impacting the desired savings coverage $s_{cov}(t)$ is calculated as a multiplier effect to the base savings coverage s_{cov} as follows:

Eq. 1.54 – Desired Savings Coverage

$$s_{cov}(t) = s_{cov} \times s(t)$$

It is worth noting, that Eq. 1.55 and Eq. 1.56 indicate that the behaviour of households in accumulating savings would be positively affected by every increase in interest rate from the banking sector (increase in interest payments), and negatively affected by any reduction in interest. Most important, the structure remains path dependent based on the performance of the economy, and lead to business cycles as well as triggering policies in both governmental and financial sector when the path dependency generates undesired conditions.

Following the same structure of the firm in calculating Retained Earnings and Retained Losses as adjustments flows to the Savings stock $\mathrm{S_H}$, Savings Deposit rate $\mathrm{SDR_H}$ and Savings Withdrawal rate $\mathrm{SWR_H}$ accumulate are calculated as the difference between all inflow and all outflows from the stock cash as follows:

Eq. 1.55 – Savings Deposit Rate

$$\mathrm{SDR_H} = \mathrm{Max}\left(0, \sum\nolimits_{CF-IN} \mathrm{cf}_i - \sum\nolimits_{CF-OUT} \mathrm{cf}_i\right)$$

Eq. 1.56 – Savings Withdrawal Rate

$$\mathrm{SWR_H} = \mathrm{Max}\left(0, -\left(\sum\nolimits_{CF-IN} \mathrm{cf}_i - \sum\nolimits_{CF-OUT} \mathrm{cf}_i\right)\right)$$

Other minor edits on households balance sheet

Another difference between Firm and Household's Balance Sheets as shown in Figure 1.15 and Figure 1.6 includes how Goods and Services have been separated between Durable Goods, which can be accounted for as property assets after purchase, and those that are instantaneously consumed or with short duration. While the cash outflow in Goods and Services payments represent the full amount of cash transferred towards the Goods and Service sector, only the fraction of durable goods is accumulated as assets in the balance sheet. As a result, when a Household defaults on their debt, goods can be impounded and sold back to the market to another purchaser. This implies the accounting of both capital and durable goods assets defaults divided between both debt and equity defaults, and finally generating an inflow to the cash of household when those assets are purchased. This is a necessary condition when household is treated as a single aggregated sector of the economy.

PRICE

The price structure represents one of the most interconnected structures in the entire model, gathering input from, and feeding back to, both the real and financial layer of each sector. This includes production and demand, labour force and wages, energy price and energy use, inflation adjusted value of assets, taxes, interest rates and risk factor for capital owners, and others. In addition, price is the variable that translates all real flows into financial flows and vice-versa, impacting revenues and all cash flows from the balance sheet, thus placing itself as the translation between financial performance and real systems. Price is an important variable in the real world for financial accounting and often used as a proxy for stability in markets.

It is worth noting that given the structure of the ERRE model, the price of capital has more than 26000 feedback loops passing through, given it is used as a decision variable for every sector of the economy. Thus the price structure can be used as a system map and reference when looking at all the sub-structures of the ERRE model. These sub-structures are Utility and Production, Real assets, Energy requirements, Assets Value, Depreciation, Adjusted Financial performance of firms, Labour and Dividends.

RATIONAL COST AND IRRATIONAL STOCK PRICE

Figure 1.18 and Figure 1.19 show the formulation for price change for each sector, with important implications both for model structure, as well as the economic theory underpinning it. It is worth noting how similar aspects have been defined in common to the nominal interest rate structure already seen in the banking sector. The price is the combined result of traders who are interested in making profit out of financial market speculation, firms' cost structure who aim at assuring profitability of their businesses while defending their risk investment preferences and paying for their cost of production, and market dynamics of supply-demand disequilibrium. In this section this formulation is described in detail, both in the cases of with and without inventory.

In both cases the cost component of price is modelled in the same way. All variable costs are calculated as sum between indicated energy payments $Ind€_i$ and indicated wage payments $IndW_i$, and the total cost of operative capital embeds the market value of capital $adjA_i(t)$ and the capital charge rate kcr_i. Unitarian costs are calculated as fraction between the variable and fixed costs and the potential production PP_i, and the desired price from cost $p_{c_i}(t)$ as the average sum between the two measured over time $AdjT_{p_{c_i}}$. It is worth noting that the mark-up on cost used to define capital owners profit is hidden under the capital charge rate formulation, being implicit of the risk investment factor of capital owners to make profit ψ_i.

Eq. 1.57 – Price from cost

$$p_{c_i}(t) = smooth\left(\frac{Ind€_i + IndW_i + kcr_i \times adjA_i(t)}{PP_i}, AdjT_{p_{c_i}}\right)$$

On the other hand, traders' stock price $p_{s_i}(t)$ is represented as the memory process on market price p_{m_i} implying adaptation and volatility in markets and calculated as follows:

Eq. 1.58 – Stock Commodity Price

$$p_{s_i}(t) = smooth(p_{m_i}, AdjT_{p_{s_i}})$$

The stock price is impacted by market price change, and feeds back to market price with potential volatility and shocks to indicated market price $Indp_{m_i}$ as follows:

Eq. 1.59 – Market Price

$$Indp_{m_\mathrm{i}} = p_{s_\mathrm{i}}(t) \times \left(\frac{p_{c_i}(t)}{p_{s_\mathrm{i}}(t)} \times \sigma_{p_i} + (1 - \sigma_{p_i})\right)$$

Where σ_{p_i} is the market price sensitivity between cost and stock price. σ_{p_i} represents the strength of producers in setting the price against the stock market. It is worth noting that when $\sigma_{p_i} = 0$, the market price would be entirely determined by stock trading, whereas in the case in which $\sigma_{p_i} = 1$ the price would be entirely determined by the rational model of firms in assuring their cost and profit purpose is met. In the real world, every industrial sector would be characterized by a different σ_{p_i} just being more or less affected by stock trading.

The market price p_{m_i} is the result of the indicated market price $Indp_{m_i}(t)$ as influenced by speculative behaviour of firms in charging additional mark-up based on their cost trend $TRNDJ(p_{c_i}(t))$, and supply-demand market forces $Mp_i(t)$ as follows:

Eq. 1.60 – Market Price

$$p_{m_i} = Indp_{m_i}(t) \times (1 + TRNDJ(p_{c_i}(t)) \times Mp_i(t)$$

In ERRE it is assumed that governments are allowed to impact the price of commodities with the addition of taxes and subsidies. As a result the final price $p_{\tau_i}(t)$ used for commodities trading with the other sectors is calculated as follows:

Eq. 1.61 – Market Price After Tax (used for trading across sectors)

$$p_{\tau_i}(t) = p_{m_i}(t) \times (1 + \tau_{p_i}(t))$$

Where τ_{p_i} represents the tax on trade applied from the government to each particular sector. It is worth noting that such an effect has been kept to a null value for the standard run scenario, and mostly used as input to the simulations in case of policy tests. Thus carbon taxes and/or subsidies to energy and food are assumed to have a direct impact on prices, with direct consequences their competitiveness and general dynamics.

MARKET FORCES IN A GENERIC FIRM

A generic firm refers to a firm which is well representative of services, construction to orders business, and manufacturing of commodities where the inventory time can be neglected or is not relevant for the purpose of ERRE. Their production responds to the management of the backlog of orders for that sector, and suppliers communicate constantly with the clients about their expected delivery delay, that is the time gap from the order registration in the backlog to the supply of that order. Firms that manage capacity dynamically are assumed to increase their delivery delay in times when their capacity is low for demand, with a tendency to increase price as an attempt to lower demand and stabilize capacity levels. Thus the market forces are assumed as being pressures to the price emerging from the ability of supply to match demand.

Figure 1.18 and Eq. 1.62 show the effects of market forces in influencing price. Both factors have multiplicative effect on the market price p_{m_i}, that would have neutral value 1 when capacity matches demand.

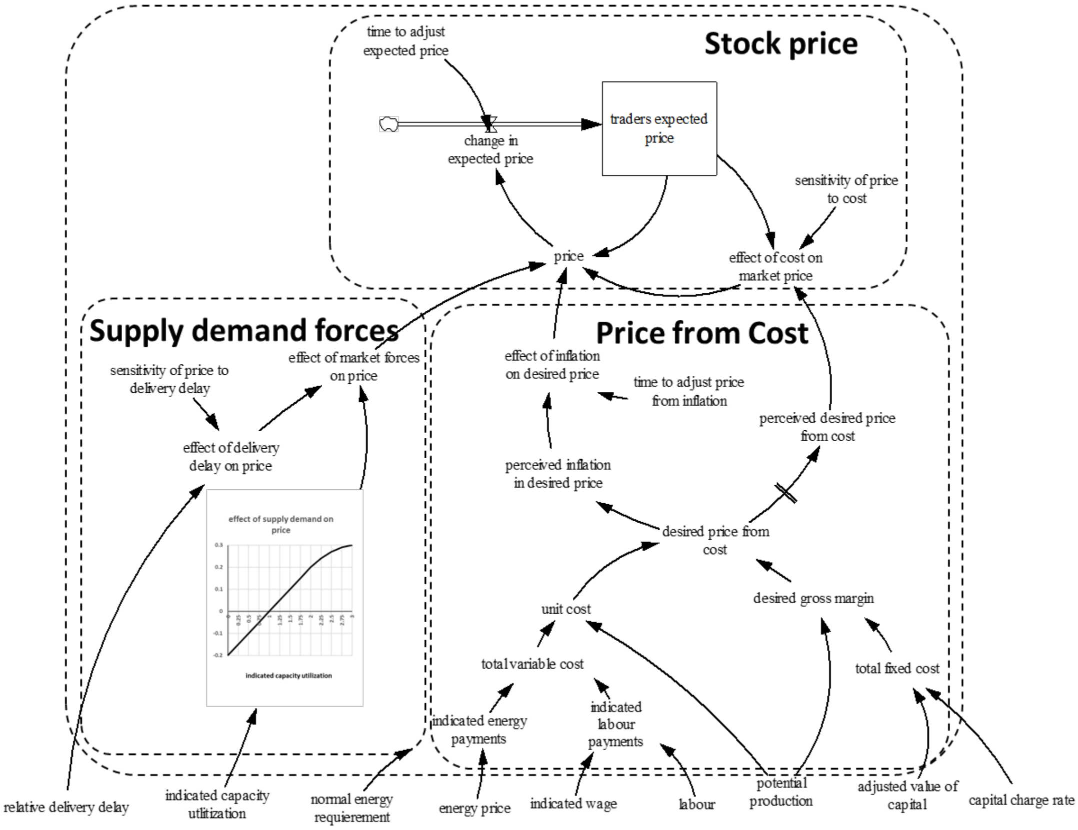
In particular, $\lambda_{SD}\left(\frac{P^*_i}{PP_i}\right)$ represents a non-linear effect dependent on the ability of potential production PP_i to meet desired production P^*_i. As described in the following section, desired production is the sum of demand and the adjustment of backlog to balance the production process of firms. The effect of λ_{SD} is assumed to be mostly linear with a slope of 20%, meaning it impacts price with a fractional decrease of -20% when desired production approaches zero and a fractional increase of +20% when desired production approaches double of current production capacity. Further increases in desired production would trigger a less than linear effect on prices, assumed to reach a +30% fractional increase when desired production is three times the potential production, staying at that level for any additional increase in desired production. Thus prices become less sensitive to supply-demand imbalance the greater the gap is.

The second term of Eq. 1.62 represents the effect of the output delivery delay assuming the normal capacity utilization $DDelNU_i$ on prices. The uncertainty behind such an effect is sufficient that the sensitivity κ_{DDel_i} has been considered as quantifying the discrepancy between average normal delivery delay $\overline{DDelNU_i}$ measured over time $AdjT_{DDel_i}$ (see Eq. 1.78) and the desired delivery delay $DDel^*_i$.

Eq. 1.62 – Market Forces in Firm without Inventory

$$Mp_{NoInv_i}(t) = \left[1 + \lambda_{SD}\left(\frac{P^*_i}{PP_i}\right)\right] \times \left(\frac{\overline{DDelNU_i}}{DDel^*_i}\right)^{\kappa_{DDel_i}}$$

Figure 1.18 – Price in Generic firm



MARKET FORCES IN A FIRM WITH INVENTORY

Figure 1.19 shows the modelling of price in the agricultural sector. Differently from the other sectors, producers of agricultural commodities include the cost of agricultural land as input to the cost structure, but maintain the same concept as all other sectors.

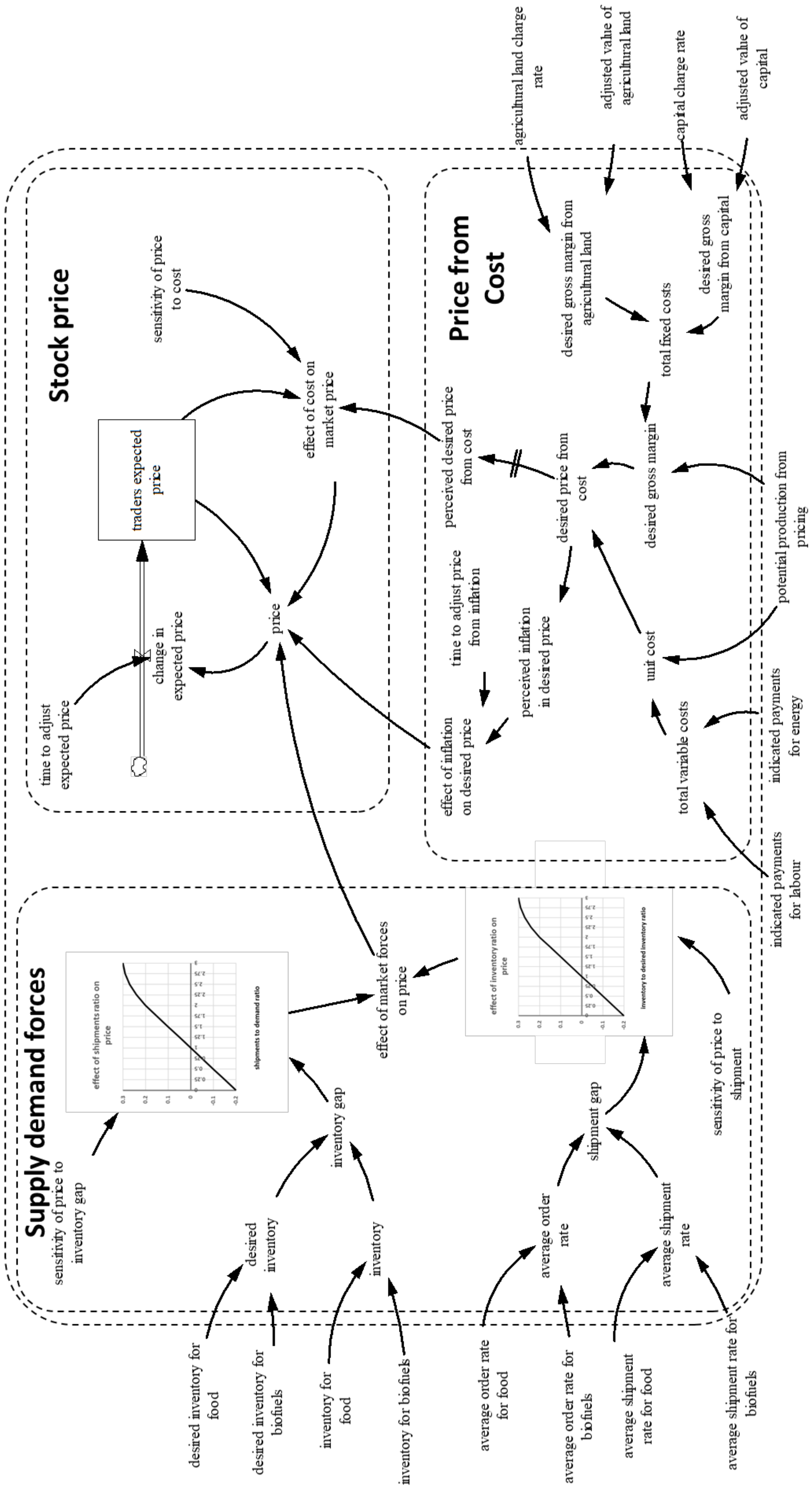
When looking at market forces the situation is completely different from the previous case, including both an effect of inventory availability of commodity $\left(\frac{{Inv^*}_{F+F_e}-Inv_{F+F_e}}{Inv_{F+F_e}}\right)$ and an effect of fractional supply-demand $\frac{\bar{O}_{F+F_e}-\bar{\Sigma}_{F+F_e}}{\bar{\Sigma}_{F+F_e}}$ (see Eq. 1.63). In the case of agriculture, all variables impacting the price are the sum of the relative variables in the Food and Biofuel sectors. For example, the inventory stock Inv_{F+F_e} that impacts the food price, is the sum between the inventory of food Inv_F and the inventory of Biofuels Inv_{F_e}. Due to the uncertainty in estimating these effects and their importance for the purpose of the ERRE model, both effects are tested with sensitivity parameters $\kappa_{O\Sigma_{F+F_e}}$ and $\kappa_{Inv_{F+F_e}}$ during calibration and testing phase of model development. At a first approximation, both effects have been considered linear with both $\kappa_{O\Sigma_{F+F_e}}$ and $\kappa_{Inv_{F+F_e}}$ assumed between 0 and 1.

Eq. 1.63 – Market Forces in Firm with Inventory

$$Mp_{Inv_{F+F_e}}(t) = \left[1 + \kappa_{Inv_{F+F_e}} \times \left(\frac{{Inv^*}_{F+F_e} - Inv_{F+F_e}}{Inv_{F+F_e}}\right)\right] \times \left[1 + \kappa_{O\Sigma_{F+F_e}} \times \left(\frac{\bar{O}_{F+F_e} - \bar{\Sigma}_{F+F_e}}{\bar{\bar{\Sigma}}_{F+F_e}}\right)\right]$$

As described in the following section, the inventory management structure can have important effects on the ability of the sector to assure demand orders O_i satisfaction via shipments Σ_i, or falter because of the low safety stock or important food loss scenarios. Thus it is assumed that the average orders $\bar{O}_i$ and shipments $\bar{\Sigma}_i$, would impact price over the short-term with potential volatility due to the stock market amplification. Given the presence of the inventory between production P_i and demand orders O_i, it is assumed that the availability of the stock of inventory Inv_i and its discrepancy with the desired inventory ${Inv^*}_i$ would retain most of the effect of supply availability to price. The more the inventory in comparison to desired levels the lower the price and vice-versa. The presence of the inventory stock would also allow for the testing of shock scenarios including a sudden drop in demand or a disruption of output given by an extreme climate scenario. It is worth noting that the testing of sensitivity parameter σ_{p_i} addressing the weighted average effect between stock volatile behaviour and rational firm choice to stick price to cost, would receive important instability from the market effects of Eq. 1.63, in particular when testing sudden losses in output.

Figure 1.19 – Price in Firm with Inventory



Idiosyncrasies in Fossil Fuel and energy price market

The fossil fuel price structure presents the same architecture as the food price with regard to the market effects on price, and the same as a generic firm for the cost structure, since it is assumed no agricultural land is involved in production.

Despite each sector providing an internal structure for price modelling, the energy market requires a redistribution of resources given the price differential between Biofuels, Fossil Fuels and Renewables. After presenting the production sectors, the energy market is described in detail.

UTILITY, DEMAND, SHIPMENTS AND PRODUCTION

Utility, demand and production represent the structures at the base of commodity trading among sectors in the ERRE model. In this section the behaviour of production and consumption, as well as the cascading demand that generates the dynamic interplay across all sectors in the model are described.

In the ERRE model there exist differences, as well as commonalities, among all production and utility calculations. In particular, variants on the Constant Elasticity of the production function are applied both as output of each firm and as utility in the households. However, whereas Capital, and Goods and Services sectors are assumed to respond to a backlog structure and not store any output in inventory, fossil fuels and food explicitly account for an inventory structure as well as cost curves dependent on depletion. All sectors plan production capacity based on extrapolative expectations formed based on past demand with anchor bias, in line with cognitive abilities of decision makers and planners to manage capacity. Adapting to demand, orders are placed for any productive factor, cascading towards additional demand for both energy and capital sectors. Households are assumed to be consumers biased towards growth, thus generating a cascading effect on the entire economy. The final consumption is constrained by supply capacity, this in a demand driven model limited by resource limits. This section describes the behaviour of production and consumption in the ERRE model.

HOUSEHOLD UTILITY AND DEMAND

Households lie at the top of the demand chain in the ERRE model. As households are not producers of any commodity, their only role is to consume the output of other sectors and generate the orders necessary to unbalance the economy towards growth.

Figure 1.20 shows the utility structure and consumption bias that generate growth in the ERRE model. In particular, a CES production function is employed to determine the value of potential utility, based on the availability of unpaid workers, capital, goods and services and food. While food and workers are assumed to generate utility by themselves, capital and goods and services are assumed to be coupled with energy requirements for the same purpose. All productive factors except workers have a counterpart cash flow in the household balance sheet (see Figure 1.15). In fact, voluntary unpaid workers represent all those people who provide value without being payed (e.g. home/family carers, etc).

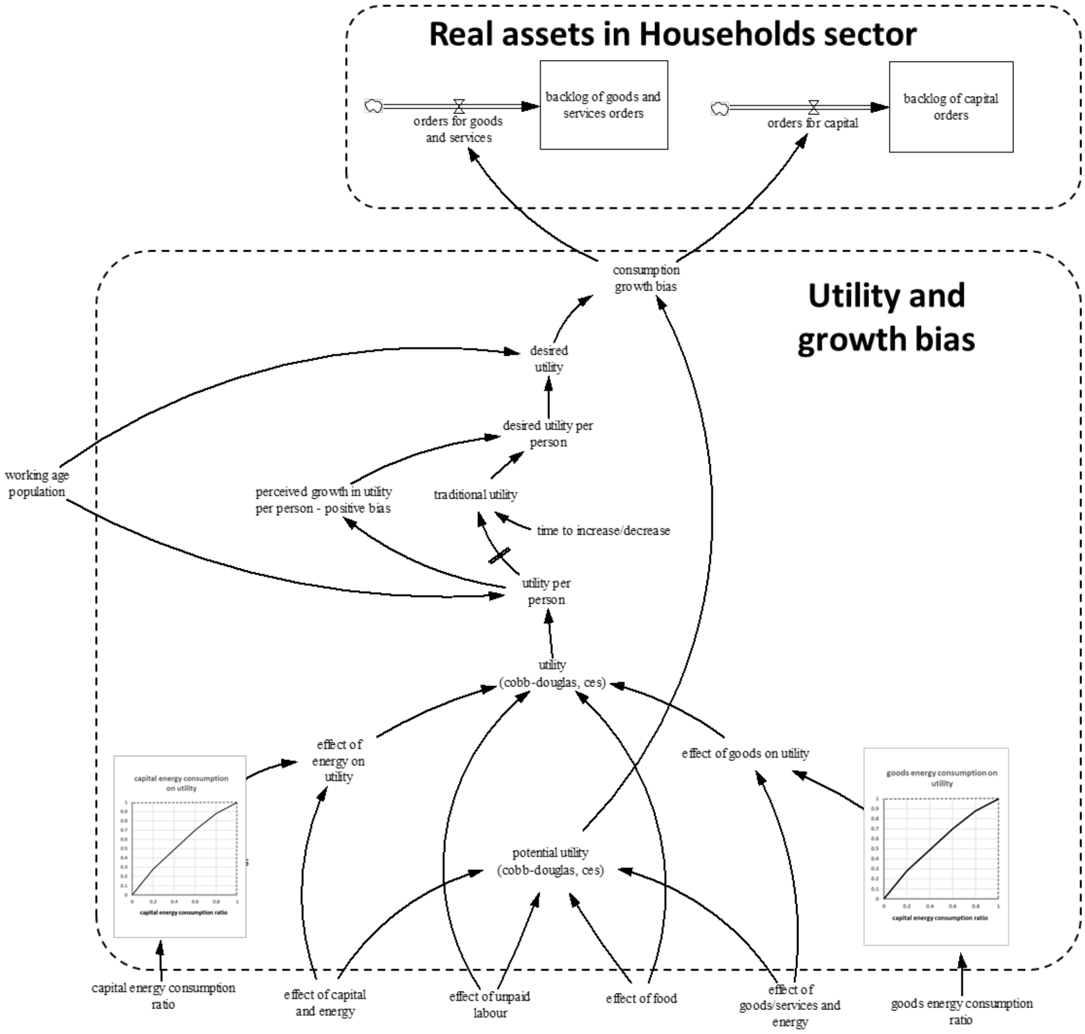
In the ERRE model, the availability of energy in case of any shock is assumed to have an impact on the effective utility that capital and goods can provide to people. This implies that Utility accounts for two non-linear effects which describe the process where the more energy reduces below the necessary level to exploit goods and capital, the more utility reduces till reaching zero when energy delivered to households approaches zero. As it will become evident in the energy market section of this appendix, the structure of energy availability impact on utility is meaningful only for extreme scenarios in which an energy shock can be severe enough to impact households. In the ERRE model it is assumed that households (for their important role in creating stability and growth in the economy) and all energy suppliers (as fundamental for energy generation), are protected by energy delivery policy from energy shortages. In particular, the energy producing sectors, would prioritize both energy suppliers and household demand to deliver energy. Whatever is the remaining energy available it is distributed to industries, agriculture and services, thus leaving all effects of energy shortages on the latter. Eq. 1.64 shows the calculation of Utility U_H, based on Potential utility PU_H and the non-linear effects of energy from capital use $\lambda_{EK}\left(\frac{E_{K_H}}{E_{K_{H_0}}}\right)$ and goods and services use $\lambda_{EGS}\left(\frac{E_{GS_H}}{E_{GS_{H_0}}}\right)$.

Eq. 1.64 – Utility

$$U_H = PU_H \times \lambda_{EK}\left(\frac{E_{K_H}}{E_{K_{H_0}}}\right) \times \lambda_{EGS}\left(\frac{E_{GS_H}}{E_{GS_{H_0}}}\right)$$

In ERRE, dynamic bias towards growth is formed assuming that every person would anchor to their material status and adjust their desired expectation towards higher consumption per person based on that status. This assumption was used in Sterman (1981) and is used in the ERRE model in order to focus the model towards the purpose of looking at the dynamics of growth in a finite world, and showing how the behaviour of people towards greater material consumption may generate environmental risks in our world.

Figure 1.20 – Utility structure and demand generation in Households



In order to serve this purpose, the utility of the global society U_H is scaled down to the expectation of the single worker (tax payer) in the economy thus determining the utility per capita Upc. Given that all working age population $WAPop$ aim to generate greater material well-being, they are assumed to perceive their status as traditional utility per person $\overline{Upc}$ calculated as smoothing average on utility per person as follows:

Eq. 1.65 – Utility per capita

$$Upc = \frac{U_H}{WAPop}$$

Eq. 1.66 – Traditional utility per capita

$$\overline{Upc} = smooth(Upc, AdjT_U(Upc, \overline{Upc}))$$

In ERRE, because households want to improve their material well-being, they are expected to increase their friction to degrowth in case of economic downturn in comparison to their behaviour when the economy does well. In fact their adjustment time in accepting their behaviour change $AdjT_U(Upc, \overline{Upc})$ is dependent on the perceived difference between their current utility Upc and their traditional utility $\overline{Upc}$ so that:

$$AdjT_U = AdjT_U(Upc, \overline{Upc}) = \begin{cases} AdjT_{U_{deg}} = 10, & Upc, < \overline{Upc} \\ AdjT_{U_g} = 2, & Upc \geq \overline{Upc} \end{cases}$$

Thus, households are assumed to adapt fast to push for growth when their utility grows (change in adjustment time for growth $AdjT_{U_g}$ =2 years), and adapt slowly in the case when the economy goes into a downturn (change in adjustment time for degrowth $AdjT_{U_{deg}} = 10$ years).

In order to determine their future consumption expectations, households are assumed to set their own trend on traditional utility $TRNDJ(Upc)$ and anchor to that if, and only if, their growth trend is positive. This can be considered as being an unconscious decision, based on the economic culture in an average capitalistic society, in which the natural friction of the economy in reaching limits and starting degrowth, would be no more than expectations for stagnation for the average citizen. Thus the household consumption bias $HCbias$ is determined as:

Eq. 1.67 – Households consumption bias

$$HCbias = MAX\big(0, TRNDJ(Upc)\big) \times AdjT_{U_g})$$

And applied as positive multiplier bias to determine desired utility U^*_H:.

Eq. 1.68 – Desired utility

$$U^*_H = WAPop \times [\overline{Upc} \times (1 + HCbias)]$$

Where $\overline{Upc} \times (1 + HCbias)$ is the desired utility per person and $WAPop$ the working age population.

The ratio between desired utility and potential utility $\frac{U^*_H}{PU_H}$ is used as a positive bias towards the ordering of new capital and goods and services in addition to the ones already owned. Its effects are described in the relative section of this appendix (see Eq. 1.166).

It is worth noting that such a structure does not imply growth in every case. For example, if the capital sector would be facing a crisis generating the collapse of capital production, the traditional utility would tend to decrease as well as the final desired utility thus following a general reduction in utility. The main effect is that desired utility U^*_H would always be greater or equal to current utility U^*_H, thus assuring that desired factor of production will always receive a positive push from household behaviour to generate orders in the capital and goods and services sectors in the economy.

DEMAND AND PRODUCTION OF GENERIC BUSINESS SECTOR

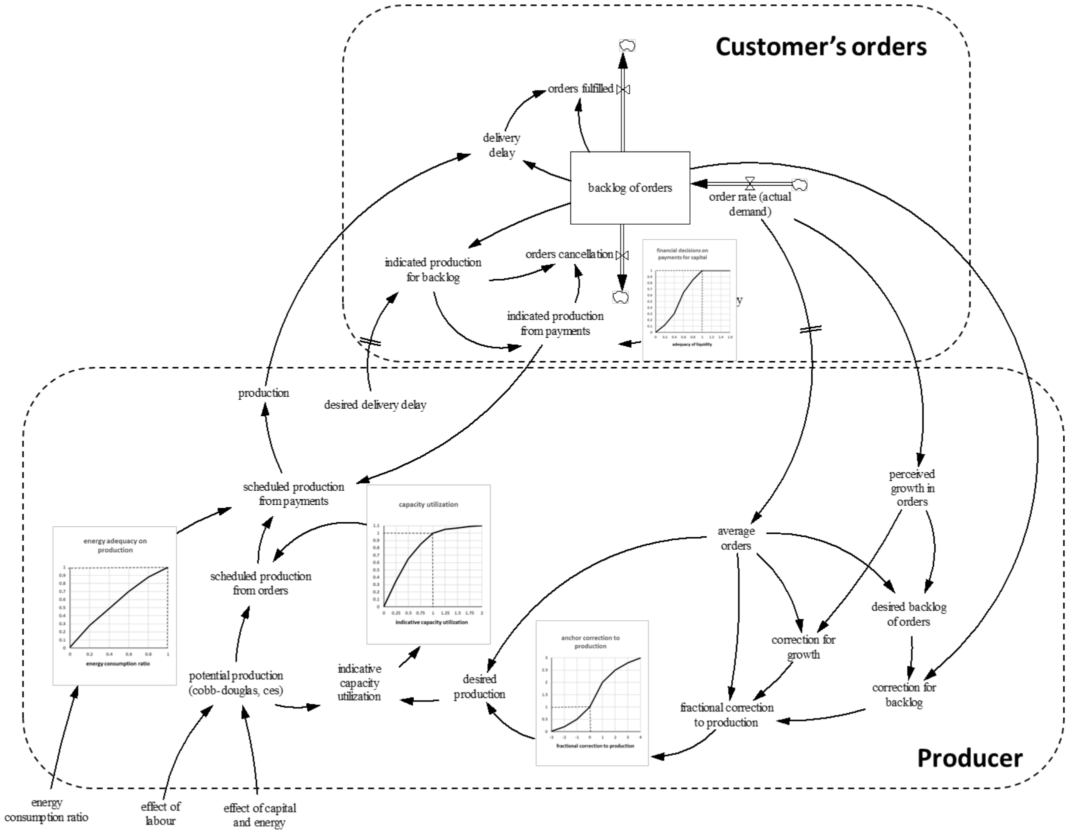
Figure 1.21 shows the generic Production structure of the Capital and the Goods and Service sectors and their interplay with their demand. In the standard formulation of the ERRE model every producer is demand driven, planning their capacity based on extrapolative adaptation and anchor adjustment on perceived demand over time. Producers are assumed to never respond to orders instantaneously, but rather define a desired delivery delay based on current level of technology, and keep the orders on hold in the backlog of orders till they can be processed and produced. The desired delivery delay is the major determinant of the desired level of backlog necessary to producers to keep their system stable, and backlog of orders is their fundamental decision variable for managing capacity.

Given that perception delays tend to differ from actual conditions, and planners do not respond to perceived information linearly, the resulting desired production can be both below or above potential production. Thus, a capacity utilization, implying extra-hours of personnel and night shifts of plants, is assumed to be applied to determine the level of actual production fulfilment based on capacity.

In order to respect both the principle of financial and energy conservation, no production is made possible without the energy sector's stability to deliver the desired energy consumption, and for the client to be capable of making payments at the time production is completed. In fact, customers are allowed to cancel their orders given their availability of liquidity, and adaptation on possible production are made given the availability of energy supplied. The production is delivered to all clients of the firm in proportion to their amount of demand. A delivery delay, based on their backlog is determined and used as a communication tool for their client who can adapt their orders based on their desired time to receive the product they need.

This picture is also affected by exogenous labour productivity growth, which is to make workers more productive with the same amount of capital. Given the model does not distinguish products based on their quality, the increased labour productivity would have an impact in terms of increased production quantity, and relative feedback to demand via prices, among others.

Figure 1.21 – Production in a Generic business sector



This rationale is captured in equations as follows. Orders demand O_i is calculated as the sum of orders from all client sectors o_j:

Eq. 1.69 – Orders for Backlog (Demand)

$$O_i = \sum_{\Pi+H} o_j$$

Each client would monitor their backlog of unfilled orders $bluo_j$ such that:

Eq. 1.70 – Backlog of Unfilled Orders accumulation

$$bluo_j = bluo_{j_0} + \int_t \left[o_j(z) - o_{fulf\,j}(z) - o_{canc\,j}(z)\right] dz$$

Where the backlog $bluo_j$ is a stock that increases with new orders $o_j(t)$ and can decrease both as fulfilled orders $o_{fulf\,j}(t)$, because of deliveries from suppliers, and orders cancellation $o_{fulf\,j}(t)$ in case their liquidity availability would not allow completion of production (See Eq. 1.79).

From the supplier perspective, the backlog of orders BLO_i to be used for managing capacity is the sum of the backlogs of unfilled orders $bluo_j$ from each clients as follows:

Eq. 1.71 – Backlog of Orders

$$BLO_i = \sum_{\Pi+H} bluo_j$$

In ERRE, desired production P^* responds to total orders O_i and backlog BLO_i control applying the so called anchor and adjustment decision rule as follows. The anchor is first determined as average orders $\overline{O_i}$ such that:

Eq. 1.72 – Average Orders rate

$$\overline{O_i} = smooth(O_i, AdjT_{O_i})$$

Thus a desired backlog $BLO^*{}_i$ is computed as the expected backlog at current growth trend $TRNDJ(O_i(t))$ measured over time $AdjT_{O_i}$, and aiming at maintaining its level to keep the delivery delay at desired level $DDel^*{}_i$:

Eq. 1.73 – Desired Backlog

$$BLO^*{}_i = (1 + TRNDJ(O_i(t)) \times AdjT_{O_i}) \times \overline{O_i} \times DDel^*{}_i$$

A desired correction factor to production $\mathrm{J}corr_{O_i}$ is determined such that:

Eq. 1.74 – Correction for growth in orders

$$\mathrm{J}corr_{O_i} = TRND\mathrm{J}(O_i(t)) \times AdjT_{O_i} \times \overline{O_i} + \frac{BLO_i - BLO^*_i}{AdjT_{BLO_i}}$$

Where $TRND\mathrm{J}(O_i(t)) \times AdjT_{O_i} \times \overline{O_i}$ represent the correction based on expected future demand, and $\frac{BLO_i - BLO^*_i}{AdjT_{BLO_i}}$ the resulting correction to keep backlog BLO_i at desired level.

Mimicking the behaviour of operations planners in adapting production to demand based on capacity inertia, the desired production P^*_i is anchored to orders $\overline{O_i}$ and adjusted to the correction $\mathrm{J}corr_{O_i}$ following the non-linear effect λ_P as follows:

Eq. 1.75 – Desired Production

$$P^*_i = \overline{O_i} \times \lambda_P \left(\frac{\mathrm{J}corr_{O_i}}{\overline{O_i}} \right)$$

The non-linear effect λ_P is formulated such that when the correction $\mathrm{J}corr_{O_i}$ is positive, P^*_i is assumed to grow proportionally to average orders $\overline{O_i}$ till the correction $\mathrm{J}corr_{O_i}$ reaches twice the average orders $\overline{O_i}$, growing below proportion if $\mathrm{J}corr_{O_i}$ grows even further. On the other hand, when correction to production $\mathrm{J}corr_{O_i}$ is negative, desired production P^*_i reduces non-linearly and monotonically till approaching zero when the correction $\mathrm{J}corr_{O_i}$ is three times in absolute value in comparison to orders O_i. In so doing, λ_P indicates the behaviour of the aggregated industry in responding to total demand. In the real world, a drop in demand would not affect all firms in one industry in the same way. For example, large businesses might be able to produce at lower cost than small companies that would be forced to shut down their operation as they are less competitive. In addition, there will always be the tendency of companies to produce and try to sell even when demand drops, in particular to pay for the operations cost and investments made. Such a behaviour would have implication for price reduction, thus feeding back to demand increase.

On the side of supply, Potential production PP_i is determined with a nested Cobb-Douglas - Constant Elasticity of Substitution production function based on current levels of capacity, energy and labour. The capacity PP_i is assumed to be relatively flexible in order to meet desired production $P^*{}_i$. The non-linear relationship λ_{CU} describes such behaviour assuming workers and machines can generate production for longer hours per day in case of capacity shortage (till +10%), or rather switch off plants in the case when production capacity PP_i is above desired $P^*{}_i$.

The resulting scheduled production from orders $schedP_i$ would imply a certain level of total energy to be consumed, but constrained by every shortage in energy availability. The non-linear relationship describing this effect λ_E is assumed to reach 1 when energy consumption matches the desired energy consumption $\frac{E_i}{EP_i{}^*} = 1$, and gradually reducing to zero monotonically and above the linear relationship. This describes a lack of energy which would correspond to a need to start saving energy by turning down auxiliary systems (e.g. heating or lighting) not directly linked to production. However, the more energy availability decreases the sharper the behaviour of firms in reducing production by switching off the main plants. The full determinant of energy consumption are described in the relative section of this appendix.

Eq. 1.76 – Scheduled Production

$$schedP_i = PP_i \times \lambda_{CU}\left(\frac{P^*{}_i}{PP_i}\right) \times \lambda_E\left(\frac{E_i}{EP^*{}_i}\right)$$

Despite the ability of the sector to produce, money and payments make the final filter to the production equation. Given the delay gap between the time in which orders are made, and the time in which production is ready to receive payments, clients might find themselves in time of liquidity shortage, needing to cancel their orders. The total indicated payments for capital $IndCFK_i$ is given by the sum of indicated payment for capital from each client $IndcfK_j$, as dependent on the expected deliveries $\frac{bluo_j}{\overline{DDelNU_\iota}}$ reduced by the inability of payments to be made $\mathfrak{f}_{CFK}\left(\frac{\Lambda}{\Lambda^*}\right)$.

Eq. 1.77 – Indicated Production from Payments

$$IndCFK_i = \sum_{\Pi+H} IndcfK_j = \sum_{\Pi+H}\left[\frac{bluo_j}{\overline{DDelNU_\iota}} \times \mathfrak{f}_{CFK}\left(\frac{\Lambda}{\Lambda^*}\right)\right]$$

As previously indicated, the delivery delay is the fundamental variable used by clients to assess their suppliers. The so called delivery delay at normal utilization $DDelNU_i$, is a given aggregated variable calculated as the ratio between total backlog BLO_i and the potential production PP_i. Thus, all clients can average such value $\overline{DDelNU_i}$ as an exponential smooth over the time $AdjT_{DDelNU_i}$.

Eq. 1.78 – Average Delivery Delay at normal utilization

$$\overline{DDelNU_i} = smooth(DDelNU_i, AdjT_{DDelNU_i}) = smooth\left(\frac{BLO_i}{PP_i}, AdjT_{DDelNU_i}\right)$$

The order cancellation $O_{canc}(t)_i$ correspond to the sum of order cancellations from each sector $o_{canc}(t)_i$ as follows:

Eq. 1.79 – Orders cancellation

$$O_{canc}(t)_i = \sum_{\Pi+H} o_{canc}(t)_i = \sum_{\Pi+H} \left[\frac{bluo_j}{\overline{DDelNU_i}} \times \left(1 - f_{CFK}\left(\frac{\Lambda}{\Lambda^*}\right)\right)\right]$$

Production P_i is the minimum between scheduled $schedP_i$ and what can actually be paid by clients $IndCFK$ as follows:

Eq. 1.80 – Production in a generic firm

$$P_i = min(schedP, IndCFK)$$

The actual delivery delay for production $DDel_i$ is determined as the ratio between total backlog BLO_i and production P_i, and provided to the customer to determine their capital construction initiation rate $CIRK_i$ as a ratio between their backlog of unfilled orders $bluo_i$ and such delivery delay $DDel_i$ as follows:

Eq. 1.81 – Delivery delay

$$DDel_i = \frac{BLO_i}{P_i}$$

Eq. 1.82 – Capital construction initiation rate

$$CIRK_j = \frac{bluo_j}{DDel_i}$$

Based on delivered production, customers pay the suppliers given their price of production p_k. It is worth noting that payments for capital cfK_i (or goods and services) is the outflow from the balance sheet for the payment of capital and goods and services output as seen in Figure 1.6 and Figure 1.15.

Eq. 1.83 – Payments for New Capital

$$cfK_j = CIRK_j \times p_k$$

Thus the supplier is allowed to collect all its revenue from sales R_i as sum of all their clients' payments cfK_j:

Eq. 1.84 – Revenue in a generic production sector

$$R_i = \sum_{\Pi+H} cfK_j$$

<u>DEMAND, PRODUCTION AND SHIPMENTS IN AGRICULTURE</u>

Food demand and shipment

Figure 1.22 shows the structure determining demand management, production and shipment of food. The food sector shares the same structure of the business sector, with the difference being in placing inventory and production development in between orders and production. In fact, a backlog of orders is maintained and both effects of capacity utilization and energy requirement are kept. Two minor differences in these structures are the financial effects of food clients (Households) that, despite maintaining the same structure as the generic business sector, have very little impact on the cancellation of orders, and the capacity utilization having lower elasticity in adaptation to desired production than in a normal generic business sector.

The indicated shipments $Ind\Sigma_F$ is determined as the minimum between the average orders $\overline{O_F}$ and expectations on payments based on the households availability of cash $IndCFF_F$. Because of the high aggregation level, this structure can be interpreted as the behaviour of a retailer who contracts farmers thus controlling their backlog of order, and sells food to households based on their demand.

Eq. 1.85 – Indicated Food shipments

$$Ind\Sigma_F = min(\overline{O_F}, IndCFF_F)$$

The actual shipments is dependent on the level of available inventory Inv_F that the retailer needs to keep close to their desired inventory $Inv^*{}_F$. In ERRE, it is assumed that if retailers are capable of processing orders to a minimum time $mopt_F$, their maximum delivery capacity would result in $\frac{Inv_F}{mopt_F}$. The non-linear correction λ_{Σ_F} assumes that the complete fulfilment of orders is possible only when the maximum shipment $\frac{Inv_F}{mopt_F}$ is 20% beyond the indicated shipments $Ind\Sigma_F$. After then, it will start decreasing non-linearly, fulfilling the 90% of indicated orders in the case that maximum orders equals the indicated shipments, and reduce to zero when inventory capacity approaches zero. Such rationale, that might look counterintuitive, is the result of aggregating all countries and all food commodities as one single global food supplier. The order satisfaction starts decreasing at the 120% ratio because it is assumed that the shortage would be the result of some countries decreasing their delivery capacity while many others would still be capable of supplying far beyond what demand requires. This rationale is described in Eq. 1.89.

Given such a non-linear relationship λ_{Σ_F}, particular importance is given to level of desired inventory $Inv^*{}_F$ that should become the convergence for actual inventory Inv. The rationale to determine desired inventory $Inv^*{}_F$ is similar to the one used in the determination of the desired backlog of orders $BLO^*{}_i$ in Eq. 1.73, with the difference that, in the case of food, orders trigger an inflow to backlog and outflow from inventory, resulting in an inversion in sign for the correction factor. In particular the average shipments $\overline{\Sigma_F}$ is calculated as smoothing average on past shipments as:

Eq. 1.86 – Average Orders rate

$$\overline{\Sigma_F} = smooth(\Sigma_F, AdjT_{\Sigma_F})$$

And desired inventory $Inv^*{}_F$ as:

Eq. 1.87 – Desired Inventory

$$Inv^*{}_F = (1 + TRNDJ(\Sigma_F(t)) \times AdjT_{\Sigma_F}) \times \overline{\Sigma_F} \times InvCov^*{}_F$$

Where $(1 + TRNDJ(\Sigma_F(t)) \times AdjT_{\Sigma_F}) \times \overline{\Sigma_F}$ represent the forecasted value of future shipments from firms, and $InvCov^*$ the desired inventory coverage, that for simplicity is kept as a parameter. In order to satisfy all orders (i.e. Σ_F=O_F), this structure implies a strict relationship between λ_{Σ_F}, $InvCov^*{}_F$ and $mopt_F$. Assuming the ideal condition in which the inventory equals desired inventory $Inv_F = Inv^*{}_F$, a temporary stability in food shipments (i.e. $TRNDJ(\Sigma_F(t)) =$

0), no limit in cash availability so that indicated shipments $Ind\Sigma_F$ correspond to average orders $\overline{O_F}$, and putting together Eq. 1.85, Eq. 1.89, Eq. 1.86, and Eq. 1.87 it is possible to show that:

Eq. 1.88 – Relationship between Inventory coverage and minimum orders processing time

$$\Sigma_F = \overline{O_F} \times \lambda_{\Sigma_F}\left(\frac{\frac{\overline{\Sigma_F} \times InvCov^*{}_F}{mopt_F}}{\overline{O_F}}\right)$$

Given λ_{Σ_F}, the full order fulfilment ratio can be assured only if its argument is beyond 120%, which means that the value $\frac{InvCov^*{}_F}{mopt_F}$>120%. In other words, if $InvCov^*{}_F > 1.2 \times mopt_F$, then the food sector will tend to fulfil all its orders, whereas in the opposite case the food sector inventory would always adjust to deficiencies in the supply, generating a persistent shortage in the food supply for the entire duration of the simulation.

The rest of the structure underpinning desired planting $Pl^*{}_F$ follows the same principles previously seen for determining desired production $P^*{}_i$ in a generic business sector (see Eq. 1.75), with the main differences that the entire structure anchors and adjusts to shipments $\Sigma_F(t)$ instead of orders $O_i(t)$, and there are more stocks to consider for the anchor and adjustment process.

Eq. 1.89 – Food shipments

$$\Sigma_F = Ind\Sigma_F \times \lambda_{\Sigma_F}\left(\frac{\frac{Inv_F}{mopt_F}}{Ind\Sigma_F}\right)$$

Thus both the desired backlog $BLO^*{}_F$ and the desired production in progress $PiP^*{}_F$ are determined based on the expected shipments $(1 + TRNDJ(\Sigma_F(t)) \times AdjT_{O_i}) \times \overline{\Sigma_F}$ as follows:

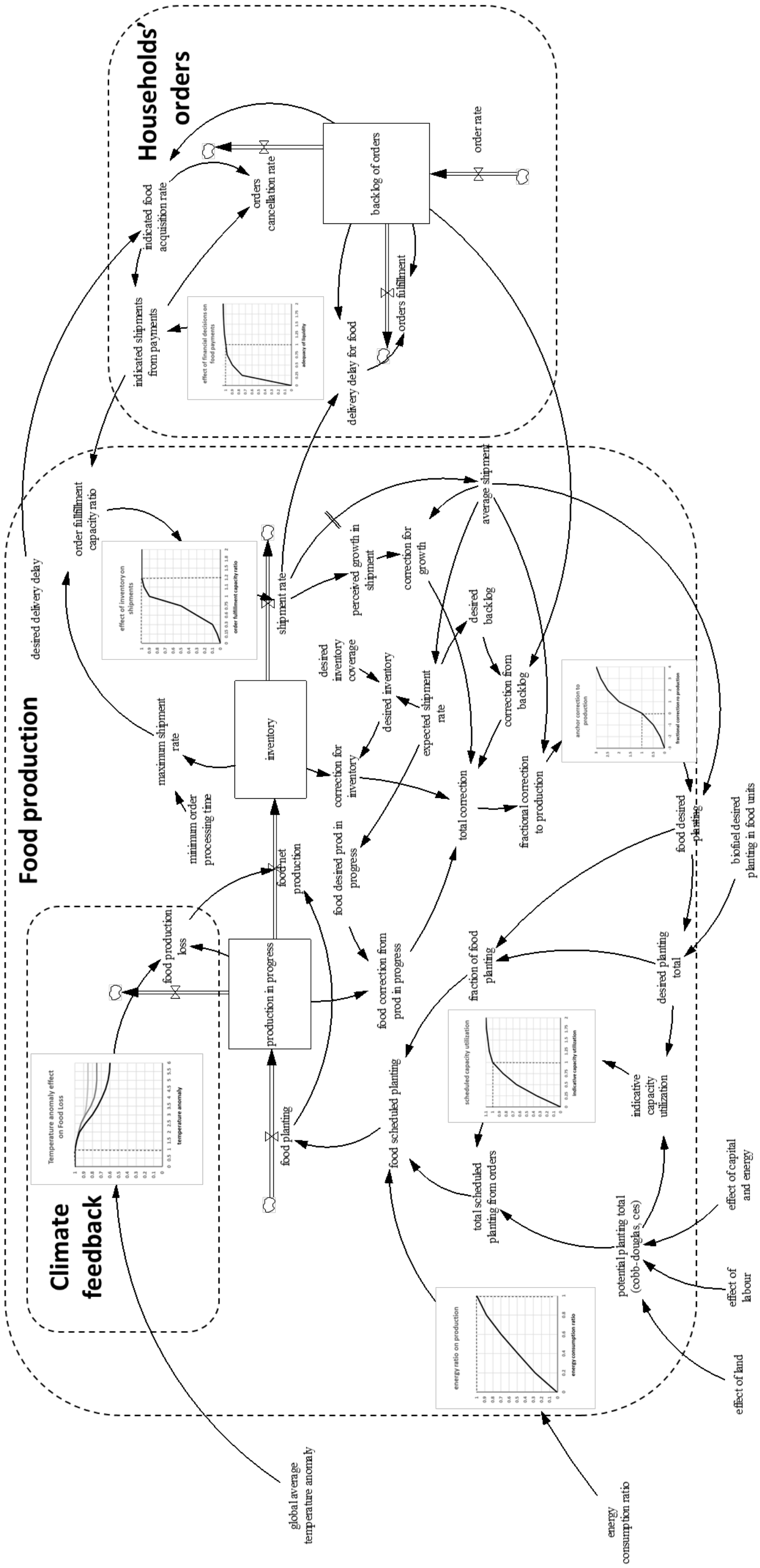
Eq. 1.90 – Desired Backlog

$$BLO^*{}_F = (1 + TRNDJ(\Sigma_F(t)) \times AdjT_{\Sigma_F}) \times \overline{\Sigma_F} \times DDel^*$$

Eq. 1.91 – Desired Production in progress

$$Pip^*{}_F = \left(1 + TRNDJ\left(\Sigma_F(t)\right) \times AdjT_{\Sigma_F}\right) \times \overline{\Sigma_F} \times FPT^*$$

Figure 1.22 – Food Production



Where $DDel^*$ is the desired delivery delay of food orders, and FPT^* the desired food production time (i.e. the time required between planting and harvesting).

The anchor correction factor is considered as:

Eq. 1.92 – Production correction factor for food

$$\text{J}corr_{\Sigma_F} = TRNDJ\big(\Sigma_F(t)\big) \times AdjT_{\Sigma_F} \times \overline{\Sigma_F} + \frac{BLO_F - BLO^*{}_F}{AdjT_{BLO_F}} + \frac{Inv^*{}_F - Inv_F}{AdjT_{Inv_F}} + \frac{PiP^*{}_F - PiP_F}{AdjT_{PiP_F}}$$

And desired planting $Pl^*{}_F$:

Eq. 1.93 – Desired planting of food in Food Units

$$Pl^*{}_F = \overline{\Sigma_F} \times \lambda_{\Sigma_F}\left(\frac{\text{J}corr_{\Sigma_F}}{\overline{\Sigma_F}}\right)$$

Where λ_{Σ_F} has the same shape for every sector in the model.

Aggregation and Disaggregation of food from biofuels for production

The Agriculture sector of ERRE is responsible of both the supply of food to households and biofuels as energy supply to every sector in the economy. However, whereas demand for both commodities comes from two sources, the effective production was maintained as aggregated, assuming that farmers would employ the same capital, fertilizers, land, and labour for the total production of agricultural output, and, secondly, separate those out among the biofuels and food.

The structure of demand and shipments of biofuels is the same used in food, with two special features of the energy system, that are:

1. Backlog is not considered for the management of orders in the energy sector
2. There is no effect of client liquidity constraint on energy production, assuming this is determined a priory.

In other words, energy is placed at a higher priority sector than all other producing sectors, as economic sustainability considerations determine that energy is a fundamental requirement of production in the other sectors (as well as energy itself).

Apart from these differences, the biofuel desired planting $Pl^*{}_{F_e}$ is calculated in the same way as food, that is:

Eq. 1.94 – Desired planting of Biofuels in Energy units

$$Pl^*{}_{F_e} = \overline{\Sigma_{F_e}} \times \lambda_{\Sigma_{F_e}}\left(\frac{\mathrm{J}corr_{\Sigma_{F_e}}}{\overline{\Sigma_{F_e}}}\right)$$

Where $\mathrm{J}corr_{\Sigma_{F_e}}$ is calculated as in Eq. 1.92 without accounting for any adjustment from backlog.

The desired planting $Pl^*{}_{F+F_e}$ for the entire agricultural sector is calculated as:

Eq. 1.95 – Desire planting in Agriculture

$$Pl^*{}_{F+F_e} = Pl^*{}_F + \frac{Pl^*{}_{F_e}}{\varsigma}$$

Where ς is the conversion factor from Food Units in Energy Units.

The response of planting is the same as that seen in the generic business sector, with the main difference that the capacity utilization curve $\lambda_{CU_{F+F_e}}$ result being less elastic than in the previous case, capturing the inertia of agriculture and dependency of their land, which gives little room for flexible production.

Eq. 1.96 – Scheduled Agricultural Planting

$$schedPl_{F+F_e} = \lambda_{CU_{F+F_e}}\left(\frac{Pl^*{}_{F+F_e}}{Pl_{F+F_e}}\right)$$

The energy availability is also assumed to impact agricultural production non-linearly λ_E with the same curve. This results in both planting equations as follows:

Eq. 1.97 – Food Planting

$$Pl_F = \lambda_{E_F}\left(\frac{E_F}{E_F{}^*}\right) \times \frac{schedPl_{F+F_e}}{Pl^*{}_{F+F_e}} \times Pl^*{}_F$$

Eq. 1.98 – Biofuels Planting

$$Pl_{F_e} = \lambda_{E_{F_e}}\left(\frac{E_{F_e}}{E_{F_e}{}^*}\right) \times \frac{schedPl_{F+F_e}}{Pl^*{}_{F+F_e}} \times \frac{Pl^*{}_{F_e}}{\varsigma}$$

Where $\frac{schedPl_{F+F_e}}{Pl^*{}_{F+F_e}}$ is the fraction of scheduled production to desired production, $Pl^*{}_F$ the desired food production, $\frac{Pl^*{}_{F_e}}{\varsigma}$ the desired biofuel production in food units, $\frac{E_F}{E_F{}^*}$ the ratio of energy available for scheduled food production to normally available, and $\frac{E_{F_e}}{E_{F_e}{}^*}$ the ratio of energy available for scheduled biofuel production to normally required.

As described in the energy market, the biofuel sector, being an energy provider, is protected by energy policy maintaining its energy availability as close to desired as possible.

As Figure 1.22 shows, the planting flows in the same structure for both food and biofuel sector accumulating in the stock Production in progress and then inventory, thus closing the loop. Then Eq. 1.99 shows the production of food P_F. In order to improve realism to the supply chain in the model a third order delay $Delay3()$ is assumed describing the relationship between production P_F and planting Pl_F as follows:

Eq. 1.99 – Food production

$$P_F = Delay3(Pl_F, FPT_{P_F})$$

The revenue of the agriculture sector is determined via the shipments of both food and biofuels as follows:

Eq. 1.100 – Agriculture Revenue

$$R_{F+F_e} = \Sigma_F \times p_F + R_{F_e}$$

Where Σ_F is the food shipments, p_F the price of food, and R_{F_e} the revenue from biofuels. This latter is described in detail in the energy market section of this appendix.

DEPLETION, PRODUCTION AND SHIPMENTS OF FOSSIL FUELS

Figure 1.23 shows the demand, production and depletion structure of the fossil fuel sector. The structure presented here is similar in general behaviour to the Energy Transition proposed in Sterman (1981), but fully reviewed and updated for the scope of ERRE, including additional non-linear effects, structure of inventory and application of production to extraction rather than discovery. The full structure is reviewed here.

On the side of production and inventory, most aspects are common to the relative structures in agriculture and generic firms. Two minor differences in the treatment of shipments are that orders are assumed to be met instantaneously given fossil fuel in inventory are burnt and used to produce energy, and no order cancellation for energy production is allowed. Thus shipments Σ_N are calculated as:

Eq. 1.101 – Fossil Fuels shipment

$$\Sigma_N = O_N \times \lambda_{\Sigma_N}\left(\frac{\frac{Inv_N}{mopt_N}}{O_N}\right)$$

Where O_N is the orders to the non-renewable sector, λ_{Σ_N} is assumed as having the same shape as in agriculture (see Eq. 1.89), $mopt_N$ is the minimum order processing time and Inv_N the available inventory. It is worth noting that all relationships hold as in the case of agriculture, including the relationship between inventory coverage $InvCov_N$ and minimum order processing time $mopt_N$. In the case of energy it is assumed that order processing remains much lower than the previous case, allowing energy shipments to be far beyond orders O_N, keeping λ_{Σ_N} in the area in which orders can be fulfilled.

Given that backlog is not considered the net desired production nP^{*}_{N} is calculated as:

Eq. 1.102 – Fossil Fuels desired net production

$$nP^{*}_{N} = \overline{\Sigma_N} \times \lambda_{\Sigma_N}\left(\frac{\mathrm{J}corr_{\Sigma_N}}{\overline{\Sigma_N}}\right)$$

Where $\mathrm{J}corr_{\Sigma_N}$ accounts for both the shipment trend $TRNDJ\big(\Sigma_N(t)\big) \times AdjT_{\Sigma_N} \times \overline{\Sigma_N}$ and the correction from inventory $\frac{Inv_N - Inv^{*}_{N}}{AdjT_{Inv_N}}$:

Eq. 1.103 – Fossil fuel correction to production

$$\mathrm{J}corr_{\Sigma_N} = TRNDJ\big(\Sigma_N(t)\big) \times AdjT_{\Sigma_N} \times \overline{\Sigma_N} + \frac{Inv^{*}_{N} - Inv_N}{AdjT_{Inv_N}}$$

In addition, fossil fuels accounts for an energy loss conversion factor used as a metric of energy efficiency in the transformation process from Gross gP^{*}_{N} to Net Energy nP^{*}_{N} equivalent. As a result the desired gross energy gP^{*}_{N} is calculated as follows:

Eq. 1.104 – Desired Gross Production of Fossil Fuel Energy

$$gP^{*}_{N} = \frac{nP^{*}_{N}}{\boldsymbol{\eta_N}}$$

The major differences that make the fossil fuel sector unique in the ERRE, are in production and depletion. On the left hand side of Figure 1.23 a dynamic representation of the McKelvey diagram (Figure 3.1) is shown. It is worth paying particular attention to the shape of the two non-linear relationships (λ_{gPP_N} and λ_{LTG_N}) that together can lead the world towards the limit to growth of resource depletion and scarcity as in Meadows et al (2003).

Potential production gPP_N is calculated in relation to the limit from proven reserves PR_N in a similar manner to shipments Σ_F from inventory Inv_N (see Eq. 1.101). This involves a correction to the normal potential production calculated with the CES production function based on the maximum deliveries defined based on the current proven reserves PR_N. Given that in today's world coal, gas and oil can be considered as having a normal reserves to production ratio of 50, and are already experiencing decreasing EROEI, it is assumed that the non-linear relationship λ_{gPP_N} determines a constraint to production, starting when the reserves to production ratio decreases below 20, and decreases linearly till reaching zero output when proven reserves are exhausted.

Eq. 1.105 – Fossil Fuel Gross Potential Production

$$gPP_N = gCESPP_N \times \lambda_{gPP_N}\left(\frac{\frac{PR_N}{mrpr_N}}{gCESPP_N}\right)$$

Thus gross production gP_N (similarly to the previous case) is modelled for both energy and capacity utilization constraints based on desired production $gP^*{}_N$, and, closing the feedback control loop to inventory, the net production nP_N can be obtained accounting for the energy efficiency factor $\boldsymbol{\eta_N}$ as follows:

Eq. 1.106 – Gross scheduled production of Fossil Fuels

$$gP_N = gPP_N \times \lambda_{CU}\left(\frac{gP^*{}_N}{gPP_N}\right) \times \lambda_E\left(\frac{E_N}{EP^*{}_N}\right)$$

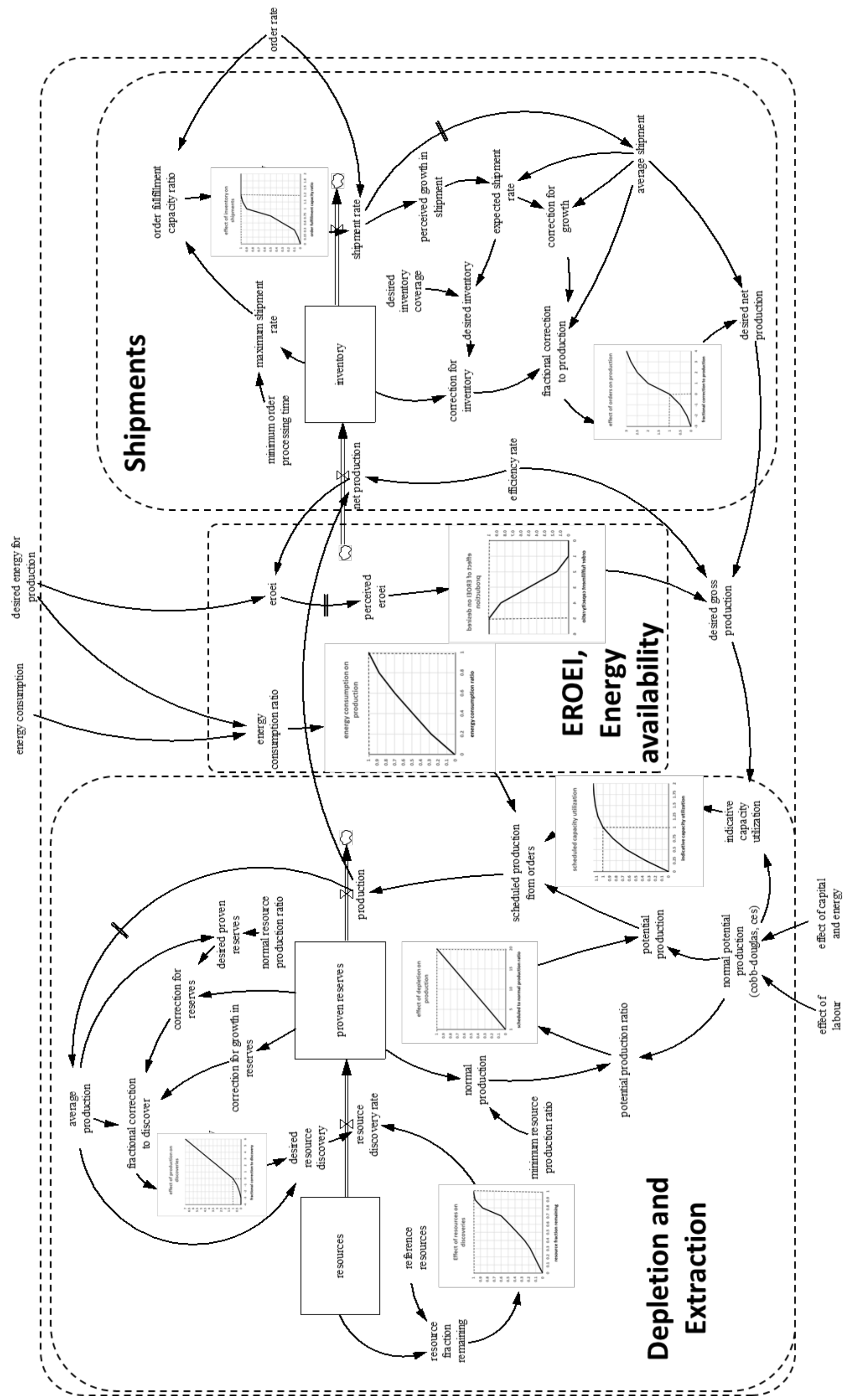
Eq. 1.107 – Fossil Fuel Net Production

$$nP_N = gP_N \times \boldsymbol{\eta_N}$$

Where gPP_N is the gross potential production, $gP^*{}_N$ the gross desired production, λ_{CU} the capacity utilization non-linear relationship, λ_E the non-linear effect of energy availability on production and $\frac{E_N}{E^*{}_N}$ the energy consumption ratio. It is worth noting that fossil fuels, similarly to households, biofuels and renewables, is a protected sector in terms of energy delivery, thus that the energy consumption ratio is normally equal to 1.

While production is extracted from the ground, proven reserves PR_N depletes, demanding geological experts to discover new resources. In the real world, the discovery process requires relatively lower capital allocation in contrast to the refineries used for extraction, and it has historically been driven by political reasons, often with unreliable and inconsistent published data.

Figure 1.23 – Production and Depletion of Fossil Fuels



In order to simplify the structure it is assumed that the discovery rate RDR_N is a process kept under the control of the fossil fuel industry in order to maintain the level of proven reserves PR_N near desired value $PR^*{}_N$ and anchored to the gross production gP_N. However, it is assumed that the total resources RS_N which exist is a in finite amount on this planet, and thus impacting the possible resource discovery rate RDR_N in a similar way to that modelled in the Limits to Growth.

Eq. 1.108 shows the desired correction to discoveries $\mathrm{J}corr_{RDR_N}$ given the information available on proven reserves PR_N, production $\overline{gP_N}$ and shipments $\Sigma_N(t)$, and Eq. 1.109 the actual resources discovery rate RDR_N given what desired $\overline{gP_N} \times \lambda_{RDR_N}\left(\frac{\mathrm{J}corr_{RDR_N}}{\overline{gP_N}}\right)$ and what actually possible given the geological constraints on depletion $\lambda_{LTG_N}\left(\frac{RS_N}{RS_{N_0}}\right)$.

Eq. 1.108 – Correction to discovery rate

$$\mathrm{J}corr_{RDR_N} = TRND\mathrm{J}\left(\Sigma_N(t)\right) \times AdjT_{\Sigma_N} \times \overline{gP_N} + \frac{PR^*{}_N - PR_N}{AdjT_{PR_N}}$$

Eq. 1.109 – Resources discovery rate

$$RDR_N = \overline{gP_N} \times \lambda_{RDR_N}\left(\frac{\mathrm{J}corr_{RDR_N}}{\overline{gP_N}}\right) \times \lambda_{LTG_N}\left(\frac{RS_N}{RS_{N_0}}\right)$$

The non-linear relationship λ_{LTG_N} apply the same concept previously seen in Limits to Growth (Meadows et al 2003). The more resources RS_N are depleted in comparison to the initial resource value $\left(\frac{RS_N}{RS_{N_0}}\right)$, the more discoveries RDR_N are reduced, reaching zero when no resources are left.

Under business as usual, shipments Σ_N would meet demand O_N (Eq. 1.101) reducing inventory Inv_N. The disequilibrium would require to generate gross production gP_N and net production nP_N thus balancing inventory (Eq. 1.106, Eq. 1.107), and cascading towards the unbalance of proven reserves PR_N, and generating the need for new discovery RDR_N to keep the system flowing and functioning. However, if RS_N depletes below a certain threshold, discoveries reduce following λ_{LTG_N} (Eq. 1.109) reducing proven reserves PR_N, which in turn, below a certain threshold defined with λ_{gPP_N}, would make potential production gPP_N less effective, thus implying additional investments in capital and energy to fulfil desired production

$gP^*{}_N$ (Eq. 1.105, Eq. 1.106), generating a self-reinforcing feedback loop that would reduce EROEI even further and exacerbate depletion. When the constrained production nP_N would not be sufficient to achieve desired inventory $Inv^*{}_N$, shipments Σ_N would reduce below what demand requires (Eq. 1.101).

The structure presented in this section describes the dynamic cascading impact of resources depletion on the economy, determined by a reinforcing feedback loop of investments increase when depletion rises increasing depletion even further, and a balancing effect emerging from the reduced supply and relative increase in prices, which determines lower demand over time, and potential rise in global inflation. In such a scenario, investing in alternative energy resources would be the only way to avoid the downturn of the entire economy thus favouring an energy transition. After a quick note on the renewable energy sector, all these dynamics are represented in the following section of this appendix, that is the Energy Market.

PRODUCTION OF RENEWABLES

The production of alternative energy follows the same rationale as in the Sterman (1981). In particular, the structure of production is equivalent to the one of a generic business sector (Figure 1.21), with the main differences that (i) no backlog of orders has been considered, (ii) an energy efficiency factor $\boldsymbol{\eta_N}$ has been added similarly to the treatment of the production of fossil fuels, (iii) energy consumption ratio is protected by energy policy in ERRE, and (iv) the capacity utilization non-linear relationship is assumed to be linear when desired production is below potential production. This latter factor is very important since no backlog and no inventory are present between production and shipments, and any additional energy production when not necessary would be lost.

No emissions or limits to resources are considered for the renewable energy sector in ERRE. It is noted that of course some emissions are generated in the production of renewable infrastructure and that some limits to resource availability, in particular rare earth metals, do indeed exist, however this is currently beyond the scope of ERRE as solutions and alternatives to these limits are myriad and complex. Hence, this sector can be considered as idealised case at present and is key for the sensitivity analysis to explore options for an energy transition away from fossil fuels.

The next section describes the energy market, inclusive of energy orders and revenue streams to all energy sectors.

ENERGY DEMAND AND ENERGY MARKET

In ERRE, energy is embedded in capital and a direct consequence of the use of that capital to generate production. The structure of energy requirements of capital at normal utilization $EReq_i$ is described in detail in the following sections. In this section, it is assumed that Energy Requirement at Normal Utilization is given, and used to determine the energy consumption and payments, as well as the distribution of energy demand across all energy suppliers.

In order to keep the model of each consumer sector consistent in terms of both physical and financial flows, the energy variables are broken down in different layers, each building up on the other to move from a normal energy demand $EReq_i$ to the final energy consumption E_i, and impacting the model at any level. These are:

- Energy requirements of Capital at Normal Utilization $EReq_i$
- Energy desired for production $EP^*{}_i$
- Desired energy (or energy demand) from each sector $E^*{}_i$
- Energy consumption from each sector E_i

Given that all suppliers respond together to demand, the energy demand from each sector is aggregated as total demand, and depending on their prices and performance orders are distributed among suppliers determining their orders for capacity and energy that cascades throughout the rest of the economy. In this section, the energy demand from each sector and the distribution of orders in the energy market are described, determining the orders for each energy supplier.

<u>TOTAL ENERGY DEMAND, TOTAL ENERGY SUPPLY AND EROEI</u>

The energy demand from each sector $E^*{}_i$ is dependent on the normal energy requirement based on known levels of productive assets, and is affected by the production λ_{CU} and financial $\mathfrak{f}_E$ decisions as described in the previous section. In particular, it is assumed that the variations in capacity utilization λ_{CU} on potential production gPP_i in comparison to desired production $gP^*{}_i$ (see Eq. 1.106) would require adjustment in the energy use. This defines desired energy for production $EP^*{}_i$ as follows:

Eq. 1.110 – Desired Energy for Production

$$EP^*{}_i = EReq_i \times \lambda_{CU}\left(\frac{gP^*{}_i}{gPP_i}\right)$$

Such information is shared with the financial dimension of the model, whom, depending on their liquidity adequacy $\frac{\Lambda_i}{\Lambda^*{}_i}$, and relative value for money generated by energy consumption $rVfMe_i$, can decide how much production they are willing to sacrifice in order to save money in energy payments.

In particular, the relative value for money for energy consumption $rVfMe_i$ is determined as:

Eq. 1.111 – Relative value for money of energy intensity

$$rVfMe_i = \frac{p_{\tau_i} \times \dfrac{\partial PP_i}{\partial E_i}}{p_e}$$

Where p_{τ_i} is the price of output of each sector, $\frac{\partial PP_i}{\partial E_i}$ is the marginal productivity of energy consumption, and p_e the price of energy. This allows to define $p_{\tau_i} \times \frac{\partial PP_i}{\partial E_i}$ as the value generated by every unit of energy consumption and p_e the cost of that same unit.

The actual demand for production $E^*{}_i$ is determined based on those information, and applying a non-linear control based on liquidity $\mathfrak{f}_E$ and marginal value $rVfMe_i$ such that:

Eq. 1.112 – Energy demand from each sector

$$E^*{}_i = EP^*{}_i \times \mathfrak{f}_E\left(\frac{\Lambda_i}{\Lambda^*{}_i}\right) \times \lambda_{rVfMe_i}\left(\overline{rVfMe_i}\right)$$

Where both effects $\mathfrak{f}_E$ and λ_{rVfMe_i} are assumed having no impact (multiplier at 1) if their argument is higher than 40%, and drastically reduce to zero when their argument approaches zero. It is worth noting that the effect of relative value is dependent on the perceived value calculated as exponential smooth on the relative value $rVfMe_i$.

Such demand is transferred by each customer to the energy suppliers. The total energy orders (demand) O_E is calculated as:

Eq. 1.113 – Total energy demand

$$O_E = \sum_{\Pi+H} E^*{}_i$$

On the other hand, the total net energy supply TNP_E is determined as follows:

Eq. 1.114 – Total Net Energy Supply

$$TNP_E = \Sigma_{F_e} + \Sigma_N + NP_{Ren}$$

Where Σ_{F_e} is the shipments of biofuels in energy units, Σ_N the shipments of fossil fuels in energy units, and NP_{Ren} the net production of renewables.

In turn it is possible to define the EROEI indicator in the ERRE model. For example, in the case of fossil fuels Eq. 1.107 (net production) and Eq. 1.110 (energy scheduled for desired production), EROEI is defines as:

Eq. 1.115 – EROEI

$$EROEI_N = \frac{nP_N}{EP^*{}_N}$$

EROEI can be considered the most important indicator for understanding the output capacity of each energy supplier in the model.

ENERGY PROTECTION AND ENERGY CONSUMPTION

As anticipated in the previous sections, an important aspect of ERRE is in the treatment of energy distribution by explicitly protecting all energy providers and the household sector in the economy to ensure its sustainability. In business as usual conditions, every time the energy supply TNP_E would not be able to match total energy demand O_E, it assumed that all these protected sectors would have priority on the others by having their total demand met. Whatever the energy supply left, that would be distributed among producers of commodities, food, services and capital.

This is achieved with the modelling of two energy availability indicators both for protected $\varkappa_{EP}$ and unprotected $\varkappa_{EnP}$ sectors as shown in Figure 1.24.

Thus the perceived fraction ϑ_{EP} of total supply TNP_E on the energy demand from protected sectors $\sum_{N+Ren+F_e+H} E^*{}_i$ is determined as follows:

Eq. 1.116 – Perceived fraction of total supply on protected energy demand

$$\vartheta_{EP} = smooth\left(\frac{TNP_E}{\sum_{N+Ren+F_e+H} E^*{}_i}, AdjT_{\vartheta_{EP}}\right)$$

In normal conditions ϑ_{EP} would be far greater than 1 thus having no impact whatsoever in constraining those sectors with any energy shortage. The non-linear effect $\lambda_{\varkappa_{EP}}$ assumes that the reduction in energy availability $\varkappa_{EP}$ below 1 would start when the ratio between energy supply and demand is 110%, and quickly decreasing till 0 when no energy supply is left.

Eq. 1.117 – Energy Availability fraction to protected sectors

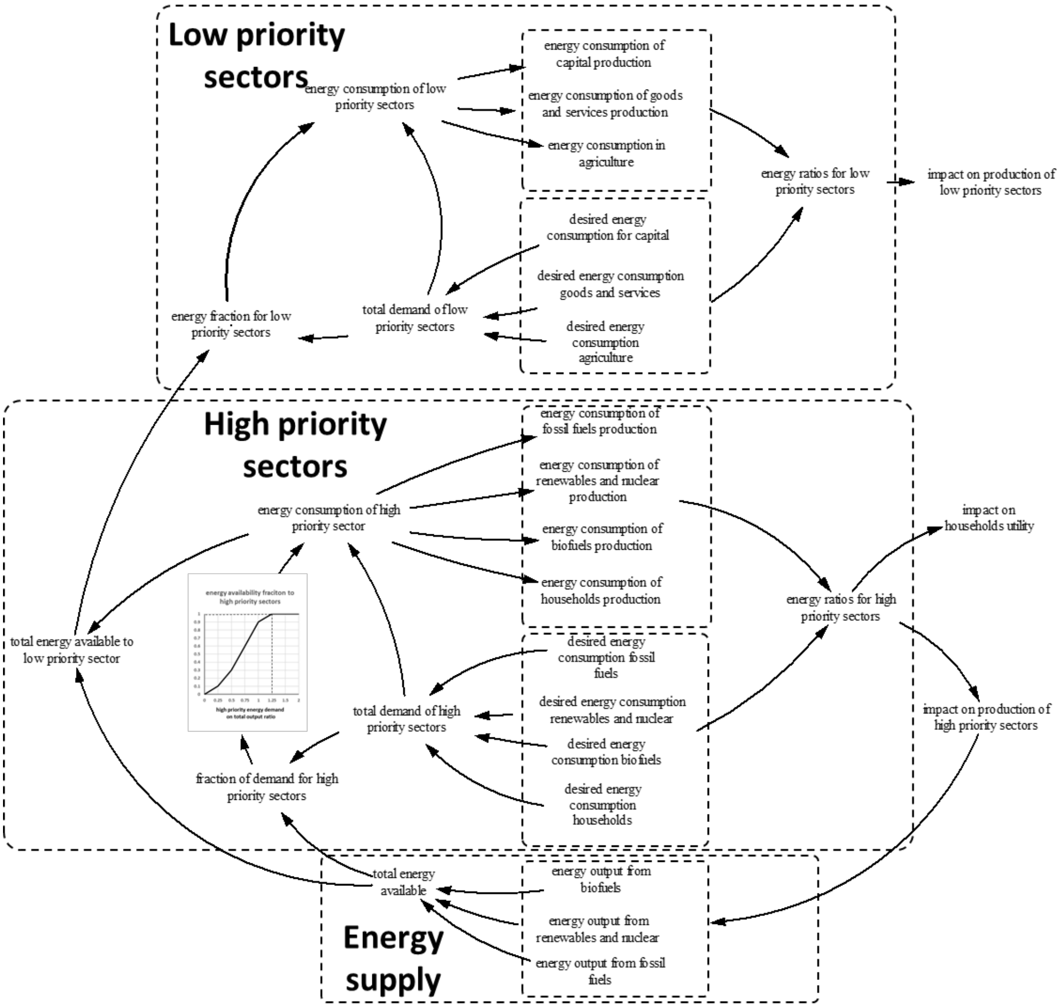
$$\varkappa_{EP} = \lambda_{\varkappa_{EP}}(\vartheta_{EP})$$

Such an effect would determine the final energy consumption of those protected sectors ${E_{EP}}_i$, applying their energy availability constraint $\varkappa_{EP}$ on their demand ${E_{EP}}^*{}_i$ as follows:

Eq. 1.118 – Energy consumption of Protected sectors

$${E_{EP}}_i = \varkappa_{EP} \times {E_{EP}}^*{}_i$$

Figure 1.24 - Energy availability and Energy protectionism



This allows the calculation of energy availability fraction of unprotected sector $\varkappa_{EnP}$ as the ratio between total energy supply left $TNP_E - \sum_{N+Ren+F_e+H} E_i$ and the energy demand of unprotected sectors $O_E - \sum_{N+Ren+F_e+H} E^*{}_i$:

Eq. 1.119 – Energy availability fraction of non-protected sectors

$$\varkappa_{EnP} = \frac{TNP_E - \sum_{N+Ren+F_e+H} E_i}{O_E - \sum_{N+Ren+F_e+H} E^*{}_i}$$

The energy consumption of each sector ${E_{EnP}}_i$ can finally be determined as the application of their available fraction $\varkappa_{EnP}$ on total demand of each sector ${E_{EnP}}^*_i$ as follows:

Eq. 1.120 – Energy consumption of unprotected sectors

$${E_{EnP}}_i = \varkappa_{EnP} \times {E_{EnP}}^*_i$$

In sum, based on the effect of energy availability on protected sectors $\lambda_{\varkappa_{EP}}$, it is assumed that all energy shortage in the model would always affect the firms till the total energy available would be 10% more of the energy required for producing energy and supporting households desires (Eq. 1.117). Below that level a bit of energy consumption of the protected sectors would be gradually reduced (Eq. 1.118) leaving more energy for firms consumption (Eq. 1.120). This would be the case till total energy production reaches zero, thus leaving the consumption of both categories at zero, and the economy left incapable of doing anything.

This closes the loop of the effect of energy availability to the production of each sector $\lambda_E\left(\frac{E_i}{EP^*_i}\right)$ as shown in Eq. 1.76 and Eq. 1.106. In the case of households Eq. 1.64 includes the denominator with normal energy requirement $EReq^*$ for both capital and goods and services.

<u>AVERAGE ENERGY PRICE, REVENUE DISTRIBUTION AND ENERGY TAX</u>

Given the structure presented above, it is possible to model the cash flows linked with energy payments and energy revenue in each sector.

The energy price used to charge every sector of the economy is the average among the three energy suppliers as follows:

Eq. 1.121 – Average Energy Price after Tax

$$p_e = \frac{\Sigma_{F_e} \times p_{\tau_{F_e}} + \Sigma_N \times p_{\tau_N} + NP_{Ren} \times p_{\tau_{Ren}}}{TNP_E}$$

Where p_{τ_i} is the price after tax per each sector, and TNP_E the total energy supply as the sum of the supply of the three producing sectors.

Thus every sector makes energy payments ϵ_i based on their consumption E_i as follows:

Eq. 1.122 – Energy payments

$$€_i = E_i \times p_e$$

The total gross revenue of the sector TR_E is given as the sum of payments $€_i$, which corresponds to the sum of gross revenue of each sector.

Eq. 1.123 – Total Gross revenue of the energy sector

$$TR_E = \sum_{\Pi+H} €_i = \Sigma_{F_e} \times p_{\tau_{F_e}} + \Sigma_N \times p_{\tau_N} + NP_{Ren} \times p_{\tau_{Ren}}$$

Finally the revenue from sales of each sector R_{E_i} is calculated as the difference between their gross revenue, and the amount due to the government given energy taxes. Eq. 1.124 shows the case of fossil fuel sector, where gross revenue is calculated as $\Sigma_N \times p_{\tau_N} - \Sigma_N$ whereas the sum devoted to government is $\Sigma_N \times p_m(t) \times \tau_{p_N}$ (see Eq. 1.61).

Eq. 1.124 – Energy Gross Revenue per each supplier

$$R_{E_N} = \Sigma_N \times p_{\tau_N} - \Sigma_N \times p_{m_N}(t) \times \tau_{p_N}$$

The total tax revenue for government T_E (see Eq. 1.3) is then calculated as follows:

Eq. 1.125 – Energy Tax Revenue for Government

$$T_E = \Sigma_N \times p_{m_N}(t) \times \tau_{p_N} + \Sigma_{F_e} \times p_{m_{F_e}}(t) \times \tau_{p_{F_e}} + NP_{Ren} \times p_{m_{Ren}}(t) \times \tau_{p_{Ren}}$$

Where p_{m_i} is the market price for each sector and τ_{p_i} is the fractional tax on price applied from the government.

<u>ENERGY ORDERS DISTRIBUTION AND MARKET LED ENERGY TRANSITION</u>

In ERRE, the energy transition is a market led phenomena that can be triggered by environmental limits. Figure 1.25 shows the structure determining the fractions of total energy orders O_E that are distributed to each energy supplier.

The fraction of orders FO_i represent the stock initialized at the desired year to the corresponding fraction based on real production data (EIA 2019). For example, in the case of fossil fuels, the fraction of orders $FO_N(0)$ would be:

$$FO_N(0) = \frac{\Sigma_{N0}}{TNP_{E0}}$$

Where $\frac{\Sigma_{N0}}{TNP_{E_0}}$ is the initial relative fossil fuel shipments relative to the total energy output.

The three sectors compete with each other for their share in the energy market, and it is therefore assumed that each sector would use current market performance and fraction of orders $FO_i(t)$ to calculate their attractiveness $Attr_{E_N}$ and determine the future fraction of orders $FO_i(t+1)$. The variation on attractiveness is based on two different market pressures:

1. Price - Given that the energy market is affected both by price and by type of technology, it is assumed that the initial energy price is anchored to the initial fraction of orders for each sector, but that every price reduction in comparison to the average price p_e would trigger and advantage to competitors.
2. Reliability – Every time a sector is not capable of fulfilling their orders, it is assumed that the market would find it less attractive, and move their orders to the competitors.

Thus a relative price $relp_{e_i}$ of each sector price p_{e_i} in comparison to average price p_e is calculated:

Eq. 1.126 – Relative energy price

$$relp_{e_i} = \frac{p_{e_i}}{p_e}$$

And the attractiveness of each sector $Attr_{E_i}$ is determined as follows:

Eq. 1.127 – Attractiveness of energy sector from price

$$Attr_{E_i} = \left(\frac{relp_{e_i}}{relp_{e_{i_0}}}\right)^{\varpi_i} \times \frac{\Sigma_i}{O_i} \times FO_i(t)$$

Where $\frac{relp_{Ni}}{relp_{Ni_0}}$ represents the relative price variations in comparison to initial conditions, ϖ_i a weighted average factor negative in value, $\frac{\Sigma_i}{O_i}$ the current performance of each sector to fulfil their orders, given Σ_i is their shipments (or net production NP_{Ren} in case of renewables) and O_i their actual orders, and $FO_i(t)$ their fraction of orders at present time.

The desired fraction of orders $FO^*{}_N$ is calculated as relative attractiveness $Attr_{E_N}$ to the sum of attractiveness of all sectors $\sum_E Attr_{E_i}$ as follows:

Eq. 1.128 – Desired Fraction of orders to each sector

$$FO^*{}_N(t) = \frac{Attr_{E_N}}{\sum_E Attr_{E_i}}$$

And the fraction of orders at the following time step as a smooth on desired fraction of orders as follows:

Eq. 1.129 – Fraction of orders

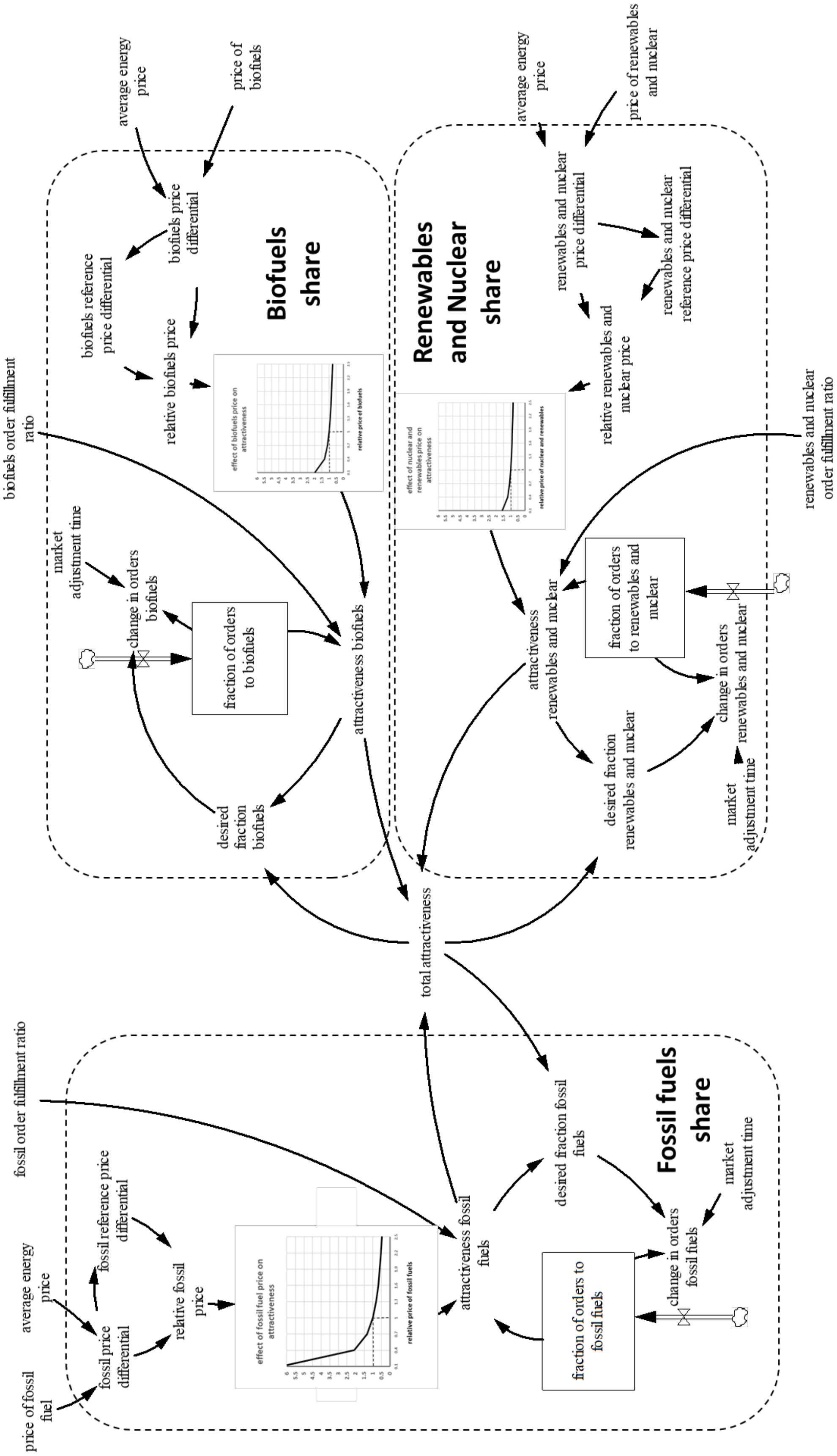
$$FO_N(t + dt) = smooth(FO^*{}_N(t), AdjT_{FO})$$

Where $AdjT_{FO}$ represents the time necessary for the market to adapt in terms of energy provider. Thus the normal orders $nO_i(t + dt)$ at the next time step can be calculated as the fraction of total orders $O_E(t + dt)$ as follows:

Eq. 1.130 – Normal orders to each sector

$$nO_i(t + dt) = FO_i(t + dt) \times O_E(t + dt)$$

Figure 1.25 – Energy orders attractiveness



ENERGY SHORTAGE, UNFILLED ORDERS DISTRIBUTION, AND ACTUAL ORDERS DEMAND PER EACH SECTOR

Finally, in the ERRE model it is assumed that those energy customers who were affected by energy shortages would find it natural to shift towards another energy supplier. For simplicity, it is assumed that there exists a certain time to switch supplier, and that their choice would be distributed depending on the competitiveness of each supplier based on their current market share $FO_i(t)$.

In particular a total energy shortfall $Oshort(t)$ is calculated as the difference between total orders $O_E(t)$ and total production $TNP_E(t)$ when production is insufficient:

Eq. 1.131 – Energy shortfall

$$Oshort(t) = max(0, O_E(t) - TNP_E(t))$$

All those customers who were affected by such a shortage would reallocate their orders $Oalloc_i(t + dt)$ towards another supplier based on current market share $FO_i(t)$, within the adjustment time $AdjT_{Ealloc}$ as follows:

Eq. 1.132 – Shortfall allocation

$$Oalloc_i(t + dt) = \ smooth(Oshort(t) \times FO_i(t), AdjT_{Ealloc})$$

Thus the energy demand for each sector at the next time step $O_i(t + dt)$ is determined as the sum of order allocations due to shortage $Oalloc_i(t + dt)$ and the normal orders $nO_i(t + dt)$ as follows:

Eq. 1.133 – Orders allocation to each sector

$$O_i(t + dt) = nO_i(t + dt) + Oalloc_i(t + dt)$$

REAL ASSETS FOR PRODUCTION AND UTILITY

<u>*FIRMS*</u>

Capital

Figure 1.26 shows the real capital sub-dimension for each producer composed of (i) a third order capital vintage structure, (ii) the representation of capital in defaults kept in idle before being reintroduced to the market, (iii) a capital under construction, and (iv) a backlog of unfilled orders for capital, representing a fraction of the total backlog of the capital producing sector.

The function of capital is to support production. The total stock of capital $K_i(t)$ is the sum of the capital vintage structure such as:

Eq. 1.134 – Capital

$$K_i(t) = Kv1_i(t) + Kv2_i(t) + Kv3_i(t)$$

Where $KvN_i(t)$ is the Nth capital vintage cohort. Given the third order structure of capital with average life of capital ALK_i, each stock is decreased as a first order delay with average cohort life $\frac{ALK_i}{3}$, such that the discarded capital vintage of the generic Nth cohort $\delta K_i N(t)$ is calculated as:

Eq. 1.135 – Capital discard rate of Nth vintage cohort

$$\delta KvN_i(t) = \frac{KvN_i(t)}{\frac{ALK_i}{3}}$$

It is worth noting that the capital discard on third position corresponds to the capital discard of the entire structure, so that it is possible to define:

Eq. 1.136 – Capital discard rate

$$\delta K_i(t) = \delta Kv3_i(t)$$

While the relationships between backlog of orders and capital construction initiation rate have been described in the relative section of this appendix (Eq. 1.82), the capital addition rate

KAR_i is modelled as a third order delay on capital initiation $CIRK_i$ over the capital construction time KCT_i, to improve realism of the construction chain, so that:

Eq. 1.137 – Capital addition rate

$$KAR_i = DELAY3(CIRK_i, KCT_i)$$

Figure 1.27 shows the capital default structure for a generic capital vintage cohort, and should be seen in conjunction with the financial defaults structures presented in Eq. 1.32. Thus the real capital defaults of an Nth cohort $KvN_i(t)$ has been calculated as:

Eq. 1.138 – Real assets default in Nth capital vintage position

$$\Omega vN_i = KvN_i(t) \times \xi_i \times \mathfrak{f}_{\nabla}\left(\frac{\Lambda}{\Lambda^*}\right)$$

Where ξ_i is the normal default on capital, and $\mathfrak{f}_{\nabla}\left(\frac{\Lambda}{\Lambda^*}\right)$ the effect of liquidity constraints on the change in defaults.

Each capital in default cohort $K\Omega vN_i$ is assumed to be productive capital kept in idle by the bank, which waits to sell those assets back to market. In order to simplify the structure, the bank, not firms, are able to reintroduce those assets to the market at market price with a certain time delay $\Omega AcqT_i$. However, the ability of the firms to pay $\mathfrak{f}_{\mathrm{K}}\left(\frac{\Lambda}{\Lambda^*}\right)$ is assumed to be constrained by the acquisitions from capital in default of each cohort $\Omega AcqvN_i$ such that:

Eq. 1.139 – Defaulted capital acquisitions

$$\Omega AcqvN_i = \frac{K\Omega vN_i(t)}{\Omega AcqT_i} \times \mathfrak{f}_{\mathrm{K}}\left(\frac{\Lambda}{\Lambda^*}\right)$$

It is worth noting that defaulted capital $K\Omega vN_i(t)$ is assumed to be not discarding till kept in idle. The total capital in default is calculated as:

Eq. 1.140 – Real Capital in Default

$$K\Omega_i(t) = K\Omega v1_i(t) + Kv\Omega 2_i(t) + K\Omega v3_i(t)$$

Figure 1.26 – Real capital

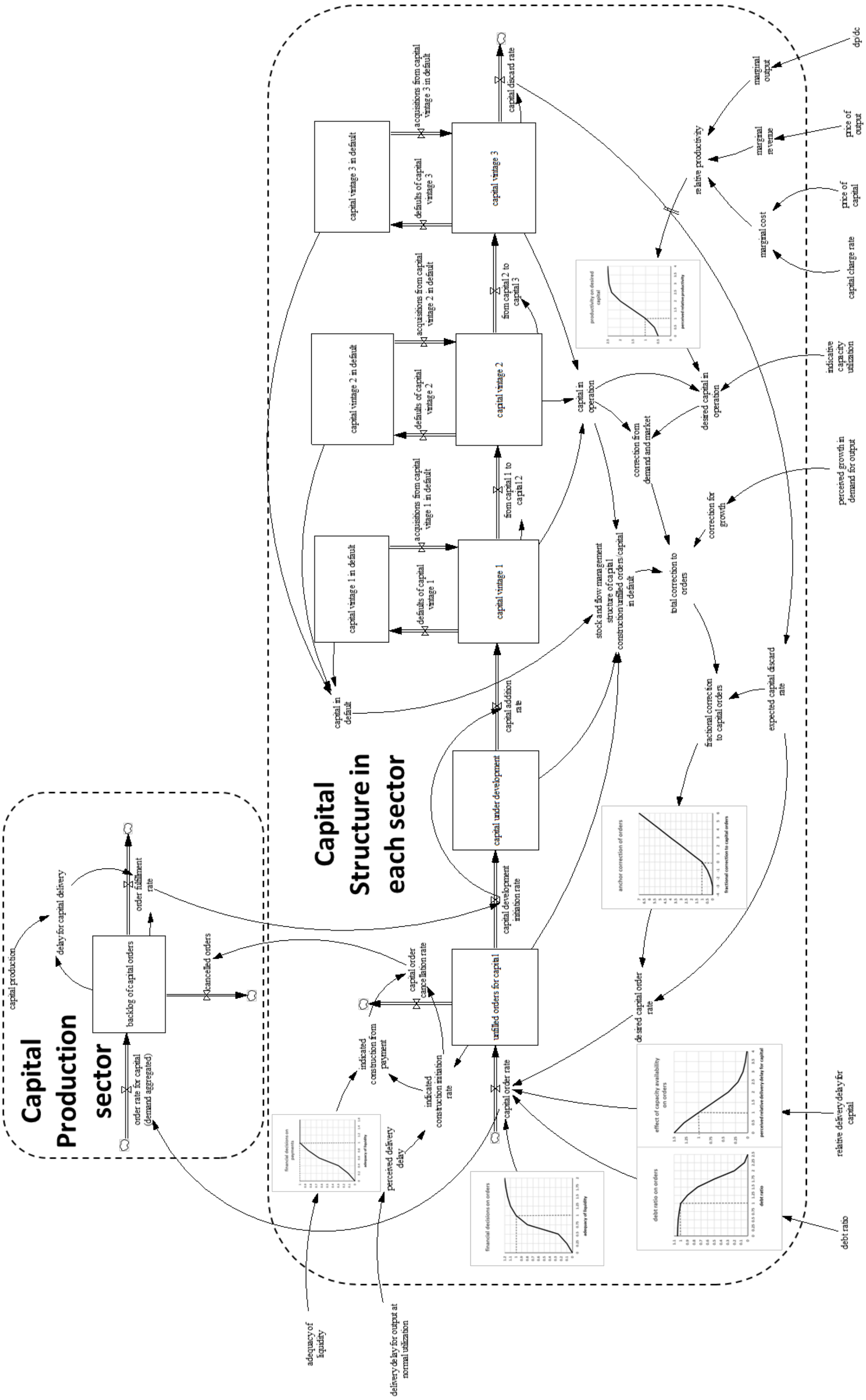
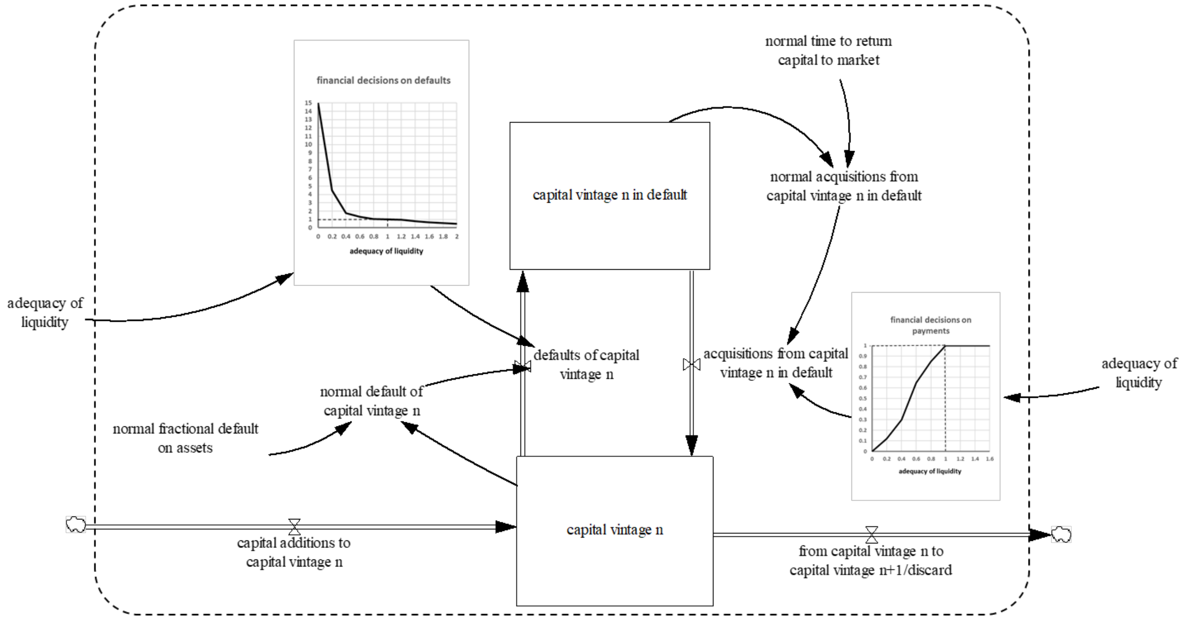


Figure 1.27 – Real Capital Default



In the entire capital structure, the capital discard $\delta K_i(t)$ represents its inherent disequilibrium, and must be taken into account with care by decision makers while performing investment decisions. As a principle, the capital structure would be stable if and only if capital orders o_{K_i} are capable of balancing the discard rate $\delta K_i(t)$, the change in demand, and the various oscillations emerging in the capital vintage structure. Thus, in the ERRE model, firms are assumed to aim for stability, while being perturbed by market forces, the perception of suppliers reliability, and financial performance. In so doing, the capital order structure is similar to the one of shipments and inventory, where capital discard $\delta K_i(t)$ is the variable to which both orders and capital structure anchors and adjusts to in order to meet desired demand and market stability.

In order to address the uncertainty dependent on markets, a firms is assumed to determine a perceived relative value for money spent for every additional unit of capital $rVfMk_i$, and uses that as a propensity to increase or decrease the amount of desired capital. The $rVfMk_i$ can be defined as the ratio between the marginal income generated from every additional capital unit added to production, and can be calculated as:

Eq. 1.141 – Relative Value for Money of Capital

$$rVfMk_i = \frac{p_{\tau_i} \times \frac{\partial PP_i}{\partial K_i}}{p_{k_i} \times kcr_i}$$

Where p_{τ_i} represents the price of output after tax, $\frac{\partial PP_i}{\partial K_i}$ the real marginal output of every unit of capital, and $p_{k_i} \times kcr_i$ the unitarian cost of any additional unit of capital, calculated as the capital charge rate kcr_i at current capital price p_k. It is worth noting that the relative value for money indicator is assumed to be among the central nodes for the initialization of the model to assure that costs and price of output do match, resulting in balancing around the value 1 and initializing the model towards equilibrium.

Thus, the desired capital $K^*{}_i(t)$ is determined as:

Eq. 1.142 – Desired capital

$$K^*{}_i(t) = \frac{P^*{}_i}{PP_i} \times K_i(t) \times \lambda_{rVfMk}\left(\overline{rVfMk}\right)$$

Where $\frac{P^*{}_i}{PP_i}$ is the desired production relative to current potential production, $K_i(t)$ the current level of capital, and $\lambda_{rVfMk}\left(\overline{rVfMk}\right)$ the non-linear effect of perceived relative value for money on capital investment. The shape of the non-linearity λ_{rVfMk} represents a detachment from the rational model of growth. In fact, when profitability is greater than cost, the effect would imply proportional increases in desires for new capital till a doubling of desired capital in which case the perceived value of money would reach 2. However, every increase beyond that would have a decreasing marginal increase in desired capital, thus addressing the aversion of investors in committing capital during bubbles. In addition, when value for money is perceived to decrease below unity, it is assumed that the effect on orders would be limited till reaching 90% of optimal capital, given the behaviour of firms would tend to outsource capital orders to cheaper economies, thus maintaining orders to high levels.

The rest of the structure leading towards desired capital orders $o_k{}^*{}_i$ is a stock management structure anchored to capital discard $\delta K_i(t)$, where the growth trend in orders is carefully considered in determining desires for new capital.

Thus, desired capital under construction $KuC^*{}_i$ is defined as:

Eq. 1.143 – Desired capital under construction

$$KuC^*{}_i = \left[\delta K_i(t) + TRNDJ\left(O_i(t)\right) \times K_i(t)\right] \times KCT_i$$

Where $\delta K_i(t)$ is the capital discard rate, $TRNDJ(O_i(t)) \times K_i(t)$ the impact of growth of capital (downstream stock for capital under construction) based on orders, and KCT_i the amount of time necessary to construct new capital.

The desired backlog of unfilled orders $bluo^*{}_i$ is:

Eq. 1.144 – Desired backlog of unfilled orders

$$bluo^*{}_i = \left[\delta K_i(t) + TRNDJ(O_i(t)) \times \left(K_i(t) + KuC_i(t)\right)\right] \times \overline{DDelNU_i}$$

Where $TRNDJ(O_i(t)) \times \left(K_i(t) + KuC_i(t)\right)$ is the impact of perceived growth on the pressures on additional orders, and $\overline{DDelNU_i}$ the perceived delivery delay at normal utilization (see Eq. 1.78).

Firms are assumed to have expectations on the investments that might default $K\Omega^*{}_i$, based on the normal default rate ξ_i and accounting for the time necessary for banks to reintroduce assets into market $\Omega AcqT_i$:

Eq. 1.145 – Expected default rate

$$K\Omega^*{}_i = K^*{}_i(t) \times \xi_i \times \Omega AcqT_i$$

And determining the overarching growth rate in capital $g_{k_i}(t)$ as:

Eq. 1.146 – Perceived Growth rate in capital

$$g_{k_i}(t) = TRNDJ(O_i(t)) \times (K_i(t) + KuC_i(t) + bluo_i(t) + K\Omega_i(t))$$

Thus the desired anchor capital correction $Jcorr_{K_i}$ can be calculated as the sum of adjustment of all stocks involved as:

Eq. 1.147 – Capital correction

$$Jcorr_{K_i} = g_{k_i}(t) + \frac{K^*{}_i - K(t)_i}{AdjT_K} + \frac{K\Omega^*{}_i - K\Omega(t)_i}{AdjT_K} + \frac{KuC^*{}_i - KuC(t)_i}{AdjT_K} + \frac{bluo^*{}_i - bluo(t)_i}{AdjT_K}$$

And the desired order rate ${o_k}^*_i$ as adjustment on discard $\delta K_i(t)$ where λ_o is assumed to be linear and proportional for any positive correction in capital, and decreasing non-linearly but never becoming negative during market recession such that:

Eq. 1.148 – Desired capital orders

$$ {o_k}^*_i = \delta K_i(t) \times \lambda_o \left(\frac{\mathrm{J}\,corr_{K_i}}{\delta K_i(t)} \right) $$

The shape of λ_o implies an important condition in ERRE both because investments can never go negative in the real world, and secondly addressing the aggregation level of the model. In particular, during market recession some companies will still make investments, leaving less competitive firms out of market. In a more disaggregated model, such non-linear effects would need to be reviewed to represent micro-level dynamics.

Finally the order rates for capital ${o_k}_i$ (and relative demand to the capital sector), are the desired orders ${o_k}^*_i$ corrected via financial decisions $\mathfrak{f}_{oK}\left(\frac{\Lambda_i}{\Lambda^*_i}\right)$, debt discrepancies on permissible levels $\lambda_{DoK}\left(\frac{D_i}{PD_i}\right)$, and supply constraints based on suppliers ability to maintain their past performance $\lambda_{DDelNUoK}\left(\frac{DDelNU}{\overline{DDelNU_i}}\right)$:

Eq. 1.149 – Capital orders

$$ {o_k}_i = {o_k}^*_i \times \lambda_{DDelNUoK}\left(\frac{DDelNU}{\overline{DDelNU_i}}\right) \times \lambda_{DoK}\left(\frac{D_i}{PD_i}\right) \times \mathfrak{f}_{oK}\left(\frac{\Lambda_i}{\Lambda^*_i}\right) $$

In particular, the effect of financial decisions $\mathfrak{f}_{oK}$ is assumed to be a positive correction for every liquidity surplus, and a negative correction in the opposite case. The debt ratio $\frac{D_i}{PD_i}$ is assumed to have little, but positive, impact when debt is low, but a drastic decrease till no order can be made in the case when debt is double the permissible level. Finally, firms are assumed to communicate their delivery delay when performing at normal utilization $DDelNU$, and based on their perception of such a delay $\overline{DDelNU_i}$ they make adjustments. In particular, if delivery delay ratio $\frac{DDelNU}{\overline{DDelNU_i}}$ is below 1, they are meant to increase pressure on supply, whereas when capacity is constrained to meet demand in comparison to past performance, customers relieve pressure decreasing their new orders.

Agricultural land

Figure 1.28 shows the stock and flow structure of the agricultural land. This sub-dimension represent one of the points in which the structures of capital ordering as proposed in Sterman (1981) and the structure of land limits as proposed in Meadows et al. (2003) merge in ERRE.

The main structural difference between the structures of agricultural land and the capital order sector seen above can be summarized as follows:

1. A backlog of unfilled orders for agricultural land is placed between agricultural land development and forest land, thus when orders for land are cancelled, they are redirected back to forest land.
2. The erosion rate of agricultural land is assumed to increase depending on the exploitation of land for production
3. Differently from Meadows et al (2003), an intermediate stock of fallow land is assumed to accumulate the eroded land, and, with a certain delay, can be reconverted to forest land.
4. Given that the capital sector is responsible for the development of agricultural land, a conversion factor between capital units and hectares is introduced to determine the flows of capital and money between the two.
5. Agricultural land is affected by depletion and increase cost for development of land in line with Meadows et al (2003).
6. The agricultural land is assumed to be exogenously affected by an increase in food productivity per hectare in line with the Solow growth model for technology change.

Given that the management of backlog, agricultural land under development and agricultural land in default, are equivalent to the analogous structure in the capital sector they will not be repeated in here.

Figure 1.29 shows all the flows involving the stock of agricultural land AL_F, and agricultural land in default $AL\Omega_i(t)$. As in the case of capital ordering, the agricultural land discard rate $\delta AL_F(t)$ represents the key variable to anchor to in order to keep productive land at desired levels, and it is calculated as:

Eq. 1.150 – Agricultural Land Discard Rate

$$\delta AL_F(t) = UILdr_F + ALer_F$$

Where $UILdr_F$ is the urban and industrial land development, and $ALer_F$ the land erosion rate. The structure of urban and industrial land development $UILdr_F$ has been slightly modified from Limits To Growth in order to assure the stock and flow consistency of the agricultural land stock, and is calculated as follows:

Eq. 1.151 – Urban and Industrial Land Development

$$UILdr_F = Max\left(0, \frac{Pop \times UILpc - UIL_F(t)}{UILDT_F}\right) \times \lambda_{UILdr}\left(\frac{AL_F(t)}{UIL_F(t)}\right)$$

Where $UILpc$ is the urban and industrial land indicated per person, Pop is the population in ERRE, $Pop \times UILpc$ the desired urban land, and $\frac{Pop \times UILpc - UIL_F(t)}{UILDT_F}$ the desired development rate given the time to develop new infrastructures $UILDT_F$. Such development is assumed to beg only positive (Max(0)), simply because it is assumed that even in the eventuality in which the population would decrease, the size of cities, for example, would not decrease, or more explicitly not return to land used for food production. Finally, the non-linear relationship λ_{UILdr} indicates that in the eventuality in which urban and industrial land would equal each other, the less agricultural land $AL_F(t)$ is available for conversion to urban and industrial area $UIL_F(t)$ the stronger is the constraint to develop further urban areas, till reaching zero when no agricultural land is left.

The agricultural land erosion rate is calculated as in the World3-03-Edited model (Pasqualino et al. 2015), as the ratio between agricultural land $AL(t)$ and the average life of land. In particular, it is assumed that the increased exploitation of land measured as the ratio between initial and current land yield LY would affect non-linearly the normal average life of agricultural land $NALAL_F$, following the non-linear relationship $\lambda_{W3-Edited}$ as a weighted calibrated average between the two extremes of the non-linearities of the World3-03 (Pasqualino et al. 2015).

Eq. 1.152 – Agricultural Land Erosion Rate

$$ALer_F = \frac{AL(t)}{NALAL_F \times \lambda_{W3-Edited}\left(\frac{LY}{LY_0}\right)}$$

As a difference to Limits to Growth, the stock of fallow land is introduced and the forest land regeneration rate $ForLrr_F$ calculated as:

Eq. 1.153 – Forest Land Regeneration Rate

$$ForLrr_F = DELAY3(ALer_F, TRFL_F)$$

Where $ALer_F$ is the agricultural land erosion rate and $TRFL_F$ the time to regenerate forest land.

The feedback loop from forest depletion to increased cost of agricultural land has been taken from the World3-03-Edited, but normalized to account for the explicit treatment of money.

Similarly to the capital orders, the feedback from cost is determined via the value for money of additional agricultural land $VfMAL_F$ as follows:

Eq. 1.154 – Value for Money of additional Agricultural Land

$$VfMAL_F = \frac{p_{\tau F} \times \frac{\partial PP}{\partial AL}}{ALcr_F \times p_{AL F}}$$

Where $p_{\tau F}$ is the food price of output after tax, $\frac{\partial PP}{\partial AL}$ is the real marginal output of agricultural land, $ALcr_F$ is the agricultural land charge rate and $p_{AL F}$ is the nominal price of agricultural land. This latter has been normalized from the World3-03 as follows:

Eq. 1.155 – Price of Agricultural Land

$$p_{AL F} = p_k \times \lambda_{LTG}\left(\frac{ForL}{FL_0}\right)$$

Where p_k is the price of capital and $\frac{FL}{FL_0}$ the fraction of forest remaining in comparison to the beginning of the simulation, and λ_{LTG} a cost curve acting as a conversion factor from hectare to capital units.

Therefore, the relative value for money $rVfMAL_F$ is the result of the normalization to initial time such that:

Eq. 1.156 – Relative Value for money for additional agricultural land

$$rVfMAL_F = \frac{rVfMAL}{rVfMAL_0}$$

It is used for the calculation of desired agricultural land $AL^*{}_F(t)$ as follows:

Eq. 1.157 – Desired Agricultural Land

$$AL^*{}_F(t) = \frac{P^*{}_F}{PP_F} \times AL_F(t) \times \lambda_{rVfMAL}\left(\overline{rVfMAL}\right)$$

Where $\overline{rVfMAL}$is the perceived relative marginal value of agricultural land, λ_{rVfMAL} the non-linear effect of perceived added value on desired land, $\frac{P^*{}_F}{PP_F}$ is the ratio between desired and potential production, and $AL_F(t)$ the current agricultural land. It is worth noting that λ_{rVfMAL} differs from the relative counterpart in the capital ordering sector such that when the relative value for money decreases below unity, its effect is to linearly decrease desired land till reaching zero, and assuming the substitutability of land for capital for increased production.

Figure 1.28 – Agricultural land structure

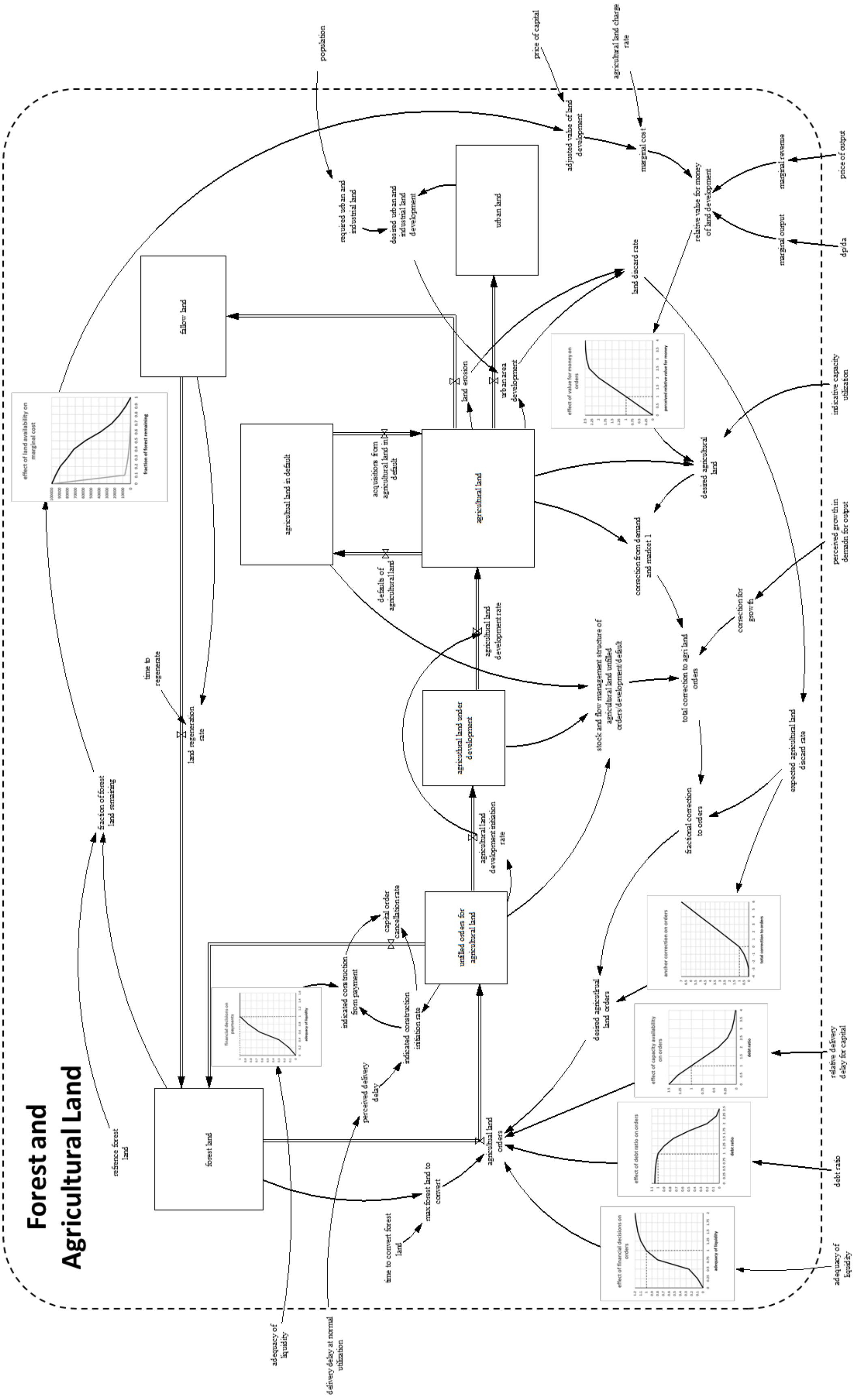
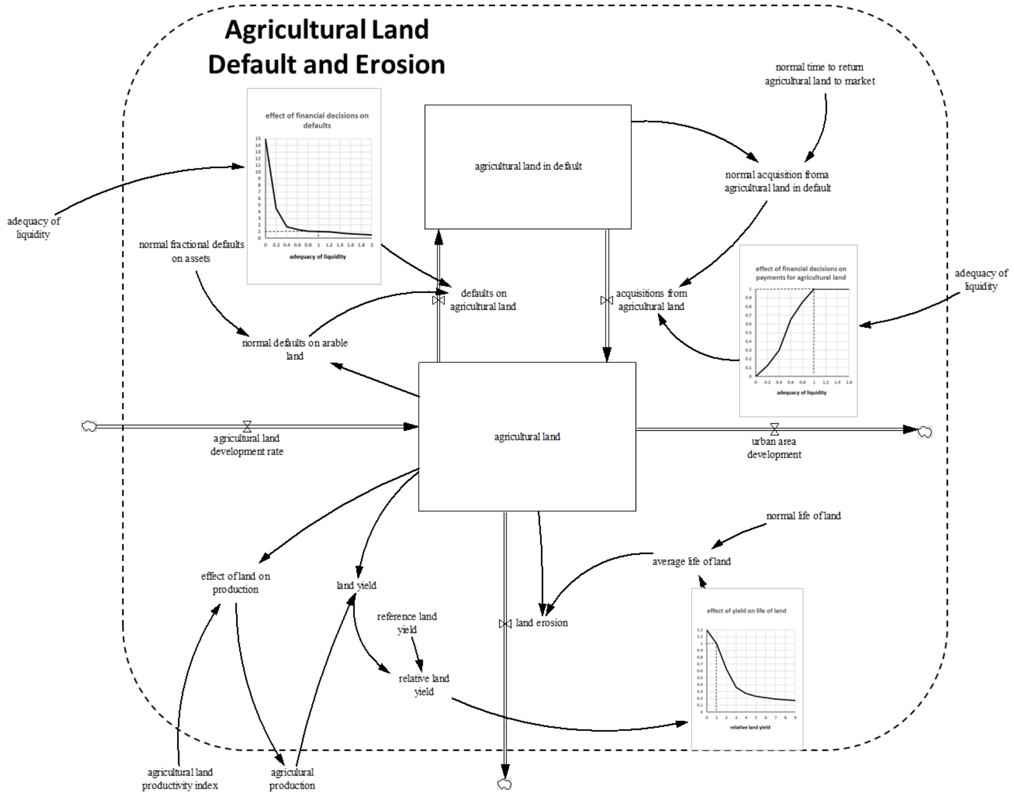


Figure 1.29 – Land erosion, Urban and Industrial Land development, and Land Defaults



Both effects of land erosion (Eq. 1.152) and land substitution (Eq. 1.157) are strictly linked to the definition of land productivity as explained below. Considering technology change is assumed to increase the amount of output possible with the same level of agricultural land, both erosion would increase, and the value for money for land would be perceived as a determinant to request less agricultural land. In addition, agricultural firms will need to make adjustments on their agricultural land stock management structure not only based on the trend in food orders but also on the trend in productive capacity over time. While the first factor would have a positive effect on the ordering decision, the second would require lowering it due to increased net capacity. In comparison to the capital ordering structure, the agricultural land orders would require the following substitution in the corresponding equations:

Eq. 1.158 – Considered correction trend with technology

$$\left[TRNDJ\left(O_F(t)\right)\right]_{Capital\ orders} \rightarrow \left[TRNDJ\left(O_F(t)\right) - TRNDJ\left(\pi_{AL}(t)\right)\right]_{Agricultural\ land\ Orders}$$

Where $TRNDJ\big(O_F(t)\big)$ is the trend, with anchor, for food orders, and $TRNDJ\big(\pi_{AL}(t)\big)$ is the trend, with anchor, for land productivity.

Finally, the payment for agricultural land development $CFAL_F$, as registered to the balance sheet of the agricultural sector and as revenue to the capital sector, takes the following form:

Eq. 1.159 – Payments for agricultural land development

$$CFAL_F = ALDIR_F \times p_{AL_F}$$

Where $ALDIR_F$ is the agricultural land initiation rate and p_{AL_F} is the price of the next hectare of agricultural land development.

HOUSEHOLDS

Value for Money of Utility factors

The most important difference between households and firms is that households do not produce and sell anything, which has profound implications on the modelling of their consumption preferences.

In particular, what households consider being 'of Value' to them is expressed by their potential utility PU_H, and the relative value for money for every utility factor $rVfM_{H_{UFj}}$, is expressed in comparison to the average perceived value for money across all utility factors $AVfM_H$.Therefore, they are assumed to be able to choose among the utility factors UF_j that generate more relative value in comparison to average.

As a result the average value for money $AVfM_H$ can be defined as:

Eq. 1.160 – Average value for Money of Utility

$$AVfM_H = \frac{PU_H}{\sum_{UF} AnExp_j}$$

Where PU_H is the potential utility, and $\sum_{UF} AnExp_j$ the total annual expenditure of the household sector for all their utility factors UF.

The value of money for every utility factor $VfM_{H_{UFj}}$ is determined as:

Eq. 1.161 – Value for Money for Each utility factor

$$VfM_{H_{UFj}} = \frac{\frac{\partial PU}{\partial UF_j}}{mExp_{UFj}}$$

Where $\frac{\partial PU}{\partial UF_j}$ is the marginal utility of each specific utility factor, and $mExp_j$ the marginal expenditure for that factor.

Thus, the relative value of each utility factor $rVfM_{H_{UFj}}$ is determined as follows:

Eq. 1.162 – Relative value of Utility for each utility factor

$$rVfM_{H_{UFj}} = \frac{VfM_{H_{UFj}}}{AVfM_H}$$

It is worth noting that the utility function is generated following the neo-classical theory and applying a nested Cobb-Douglas - CES production function, requiring that the sum of exponents of each utility factor equals 1. For simplicity, in ERRE it is assumed that households aim at maximising their utility based on utility factors that are perceived just for their monetary value, allowing all exponents $\varepsilon_{H_{UFj}}$ to be initialized as:

Eq. 1.163 – Exponents of the utility function

$$\varepsilon_{H_{UFj}} = \frac{AnExp_{UFj}}{\sum_{UF} AnExp_{UFj}}$$

Where $AnExp_{UFj}$ is the annual expenditure for each utility factor UF_j and $\sum_{UF} AnExp_{UFj}$ the total annual expenditure.

Such an assumption, together with the definition of the value share of energy in capital, assures that the relative value for money of each utility factor $rVfM_{H_j}$ is initialized in proximity of 1 (balanced model) at the beginning of the simulation. Each utility factor and their costs are described in the relative section of this appendix.

Capital

The capital ordering structure for households follows the same principles as every other firm with the following three differences:

1. They have a simplified accounting structure for the marginal cost of capital
2. The effect of value for money on desired capital is calculated in comparison to the average value among all utility factors
3. A consumption bias is used in substitution for the ratio between desired and potential production to determine desired capital
4. A trend in desired utility is used in substitution for the trend in orders to address the capital stock management structure.

Thus the marginal capital expenditure $mExp_K$ is determined as follows:

Eq. 1.164 – Marginal cost of capital for households

$$mExp_K = p_k \times \left(i_{rp_H} - \gamma + \frac{1}{ALK_H} \right)$$

Where p_k is the current market price of capital, i_{rp_H} the interest rate to service loans, γ the average inflation rate, and $\frac{1}{ALK_H}$ the capital discard rate is the inverse of the average life of capital.

This allows the calculation of the relative value for money of capital $rVfMK_H$ as:

Eq. 1.165 – Relative value for money of capital in Households

$$rVfMK_H = \frac{\frac{\frac{\partial PU}{\partial K_j}}{mExp_K}}{AVfM_H}$$

Where the marginal value of capital is the ratio between the marginal utility of capital $\frac{\partial PU}{\partial K_j}$, and its marginal expenditure $mExp_K$, and $AVfM_H$ the average value across all utility factors. The desired value of capital $K^*{}_H$ in households is calculated as:

Eq. 1.166 – Desired Capital in Households

$$K^*{}_H = K_H \times \frac{U^*{}_H}{PU_H} \times \lambda_{rVfMK}\left(\overline{rVfMK_H}\right)$$

Where K_H is the current level of capital, $\frac{U^*{}_H}{PU_H}$ the consumption bias, and λ_{rVfMK} represents the non-linear behaviour on capital desired based on its actual perceived monetary value.

As in the firm sector, the desired capital is used as a correction factor to the orders of new capital. However, the stock management structure accounts for the trend, with anchor, in desired utility $TRNDJ\left(U^*{}_H(t)\right)$, instead of the trend, with anchor, of customer orders $TRNDJ\left(O_i(t)\right)$ as follows:

Eq. 1.167 – Correction to trend in Capital for Households

$$\left[TRNDJ\left(O_i(t)\right)\right]_{Producers} \rightarrow \left[TRNDJ\left(U^*{}_H(t)\right)\right]_{Households}$$

All other effects and non-linear relationships are assumed to be the same as in a generic producer sector, so they are not repeated here.

For the purpose of initialization, the exponent of capital on utility ε_{H_K} is calculated as follows:

Eq. 1.168 – Capital Exponent in Households

$$\varepsilon_{H_K} = \frac{K_H(t) \times mExp_K + EReqK_H(t) \times p_e}{\sum_{UF} AnExp_{UFj}}$$

Where $K_H(t)$ is the total real capital, $mExp_K$ is the marginal expenditure of every unit of capital, $EReqK_H(t)$ is the energy necessary to operate capital, p_e is the energy price, and $\sum_{UF} AnExp_{UFj}$ the total annual expenditure of utility. Therefore $K_H(t) \times mExp_K$ represents the annual expenditure of capital, and $EReqK_H(t) \times p_e$ the annual expenditure of energy use due to capital utilization.

Goods and Services

Figure 1.30 shows the structure of goods and services in the household sector. Their aggregation as a single producing entity was necessary due to the limited scope of the ERRE model in this specific area. As ERRE is focussed on the resource limits and financial risk component between food and energy, no further detail on the goods and services structure was necessary for such a purpose. However, from the household perspective, goods are separated between those that both require energy for their utilization, and, for accounting reasons, retain value and can be impounded in case of household default (named Valuable Goods), and those that are not valuable and/or their energy consumption can be neglected. Services, are interpreted as all those needs provided by third parties (Goods and Services sector's output) which provide value by consuming energy, but do not imply any direct ownership from the household themselves (for example, travelling by flight or train, or the use of the internet). Thus, services can be modelled similarly to the non-valuable goods.

As a result, the goods and services can be separated between the accumulation of Valuable Goods, with relative accounting value, and all Non-Valuable Goods as well as all Services that provide utility without being considered by banks in the case of defaults. For simplicity, a fractional parameter is assumed to split all acquisitions from the Goods and Services sector such that it equals to the sum of the inflows to the two stocks. The default structure is applied to the Valuable Goods only, for the same principle as seen in the modelling of capital. The discard rate of both stocks is assumed to be equal and have a constant average between the two. For the case of services, this can be seen as the amount of times in which those services are acquired in a unit of time, determining their intensity. Both accumulations are modelled as a first order delay due to their short life.

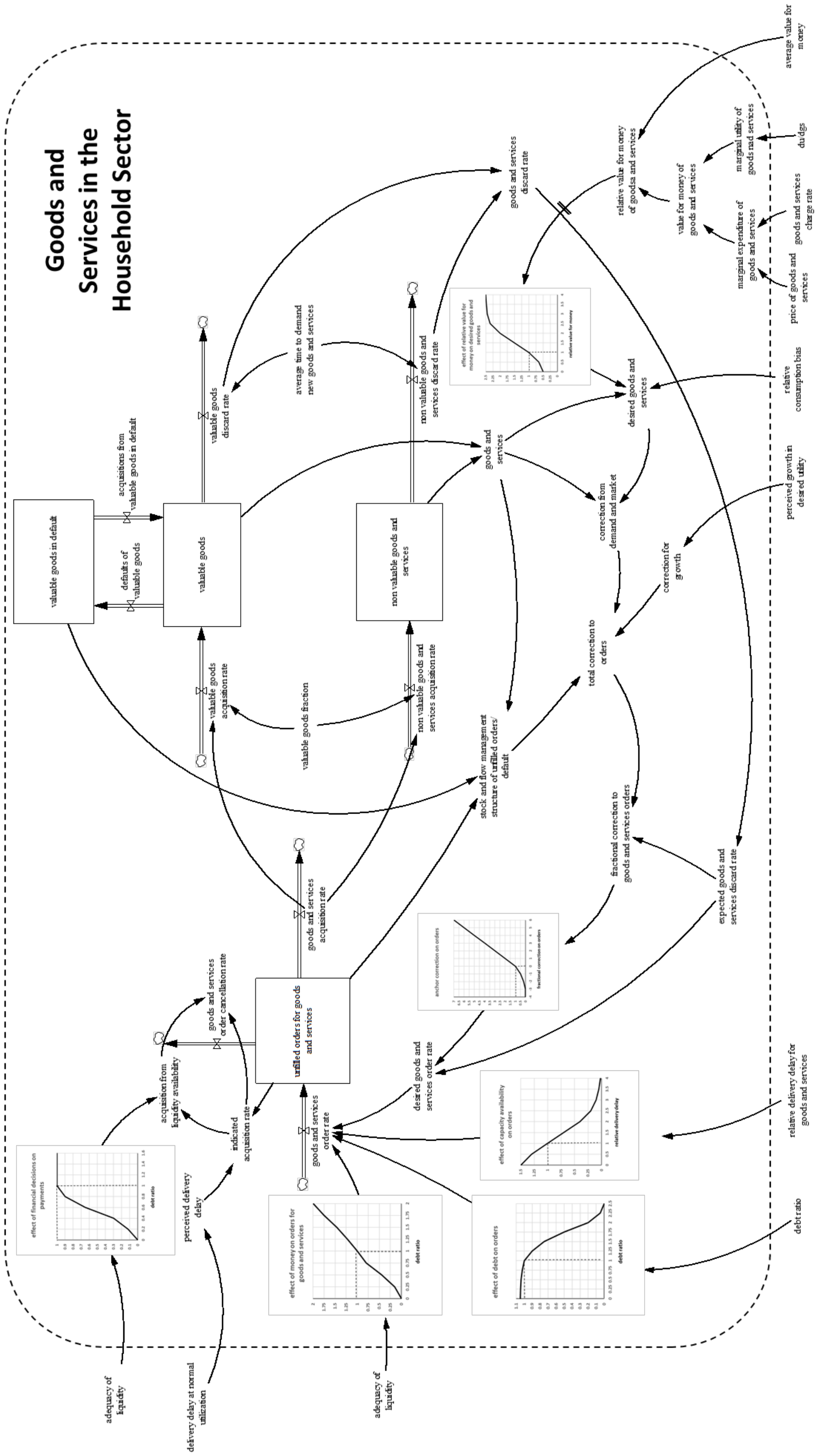
As a result, goods and services $GS_H(t)$ are assumed (i) to consume energy $EReqGS_H(t)$ for their operation, to be paid at an average price p_{GS} to the Goods and Services sector, and (ii) to discard at a certain average constant rate $\frac{1}{ALGS_H}$. This allows the determination of their marginal cost, desired value, and exponent for effect on utility, with the same formula as in capital, while substituting their relative parameters in the Eq. 1.164, Eq. 1.166, and Eq. 1.168.

Their impact on utility is determined by the sum between valuable $VG(t)$, and non-valuable and services $NVG\&S_H(t)$ as follows:

Eq. 1.169 – Goods and Services

$$GS_H(t) = VG(t) + NVG\&S_H(t)$$

Figure 1.30 – Goods And Services in the Household Sector



Food

The structure for food ordering is assumed to be similar to the one applied to the capital ordering with the following differences:

1. The structure is a first order rather than a third order delay for food consumption.
2. The average life of food is assumed to be one year.
3. Households are assumed to be sensitive to food price in a similar way as that depicted in the value for money of other utility factors, but their impact on desired food has lower elasticity.
4. Population growth, instead of utility gap, is assumed to be the main determinant for food consumption growth, and it is both accounted for in the calculation of desired food and in the stock management structure in substitution to the trend in desired utility.
5. Household income growth is assumed to impact on food preferences, resulting in increased food consumption per household over the time of the simulation.
6. Debt and delivery delay are neglected.

Thus the marginal cost of food $mExp_F$ is determined by the food price p_F such that:

Eq. 1.170 – Marginal cost of food

$$mExp_F = p_F$$

This allows the calculation of the relative value for money of food $rVfMF_H$ in the standard formulation:

Eq. 1.171 – Relative value for money for food

$$rVfMF_H = \frac{\frac{\frac{\partial PU}{\partial F_j}}{mExp_F}}{AVfM_H}$$

Therefore, the desired food for ordering is calculated as follows:

Eq. 1.172 – Desired food

$$F^*{}_H = F_H \times (1 + TREND(Pop)) \times \lambda_{rVfMF}\left(\overline{rVfMF_H}\right) \times (1 + TRNDJ(NIpc_H))^{\kappa_F}$$

Where F_H is the current level of food consumption, $(1 + TREND(Pop)) = \frac{P_{t+1}}{P_t}$ represents the contribution of additional population over the year, λ_{rVfMF} represents the non-linear relationship describing the sensitivity of household's food consumption to price, and $(1 + TRNDJ(NIpc_H))^{\kappa_F}$ is the eventual effect of income growth on food consumption growth. It is worth noting that food is an aggregated sector representing all food commodities. Thus, this latter relationship aims at quantifying, by mean of sensitivity analysis based on the exponent parameter κ_F, the effect of income growth on food intensity. For example, the rise of per capita income in China and cascading consequences for increased red meat consumption, are considered.

As Figure 1.31 shows, the final determinant of the food stock management structure is influenced by liquidity, both for the case of food ordering and food orders cancellation. However, because of the importance for life, the elasticity on those is considered to be low, keeping their effects not significant until the liquidity reaches very low levels.

The exponent of food impact on utility is determined as:

Eq. 1.173 – Annual expenditure of goods and services

$$\varepsilon_{H_F} = \frac{F_H(t) \times p_F}{\sum_{UF} AnExp_{UFj}}$$

Where $F_H(t)$ is the amount of food consumed in one year, p_F the price of food, and $\sum_{UF} AnExp_{UFj}$ the total expenditure for utility.

Figure 1.31 – Food ordering and consumption

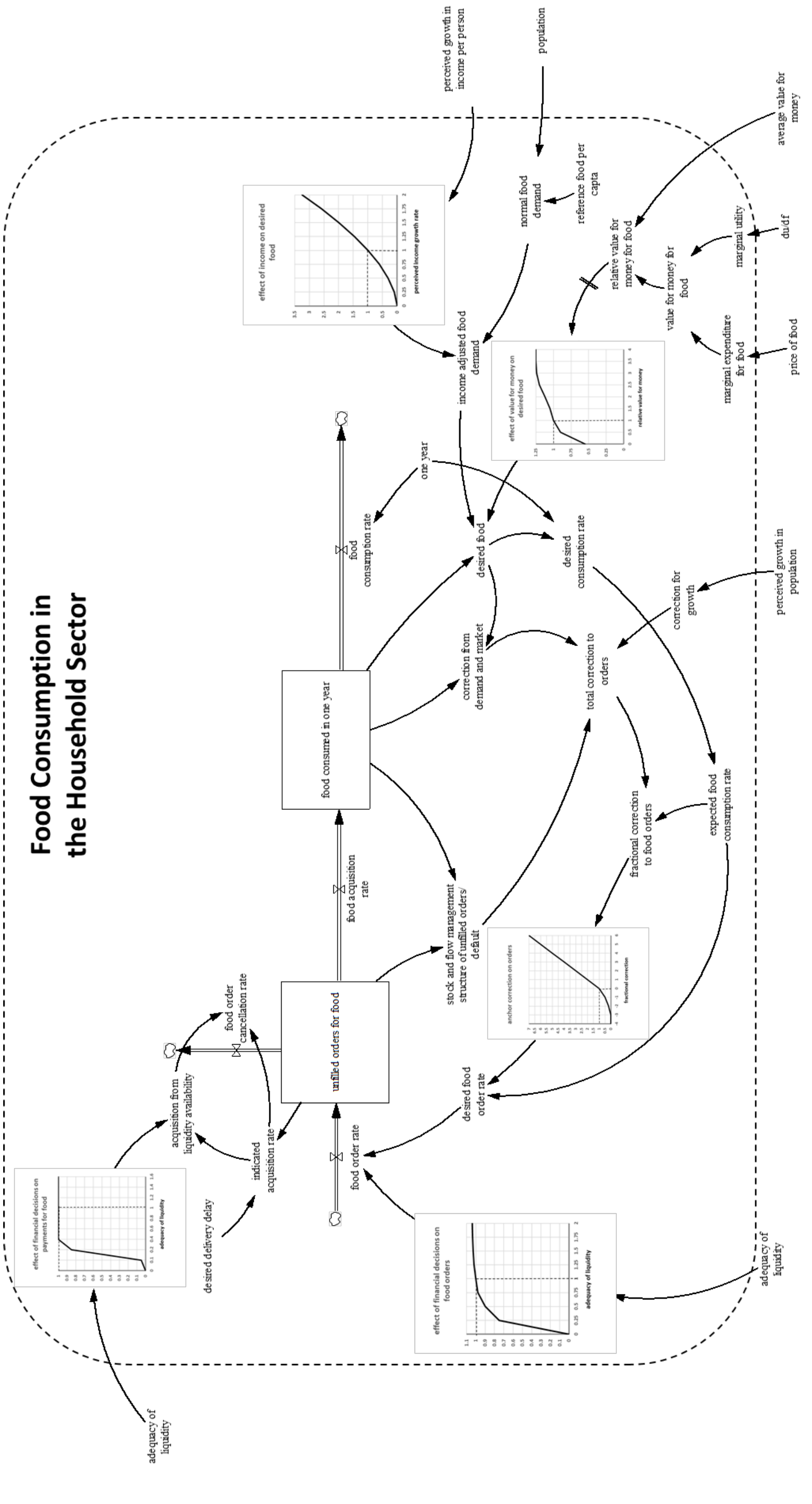
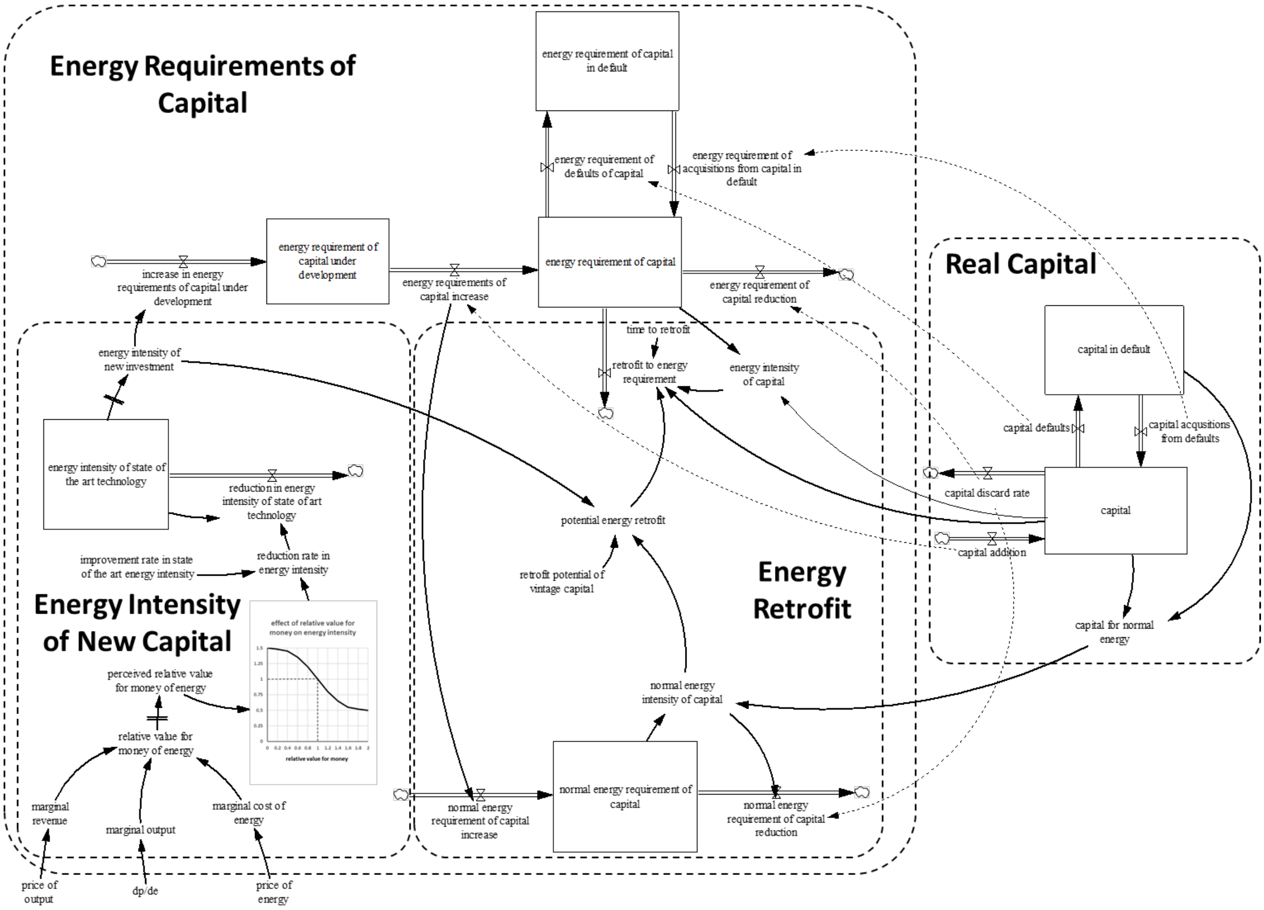


Figure 1.32 shows the modelling for energy requirements for both goods and capital at normal utilization $EReq_i$, in every sector of the economy. This structure is similar to the one used in Sterman (1981), and modified for the treatment of the evolution of state-of-the-art energy intensity and energy retrofit. In this section we shall define the use of the co-flow archetype, the four layers of energy intensity, the energy retrofit structure, and the energy requirements of capital and goods.

Figure 1.32 – Energy requirement of capital

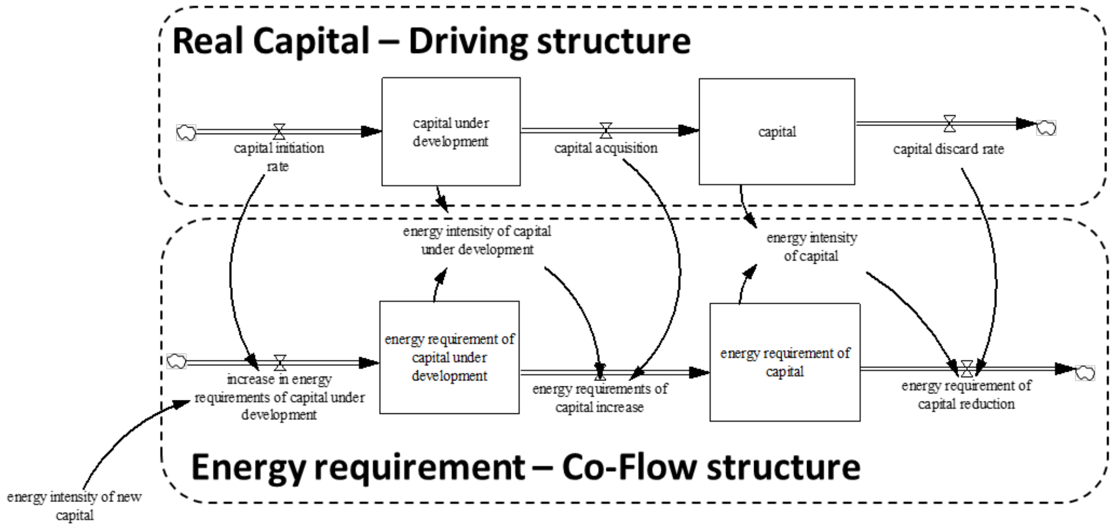


ENERGY INTENSITY AND ENERGY REQUIREMENT

Figure 1.33 shows an example of the structure underpinning the co-flows between energy and capital.

While capital moves through their stock vintage structure, it is important that the accounting of energy sticks to that capital structure. As a result, an energy intensity capital must be defined for each stock, and used as a metric to determine the energy stock outflow in order to be proportional to the outflow from the capital stock.

Figure 1.33 – Co-Flow structure in the energy sector



We can define the increase in energy requirement of capital under development $IEReqKuD_i$ as:

Eq. 1.174 – Increase in energy requirement of capital under development

$$IEReqKuD_i = CIRK_i \times eiNK_i$$

Where $CIRK_i$ is the capital construction initiation rate, and $eiNK_i$ the energy intensity of new capital. The increase in energy $IEReqKuD_i$ accumulates in the stock $\mathrm{EReqKuD}_i$ and is used to determine the unitarian energy intensity for that specific stock $eiKud_i$ as:

Eq. 1.175 – Energy intensity of capital under development

$$eiKuD_i = \frac{\mathrm{EReqKuD}_i}{KuD_i}$$

This allows the determination of the increase in energy requirement of the first cohort of capital vintage $IEReqKv1_i$ as:

Eq. 1.176 – Increase in energy requirement of capital

$$IEReqKv1_i = KAR_i \times eiKuD_i$$

Where KAR_i is the capital addition rate and $eiKuD_i$ is the energy intensity of capital under development. Following the same rationale, the decrease in energy requirement of the Nth capital vintage cohort $\delta EReqKvN_i$ is calculated as:

Eq. 1.177 – Decrease in energy requirement of Nth capital vintage cohort

$$\delta EReqKvN_i = \delta KvN_i \times \frac{EReqKvN_i}{KvN_i}$$

Where δKvN_i is the discard of the Nth capital vintage cohort, $EReqKvN_i$ the energy requirement of every capital vintage cohort, and KvN_i the capital vintage cohort. It is worth noting that the ratio $\frac{EReqKvN_i}{KvN_i}$ correspond to the energy intensity $eiKvN_i$ of that Nth capital cohort.

The same rationale is applied to the entire capital cohort structure, including all capital in defaults, and is used to determine energy requirements of valuable goods in each sector of the model. Given that the capital vintage structure is modelled as a third order delay, this must be reflected in a third order for energy requirements as well. In so doing, the energy requirement for capital at normal utilization $EReqK_i$ can be defined as:

Eq. 1.178 – Energy requirement of capital

$$EReqK_i = EReqKv1_i + EReqKv2_i + EReqKv3_i$$

Where each $EReqKvN_i$ represents the energy requirement for each capital cohort. This allows the calculation of the energy intensity of capital eiK_i as the average between all cohorts as:

Eq. 1.179 – Energy intensity of Capital

$$eiK_i = \frac{EReqK_i}{K_i}$$

Where $EReqK_i$ is the energy requirement and K_i the capital.

The structure presented in Figure 1.32 is indicative for the capital and durable goods, with the main difference that capital employs a third order vintage structure, whereas goods are modelled as a first order delay. The next section describes how the retrofit structure and evolution of state of the art intensity is determined, thus developing a dynamic theory of energy consumption in a generic sector of the economy.

STATE OF ART ENERGY INTENSITY AND RETROFIT

The modelling of energy retrofit required the addition of a secondary co-flow structure as a reference for comparison to the actual energy requirement. This resulted in modelling the energy intensity in five different levels:

1. Energy intensity of state of art technology – determined by an exogenous element describing its evolution, accompanied by an endogenous influence from price change
2. Energy intensity of new capital – the level of energy intensity actually applicable to the real assets, calculated as a delay on state of art energy intensity
3. Reference maximum energy intensity - the energy intensity that could be obtained in case no retrofit was considered
4. Potential Retrofit of Energy Intensity - the maximum possible retrofit on energy intensity
5. Energy Intensity - the energy intensity of goods and capital determining the actual energy demand.

The historical evolution of global energy intensity, measured as the fraction between global GDP and global energy consumption, has shown a steady decline over the last 40 years (EIA 2019). As a result the structure proposed in Sterman (1981), that was suitable for the historical data of US of the 1980s, required amends, with cascade modification to the energy retrofit structure.

The energy intensity of capital at state of art technology $eiKSoA_i$ has been determined as follows:

Eq. 1.180 – Energy Intensity of State of the Art Technology

$$eiKSoA_i = eiK_{i_0} + \int_t \left[-\delta_{eiKSoA_i}(z) \times eiKSoA_i(z)\right] dz$$

Where eiK_{i_0} is the energy intensity of capital at initial conditions, $-\delta_{eiKSoA_i}(t)$ represents the fractional reduction on state of art energy intensity $eiKSoA_i(t)$. In turn:

Eq. 1.181 – Fractional reduction in state of art energy intensity

$$\delta_{eiKSoA_i}(t) = \delta_{eiKSoA_i} \times \lambda_{rVfMEK}\left(\overline{rVfMEK_i}\right)$$

Where δ_{eiKSoA_i} represents the exogenous parameter setting the trend of energy intensity reduction, and $\lambda_{rVfMEK}\left(\overline{rVfMEK_i}\right)$ the non-linear effect of value for money on energy intensity. This non-linear effect acts to increase the propensity of improving energy efficiency until it reaches double its initial value at the point when value for money is lower than cost, and relieves pressure on improvement until it halves in the opposite case.

As it takes time for technology to spread in the world, it is assumed that the energy intensity of new investment $eiKn_i$ is a delay function on state of the art energy intensity $eiKSoA_i$ with time $eiKnAdjT$ such that:

Eq. 1.182 – Energy intensity of new investments

$$eiKn_i = smooth(eiKSoA_i, eiKnAdjT_i)$$

Thus energy intensity of new investment $eiKn_i$ is considered as the minimum energy intensity possible, and is used to determine the actual possible energy retrofit.

In order to do so, a reference energy intensity value was calculated with a co-flow on both capital $KvN_i(t)$ and defaulted capital $K\Omega vN_i(t)$, as if no energy retrofit had been performed. Thus, the reference energy intensity of each capital cohort $eiRefKvN_i$ represents the maximum possible energy intensity and can be calculated as:

Eq. 1.183 – Reference energy intensity of capital Nth

$$eiRefKvN_i = \frac{EReqRefKvN_i}{KvN_i(t) + K\Omega vN_i(t)}$$

Where $EReqRefKvN_i$ is the total energy requirement of both operating and defaulted capital.

The minimum possible energy intensity $eiMinKvN_i$ that each stock of capital can aim to is:

Eq. 1.184 – Minimum Energy intensity possible

$$eiMinKvN_i = eiRefKvN_i - Max(0, (eiRefKvN_i - eiKn_i) \times RetPot_i)$$

Where $eiRefKvN_i$ is the reference energy intensity, $eiKn_i$ is the energy intensity of new capital, $RetPot_i$ is a parameter between 0 and 1 representing a fractional retrofit potential for that capital. The result is that $(eiRefKvN_i - eiKn_i) \times RetPot_i$ represents the maximum possible retrofit for each capital cohort. The Max(0,) constraint has been applied to allow users willing to test the alternative hypothesis of increased energy intensity, and making sure that old capital does not adapt to increased energy intensity.

The energy retrofit for each capital cohort $ERetKvN_i$ can be calculated as the adjustment on current energy intensity $eiKvN_i$ in comparison to the minimum achievable energy intensity $eiMinKvN_i$ over the time $AdjRetT_i$, such that:

Eq. 1.185 – Energy retrofit for capital Nth

$$ERetKvN_i = \frac{eiKvN_i - eiMinKvN_i}{AdjRetT_i} \times KvN(t)$$

Where $\frac{eiKvN_i - eiMinKvN_i}{AdjRetT_i}$ represents the energy intensity retrofit, and $KvN(t)$ the cohort of capital.

This structure allows the model to assure that the normal energy demand $EReqK_i$ is always consistent with physical flows, and represents one of the contribution of the ERRE model to neo-classical and Integrated Assessment models.

FINANCIAL ASSETS AND RETURNS ON INVESTMENTS

In this section, all real assets are transformed to financial assets equivalent, in order to (i) be used for financial decision making, (ii) trade real assets in monetary terms, (iii) allow banks to assess a borrower viability for lending, and (iv) allow tax payments. Many of those elements have been presented in the treatment of the ERRE model so far (i.e. Eq. 1.26, Eq. 1.31, Eq. 1.33, Eq. 1.39, Eq. 1.57 among others). These structures are applied similarly to both goods, capital and agricultural land.

Whereas households measure their assets value as reference for loans and payments, firms account for these as a basis for their assessment for depreciation, to pay taxes, and to evaluate their return on investments, and to define payments for labour and dividends as well. In this section these structures are described in detail.

VALUE OF ASSETS

In this section, the assets that generate production and utility in both firms and households, and their default of assets structure, are presented.

Capital

Figure 1.34 shows a generic capital value vintage structure, based on a co-flow archetype as seen in the energy requirement section (see Figure 1.33). The main differences between the two are (i) to use capital value (i.e. capital price p_k) in substitution to the energy intensity of new capital (see Eq. 1.182), and (ii) to account for a default inflationary and discount factor for purchases from defaulted assets. Thus, the historical value of capital carries the value of real capital along its vintage structure, and applies a correction derived from market and discount rate for those assets purchased from defaults.

Figure 1.34 – Historical Value of Capital

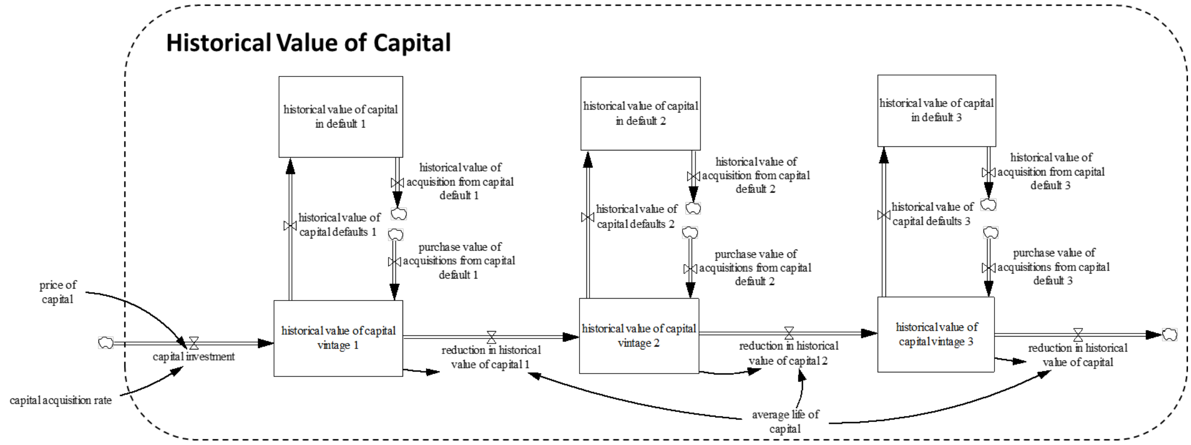
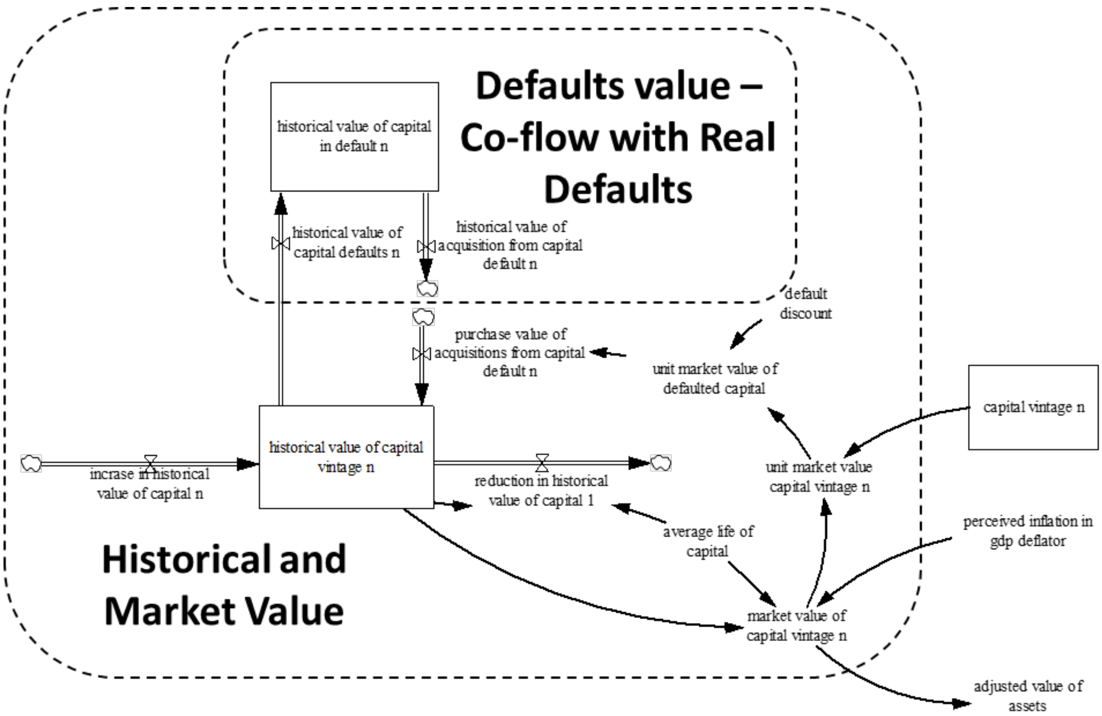


Figure 1.35 shows the modelling of market value of a generic capital cohort, the existing capital market price, and the potential selling price for defaulted capital to potential acquirers. The market value of capital is used both for the calculation of financial flows relative to capital transactions, and for the accounting of inflation adjusted value of assets from the investors assessment of the firm performance.

Figure 1.35 – Market Value of Capital



Capital investment $K\Psi_i(t)$ is thus determined as:

Eq. 1.186 – Capital investments

$$K\Psi_i(t) = p_k \times KAR_i$$

Where p_k is the price of capital and KAR_i the capital acquisition rate. Capital investments $K\Psi_i(t)$ accumulate as the historical value of capital vintage activating the first co-flow structure $HKvN(t)$, which, being equivalent to the one seen in the energy requirement section, is not repeated again here.

The market value of capital for each cohort $MKvN_i(t)$ can be calculated as:

Eq. 1.187 – Market Value of the Nth Cohort

$$MKvN_i(t) = HKvN(t) \times \left(1 + \gamma(t) \times \frac{ALK_i}{3}\right)^N$$

Where $HKvN(t)$ is the historical value of the Nth capital cohort, $\gamma(t)$ is the inflation rate calculated as trend, with anchor, on the GDP deflator, and $\frac{ALK_i}{3}$ is the average life of capital in each cohort.

The market adjusted price of the capital in operations $mAdjp_{k_i}$ (see Eq. 1.33) is determined as:

Eq. 1.188 – Marked adjusted price of capital

$$mAdjp_k vN_i = \frac{MKvN_i(t)}{KvN_i(t)}$$

Where $MKvN_i(t)$ is the market value of each capital cohort, and $KvN_i(t)$ the relative real capital. The market price of assets is the key determinant for various cash flows as registered in the balance sheet. These include the defaults on assets ∇_{A_i}, the distribution of assets defaults between liabilities ∇_{D_i} and equity $\nabla_{\in_i}$, and the purchases of defaulted assets ∇Acq_{A_i}. In detail, defaults on assets for balance sheet ∇_{A_i} (see Eq. 1.33) are determined as:

Eq. 1.189 – Defaults on Assets

$$\nabla_{A_i} = \sum_{N} \Omega vKN_i \times mAdjp_k vN_i$$

Where ΩvKN_i represents the defaults on real assets, and $mAdjp_k vN_i$ the adjusted market price of each capital cohort. As described in Eq. 1.34 and Eq. 1.35, those assets are redistributed between those allocated to debt money ∇_{D_i}, and those allocated to equity $\nabla_{\in_i}$.

The purchases of assets from defaults ∇Acq_{A_i}. are calculated as:

Eq. 1.190 – Purchase of assets from defaults

$$\nabla Acq_{A_i}. = \sum_{N} \Omega AcqvN_i \times mAdjp_k vN_i \times \nabla_i$$

Where $\Omega AcqvN_i$ is the capital acquisition from each capital cohort (see Eq. 1.139), and $mAdjp_k vN_i \times \nabla_i$ is the market price of that acquisition accounting for a possible discount rate on defaulted capital ∇_i. Each of the elements $\Omega AcqvN_i \times mAdjp_k vN_i \times \nabla_i$ is an inflow to the relative historical value of capital cohort as shown in Figure 1.35. The purchases of assets ∇Acq_{A_i} are an outflow from the balance sheet of each acquirer, and are distributed to, and recorded as inflows to the balance sheets of, the bank ∇Acq_{B_i}. and the borrower $\nabla Acq_{H+\Pi_i}$ that own those assets. These are calculated as:

Eq. 1.191 – Liquidity from defaults to banks

$$\nabla Acq_{B_i} = \nabla Acq_{A_i} \times \alpha_i$$

Eq. 1.192 – Liquidity from defaults to borrower

$$\nabla Acq_{H+\Pi_i} = \nabla Acq_{A_i} \times (1 - \alpha_i)$$

Where α_i is a parameter between 0 and 1 that represents the average ownership fraction of the defaulted assets at the time of insolvency.

All of these elements contribute to determining the inflation adjusted value of assets $AdjA_i(t)$ used as an input to various sub-dimensions of both firms and household sectors, including price formation, borrowing and returns on investments. Its calculation is:

Eq. 1.193 – Inflation Adjusted Value of Capital

$$AdjA(t) = MKv1_i(t) + MKv2_i(t) + MKv3_i(t)$$

Where $MKvN_i(t)$ is the market value of each capital cohort.

Valuable Goods

Valuable goods value follows the same treatment as capital value, with the only difference of not considering any vintage structure. The implication is to simplify the above described equations as if composed of a first order capital vintage structure only.

Agricultural Land

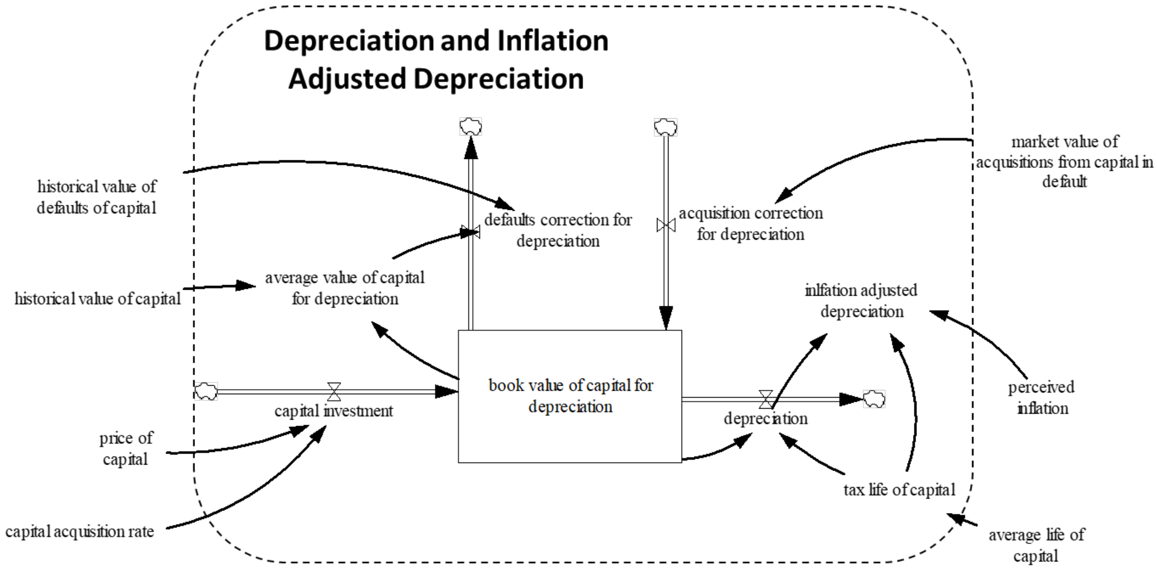
Agricultural land value is modelled as a first order delay as well. However, as a long term asset (normal average life of agricultural land is assumed to be 1000 years), the inflation adjusted structure is substituted with the historical value as dependent on capital price and land depletion cost curve.

<u>DEPRECIATION OF ASSETS</u>

The depreciation is modelled as an accounting measure necessary for tax payments for every asset used in firms. As Figure 1.36 shows, depreciation requires a first order delay structure on capital investments, no matter the vintage structure adopted to model the corresponding real assets. In fact, depreciation rate is the segmentation of an initial investment in equal parts, each accounted for the time unit in which taxes are meant to be paid.

Both defaults on assets and acquisitions from defaults are assumed to have an impact on depreciation. All defaults correspond to firms breaking their contracts for depreciation, and all acquisitions are assumed to reintroduce those contracts into operation for the remaining tax life of those assets.

Figure 1.36 – Capital Depreciation



A co-flow between depreciation and historical value structure was used to account for such a correction. In particular, the historical value of defaulting assets ΩH_i for a generic capital vintage cohort is determined as:

Eq. 1.194 – Historical value of defaulting assets

$$\Omega H_i = \sum_N \Omega vKN_i \times \frac{HKvN_i(t)}{KvN_i(t)}$$

Where $\frac{HKvN_i(t)}{KvN_i(t)}$ represents the average historical price of the assets on each cohort, and ΩvKN_i the real assets defaults from that cohort. Thus, the corresponding correction to the capital stock for depreciation $\Omega \Delta corr_i$ is modelled as:

Eq. 1.195 – Defaults and acquisitions correction to Depreciation

$$\Omega \Delta corr_i = \nabla Acq_{A_i} - \frac{\Delta K(t)_i}{HK(t)_i} \times \Omega H_i$$

Where ∇Acq_{A_i} is the market value of acquisitions from defaults, inflow to the capital for depreciation $\Delta K(t)_i$, and $\frac{\Delta K(t)_i}{HK(t)_i} \times \Omega H_i$ the relative outflow.

As a result, depreciation Δ_i is calculated as the ratio between the assets $\Delta K_i(t)$ and the average tax life of capital τALK_i, as:

Eq. 1.196 – Depreciation of capital

$$\Delta_i = \frac{\Delta K_i(t)}{\tau ALK_i}$$

For simplicity, the tax life of capital τALK_i is made equal to the average life of capital ALK_i, assuming consistency between taxed and real assets. Depreciation Δ_i is an input to the balance sheet of each firm, and is used to determine both revenue and taxes (see Eq. 1.26)

In the ERRE model, firms use depreciation Δ_i to pay taxes, but account for inflation in order to assess their investment returns, and define payments both to labour and shareholders. As a result, the inflation adjusted depreciation $Adj\Delta_i$ is determined as follows:

Eq. 1.197 – Adjusted Value for Depreciation

$$Adj\Delta_i = \Delta_i \times (1 + \gamma(t) \times \tau ALK_i)$$

Where Δ_i is the accounted depreciation, $\gamma(t)$ the global inflation rate, and τALK_i the tax life of that capital.

Whereas depreciation Δ_i is calculated in the same way for both capital and agricultural land in every firm, agricultural land adjusted value of depreciation $Adj\Delta_{ALi}$ is assumed to correspond to the actual depreciation, and no inflation adjustment is considered.

RETURN ON INVESTMENTS AND ADJUSTED RETURNS

All the above allows the determination of the return on investments ROI_i in its standard formulation as follows:

Eq. 1.198 – Return on Investments

$$ROI_i = \frac{NI_i}{AdjA_i(t)}$$

Where NI_i is the net income after tax (see), and $AdjA_i(t)$ the total value of a firm's assets (see Eq. 1.197).

Despite its simplicity in measuring a firm performance, decision makers adopt two additional derivative indicators to take decisions that impact their business. These are (i) the inflation adjusted return on investments $AdjROI_i$ and (ii) the rationally expected goal for return on investments $gROI_i$, as described below.

The adjusted return on investments $AdjROI_i$ is determined as:

Eq. 1.199 – Adjusted Return on Investments

$$AdjROI_i = \frac{AdjNI_i}{AdjA_i(t)}$$

Where $AdjA_i(t)$ is the inflation adjusted assets value, and $AdjNI_i$ is the inflation adjusted net income. This latter is represented as a net income that accounts for the inflation adjusted depreciation $Adj\Delta_i$ as follows:

Eq. 1.200 – Adjusted Net Income

$$AdjNI_i = R_i - W_i - \epsilon_i - \Upsilon_i - Adj\Delta_i - T_i$$

Where R_i is the revenue stream, W_i is the labour payments, ϵ_i the energy payments, Υ_i the interest payments, $Adj\Delta_i$ the inflation adjusted value of depreciation, and T_i the taxes payments (see Eq. 1.26 and Eq. 1.31).

In so doing, the adjusted net income $AdjNI_i$ measures the actual performance of firms while paying all costs of business, and must be compared to a 'goal' in order to determine success against expectations. Such a goal is formed rationally, based on the desired price from the cost model $p_{c_i}(t)$ (see Eq. 1.57) to achieve profit at a desired profit rate ψ_i (see **Error! Reference source not found.**).

Thus, starting from Eq. 1.57, it is possible to obtain the following equivalence:

Eq. 1.201 – Rational assumption of firm to measure investment performance

$$R_i - W_i - \epsilon_i = kcr_i \times AdjA_i(t)$$

Where R_i is the total revenue, W_i is the payments to labour, ϵ_i is the payments for energy, and $kcr_i \times AdjA_i(t)$ is the total inflation adjusted cost of capital assets.

LABOUR AND LABOUR MARKET

In the ERRE model, workers are modelled in four major systems, interconnected one to another, and distributed across firms and household sectors. These are:

1. Labour employed and labour demand (each firm sector)
2. Labour market (in between all firms and household's sectors)
3. Labour supply (each firm sector)
4. Voluntary unemployed people (household sector)

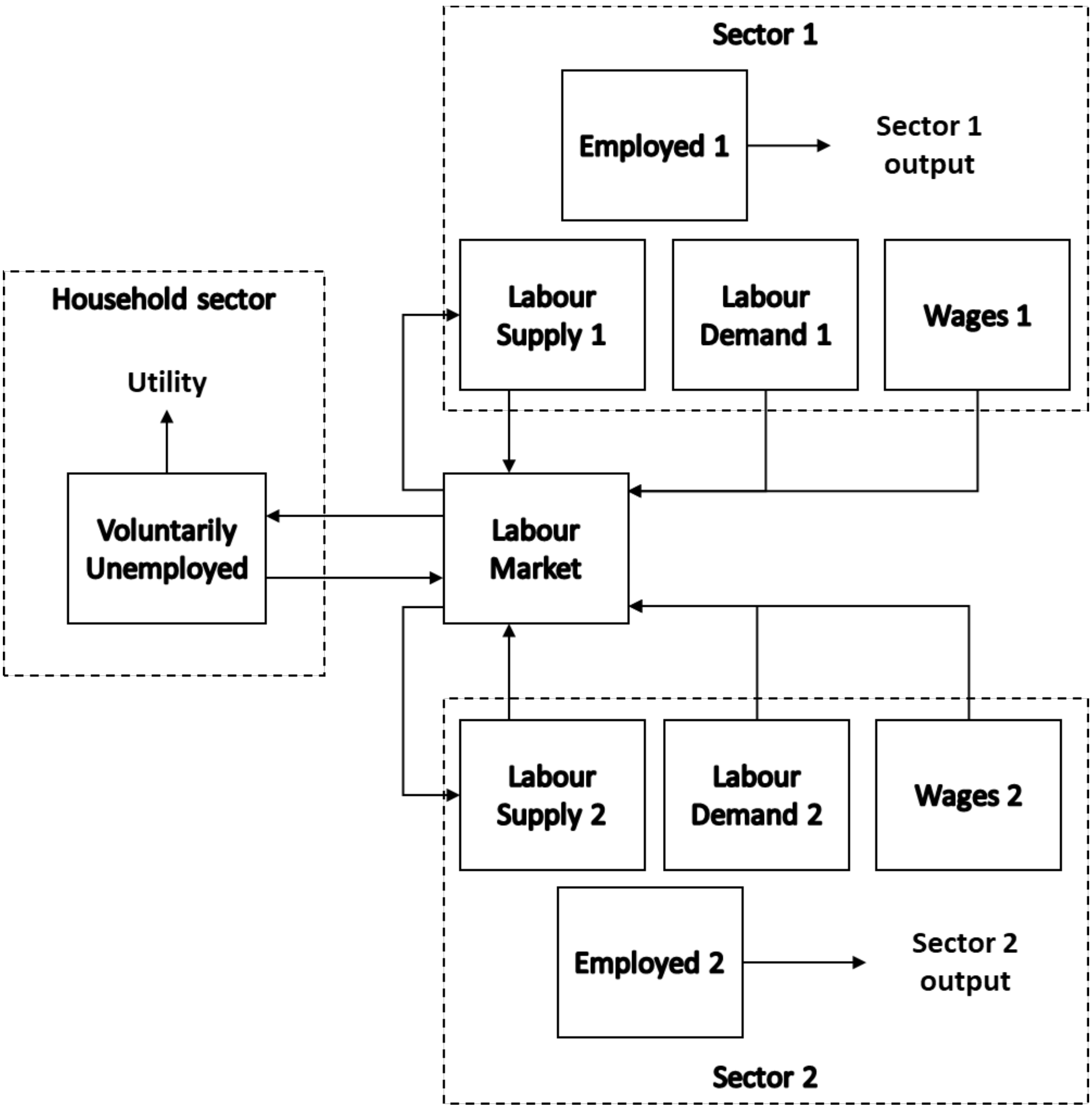
Figure 1.37 shows the relationships between those elements.

Firms employ workers to generate output for the economy, and set wages to determine the actual cost of labour. Each firm sector is assumed to hire people relying on a labour supply for that specific sector, which represents both employed and unemployed people that are willing to work for them. Labour productivity is assumed to be exogenous to the model, and represents the amount of real output that can be generated by each worker.

Both wages and labour supply are determined by comparing the information available in the labour market across all sectors. Workers are assumed to express their preferences based on wages and labour demand for each sector, and to move between those. Firms, can increase or decrease wages, in order to attract the desired amount of workers.

Voluntary unemployed workers are those who spend their time providing utility to households without receiving wages. They are assumed to take information from the job market, and, can decide to move as labour supply (go in search for jobs) or return as voluntary unemployed based on their level of indebtedness, and the perceived comparative value between receiving a wage or generating utility for themselves.

Figure 1.37 – Relationships in the Labour sector



In ERRE, the labour market does not differentiate between skillsets, experience and age. The only differences among sectors are (i) a preliminary defined target wage gap, and (ii) the level of labour productivity between those. Thus all people are seen as capable of moving across sectors without distinction. For the purpose of the model, this level of granularity is felt to be sufficient.

LABOUR FORCE AND LABOUR DEMAND

Figure 1.38 shows the structure of labour force used to generate production within firms. Labour decreases at the end of service of employment, and when there is a loss of jobs due to defaults of firms. Firms create vacancies based on the substitution for labour leaving their jobs, and those who lose their job for defaults are assumed to find their way back to job market in a simpler way than all other job seekers. Both a normal and a minimum unemployment rate are assumed to bound the labour supply to provide suitable people to the open vacancies of firms. Data show that approximately 6% of the global labour force is unemployed today (normal unemployment rate). Thus, it is assumed that firms face greater and greater difficulties in the hiring process, and push firms to reduce the average duration of employment beyond normal values, the more the employed labour reaches full employment. This can be seen as the tendency in substituting people with the right skills when the most of the labour supply is employed.

The structure underpinning this rationale is similar to the one adopted in the modelling of capital. In fact, the normal hiring rate NHR_i is determined as the sum between those who end their employment LES_i, and those who lose their jobs LLR_i. This is used as a similar concept to the capital discard rate in the capital ordering sub-system. Thus, based on a labour correction fraction emerging from the calculation of desired labour and labour demand, an anchor and adjustment archetype is applied simulating the ability of firms to hire people. However, limits to hiring is necessarily based on the labour supply.

The labour layoff rate LLR_i is determined as:

Eq. 1.202 – Labour layoff rate

$$LLR_i = L_i \times \frac{\sum_N \Omega v N_i}{K_i(t)}$$

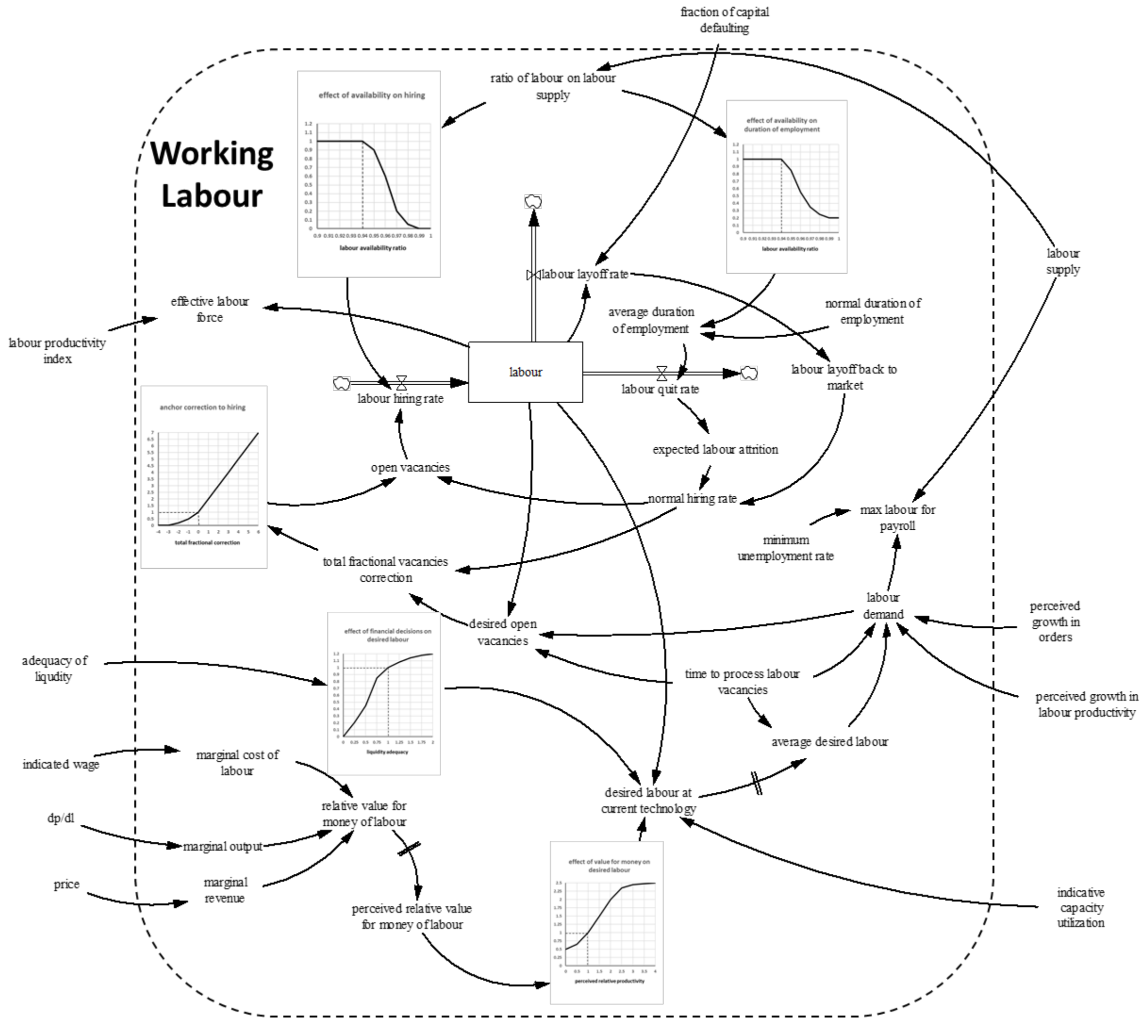
Where L_i is the current labour force, $\sum_N \Omega v N_i$ is the total real capital default on all capital vintage, $K_i(t)$ is the total capital in operation. Such an equation assumes that the fraction of assets defaulting is always proportional to the fraction of labour losing their job.

The labour end of service LES_i is calculated as:

Eq. 1.203 – Labour end of service

$$LES_i = \frac{L_i}{NADE_i \times \lambda_{LES}\left(\frac{L_i}{LS_i}\right)}$$

Figure 1.38 – Productive Labour force within firms



Where L_i is the labour force, and $NADE_i \times \lambda_{LES}\left(\frac{LS_i}{L_i}\right)$ the average duration of employment. It is worth noting that labour is modelled as a first order delay, assuming a varying duration of employment. In fact, the normal duration of employment $NADE_i$ can be impacted by the availability of people working in that particular sector represented by the labour supply LS_i. The non-linearity λ_{LES} captures the behaviour of firms and employment to work beyond their normal length of contract when no other people are available to take that particular job. The non-linearity is assumed to have no effect (multiplier as 1) on the duration of employment $NADE_i$ until the employment rate $\frac{L_i}{LS_i}$ is below the 94% (the current global employment rate), and decreases to 20% when labour employment reaches the 99% of total.

Thus, assuming that firms would have the tendency to open up positions based on workers leaving employment with a management delay $AdjTLES_i$, and those losing their jobs would have an easier chance in finding their way back to the market assuming a delay time $AdjTLLR_i$, a normal hiring rate NHR_i can be calculated as:

Eq. 1.204 – Normal hiring rate

$$NHR_i = smooth(LES_i, AdjTLES_i) + smooth(LLR_i, AdjTLLR_i)$$

On the other hand, firms would need to plan their labour force according to demand and other performances, and adjust their hiring rate accordingly. Thus, desired labour $L^*{}_i$ at current technology level is determined as follows:

Eq. 1.205 – Desired labour

$$L^*{}_i = L_i \times \frac{P^*{}_i}{PP_i} \times \mathfrak{f}_\nabla\left(\frac{\Lambda_i}{\Lambda^*{}_i}\right) \times \lambda_{rVfMAL}\left(\overline{rVfML_\iota}\right)$$

Where L_i is the current labour force, $\frac{P^*{}_i}{PP_i}$ is the ratio between desired and potential production, $\mathfrak{f}_\nabla\left(\frac{\Lambda_i}{\Lambda^*{}_i}\right)$ the effect of liquidity imbalance on desired labour, and $\lambda_{rVfMAL}\left(\overline{rVfML_\iota}\right)$ the effect of the value for money perceived by firms in hiring an additional worker. This latter is calculated in the same way as the other productive factors as:

Eq. 1.206 – Value for money of labour

$$rVfML_i = \frac{p_{\tau_i} \times \frac{\partial PP_i}{\partial L_i}}{w_i(t)}$$

where p_{τ_i} is the price of output, $\frac{\partial PP_i}{\partial L_i}$ the marginal output of labour, and $w_i(t)$ the wage payment to the single worker. The non-linear effect λ_{rVfMAL} is assumed to be proportional to the relative value when increasing beyond unity, and half the desired labour when relative value approaches zero. In so doing, in ERRE it is assumed that there will always be some people that need to be hired to allow a business to exist.

As it takes time to process vacancy orders, and accounting for the effects of labour productivity and orders' trends, it is assumed that the labour demand LD_i for the sector is determined as follows:

Eq. 1.207 – Labour demand

$$LD_i = smooth(L^*{}_i, AdjTL_i) \times \left[1 + \left(TRND\mathrm{J}(\mathrm{O}_i) - TRNDJ(\pi_{L_i})\right) \times AdjTL_i\right]$$

Where $L^*{}_i$ is the desired labour averaged over the adjustment time $AdjTL_i$, $TRNDJ(\mathrm{O}_i)$ is the trend, with anchor, in orders, and $TRNDJ(\pi_{L_i})$ is the trend, with anchor, in labour productivity growth for that sector. It is worth noting, similarly to the treatment of agricultural land orders, that the trend in productivity growth requires a reduction in total demand for labour, since each person would produce more output. Labour demand is also used to determine desired payments for labour (feeding back to the desired liquidity $\Lambda^*{}_i$), and is provided as information to the labour market.

The anchor and adjustment archetype is applied, assuming the same behaviour is adopted in the ordering of capital. In particular, a correction factor $\mathrm{J}corrL_i$ is determined as:

Eq. 1.208 – Desired hiring rate

$$\mathrm{J}corrL_i = \frac{LD_i - L_i}{AdjTL_i}$$

That allows the determination of the desired hiring rate $HR^*{}_i$ (i.e. the open vacancies to fulfil) as:

Eq. 1.209 – Desired hiring rate

$$HR^*{}_i = NHR_i \times \lambda_{HR}\left(\frac{\mathrm{J}corrL_i}{NHR_i}\right)$$

Where NHR_i is the normal hiring rate, $\mathrm{J}corrL_i$ the desired correction, and λ_{HR} a non-linear relationship to describe the interest of firms to hire people. In fact, when the correction $\mathrm{J}corrL_i$ is positive, desired hiring is assumed to fulfil all normal vacancies and accommodate all new positions opened for growth. However, if the correction $\mathrm{J}corrL_i$ is negative (need to decrease labour), it is assumed that large firms would still hire people, highlighting an implicit inequality between large and small firms, with large firms able to hire workers during recession. The adjustment follows a non-linear relationship, slowly approaching zero when the negative correction $\mathrm{J}corrL_i$ is three times greater than the normal hiring rate NHR_i in absolute value.

In ERRE, the hiring rate correspond to the desired hiring rate $HR^*{}_i$ when the employment rate $\frac{L_i}{LS_i}$ is lower than 94%, and non-linearly drops to zero when the employed labour L_i approaches 99% of the labour supply LS_i. Such a relationship, implies the increasing difficulty of firms to hire people with the right skillset when there are the less people available in the job market, and is described by the non-linear relationship λ_{HR} as follows:

Eq. 1.210 – Hiring rate

$$HR_i = HR^*{}_i \times \lambda_{HR}\left(\frac{L_i}{LS_i}\right)$$

This closes the feedback controlling the labour force L_i. The effect of labour on production is determined based on the Solow growth model as shown in **Error! Reference source not found.**.

A desired payments of labour for payroll W^*_i is defined as:

Eq. 1.211 – Desired payments for labour for payroll

$$W^*_i = LD_i \times w_i(t)$$

Where $w(t)$ is the wage, and LD_i the actual demand for labour. Finally the payments for labour W_i are determined as:

Eq. 1.212 – Wage payments

$$W_i = L_i \times w_i(t) \times \mathfrak{f}_W\left(\frac{\Lambda_i}{\Lambda^*_i}\right)$$

Where L_i is the employed labour force, $w_i(t)$ the average wage across them, and $\mathfrak{f}_W\left(\frac{\Lambda_i}{\Lambda^*_i}\right)$ is a non-linear constraint that firms adopt in times of liquidity shortage in order to remain solvent.

The following sections describe the modelling of wages and labour supply.

WAGE

The wage $w_i(t)$ represents the average payment due to one single worker in one firm sector. Wages are assumed to be impacted by six factors, which can be differentiated between those that are market and performance driven, and those that relate to the labour market itself.

The wage corrections based on firm and economic performance are:

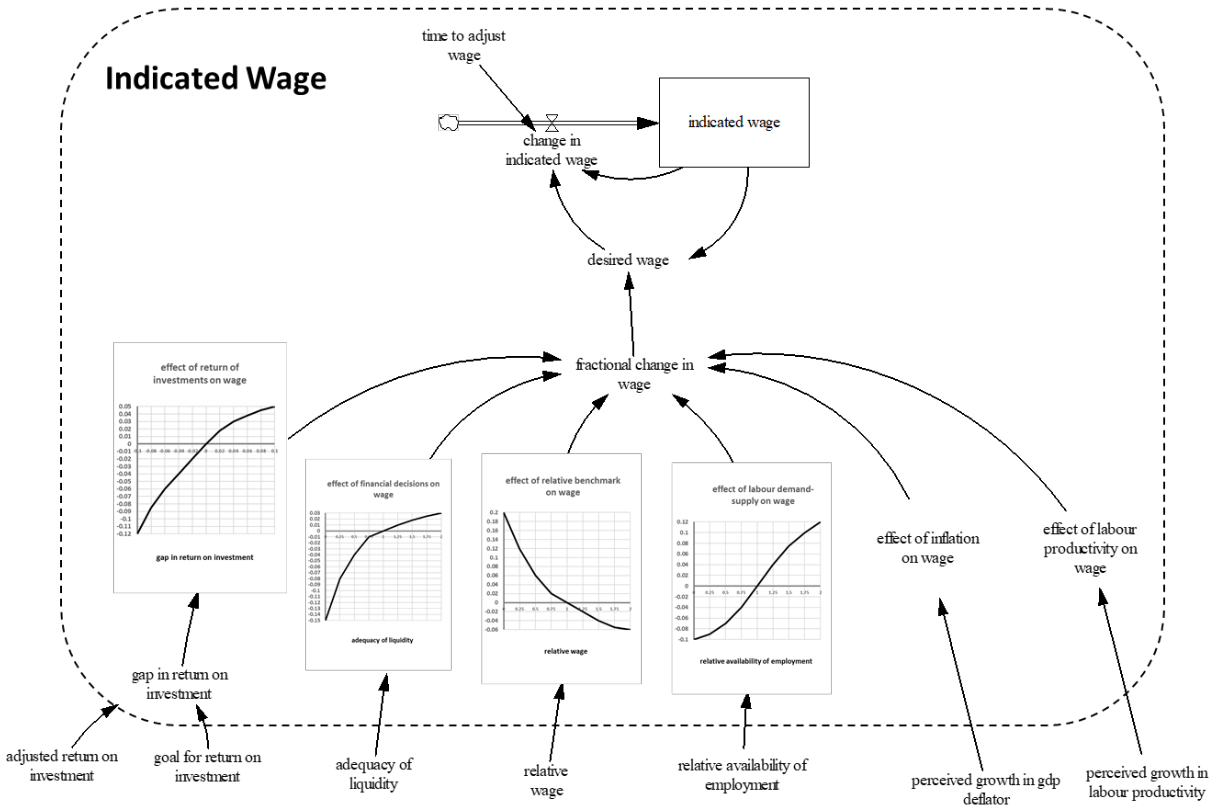
1. Inflation
2. Labour productivity growth
3. Availability of liquidity in the financial sector
4. Return on investment gap.

The corrections based on labour market are:

5. Relative wage in comparison to the other sectors of the economy
6. Ratio between demand and supply of labour.

Figure 5.56 shows the modelling of wage in ERRE. All effects are assumed to have a fractional impact on wage, and their total effect is the sum among all of them. Based on the non-linear relationships chosen, all effects are null at ideal condition (i.e. adjusted return on investments equals goal for returns, desired liquidity equals actual liquidity, etc), and starts adjusting wage non-linearly the more the system measured moves away from the ideal state.

Figure 1.39 – Wage within firms



Eq. 1.213 shows the change in wage from firm's economic performance $wcFEP_i$ as composed of its four elements:

Eq. 1.213 – Desired fractional change in wage from economic performance (reinforcing)

$$wcFEP_i = \lambda_{\Lambda_i}\left(\frac{\Lambda_i}{\Lambda^*_i}\right) + \lambda_{roi}(GapROI_i) + \gamma(t) + TRNDJ(\pi_{L_i})$$

Such a relationship assumes that while a firm registers any increase in inflation $\gamma(t)$ and productivity of labour π_{L_i}, they would proportionally increase wage. In addition, both effects of liquidity adequacy λ_{Λ_i} and return on investment gap λ_{roi}, would non-linearly increase wage with decreasing marginal effect the more they grow beyond the ideal condition. On the other hand, the more they decrease, the sharper their negative effect would be on wage. All these effects reinforce wage growth the more economic and firm performance grows.

However, firms are assumed to benchmark the labour market in order to balance their effects on wages. The effects of relative wage gap $\frac{w_i(t)}{Avew}$ across all sectors, and labour availability $\frac{LD_i}{LS_i}$ for each sector, are normalized to initial conditions to determine their effects on wage change from labour market $wcLM_i$ as follows:

Eq. 1.214 – Desired fractional change in wage from labour market benchmark (balancing)

$$wcLM_i = \lambda_{LSD}\left(\frac{\frac{LD_i}{LS_i}}{\frac{LD_{0_i}}{LS_{0_i}}}\right) + \lambda_{wgap}\left(\frac{\frac{w_i(t)}{Avew}}{\frac{w_{0_i}(t)}{Avew_0}}\right)$$

Where LD_i is the labour demand for each sector, LS_i is the labour supply for each sector, $w_i(t)$ is the wages paid from each sector, and $Avew$ is the weighted average wage across all sectors. This latter is determined as follows:

Eq. 1.215 – Average Wage

$$Avew = \sum_{\Pi} \frac{L_i \times w_i(t)}{\sum_{\Pi} L_i}$$

Where L_i is the labour for each sector, $\sum_{\Pi} L_i$ the total labour force, and $w_i(t)$ the wage for each sector.

According to the non-linearities of Eq. 1.214, these are assumed to have no effect when the employment rate $\frac{LD_i}{LS_i}$ matches the initial employment rate $\frac{LD_{0i}}{LS_{0i}}$, and when the wage gap $\frac{w_i(t)}{Avew}$ matches the initial wage gap $\frac{w_{0i}(t)}{Avew_0}$.

Thus λ_{LSD} acts to increase wages when labour supply is in shortage, and decreases it in the opposite case. It is worth noting that the combined effect of labour productivity π_{L_i} growth on wages would tend to balance out. In fact, every increase in productivity would generate a corresponding reduction in labour demand for that sector, leaving more people without jobs.

The non-linear effect of λ_{wgap} acts as balancing force to keep the wage gap $\frac{w_i(t)}{Avew}$ constant among all sectors. If this effect would be neglected, it would be possible to look at wage inequality dynamics between different industries due to the reinforcing effects generated by differences in firm performance.

All these effects are used to model the indicative wage as:

Eq. 1.216 – Indicated wage

$$Indw_i = w_i(t) \times (1 + wcLM_i + wcFEP_i)$$

This allows the determination of wages $w_i(t)$, considering the time required to adjust those $AdjTw_i$, as:

Eq. 1.217 - Wage

$$w_i(t) = smooth(Indw_i, AdjTw_i)$$

LABOUR SUPPLY AND LABOUR MARKET

Figure 1.40 shows the modelling of labour supply LS_i in each firm sector, and the way in which workers move from sector to sector in the ERRE model. In the ERRE model, total labour force is represented by working age population WAP, a fraction of total population, which in turn, is an exogenous variable to the system. The labour supply LS_i of each sector is considered to be a fraction FL_i. of total working population. Thus, the modelling of labour supply and labour market is the definition of a dynamic theory explaining the mobility of workers among sectors, as the description of the variability of such a fraction FL_i. Thus, labour supply LS_i is determined as follows:

Eq. 1.218 – Labour Supply

$$LS_i(t) = FL_i(t) \times WAP$$

Where WAP is the exogenous working age population, and $FL_i(t)$ the stock describing the fraction of labour willing to work in that particular sector.

The fraction for labour in each sector is assumed to decrease because of the fraction of workers departing from that sector FDR_i, and increase because of the fraction of workers arriving to that sector FAR_i. Considering that the majority of workers leaving a certain firm would seek a job in a company working in the same sector, it is assumed that, in general, those arriving are anchored to those leaving. In, particular the fractional departure rate FDR_i is calculated as follows:

Eq. 1.219 – Fractional departure rate

$$FDR_i = FL_i(t) \times NDR_i \times \lambda_{FDR-w}\left(\frac{\frac{w_i(t)}{Avew}}{\frac{w_{0_i}(t)}{Avew_0}}\right) \times \lambda_{FDR-LSD}\left(\frac{\frac{LD_i}{LS_i}}{\frac{LD_{0_i}}{LS_{0_i}}}\right)$$

Where $FL_i(t)$ is the fraction of labour in each sector, NDR_i is the normal fractional departure rate parameter, $\frac{w_i(t)}{Avew}$ is the wage gap, $\frac{LD_i}{LS_i}$ is the labour demand to supply ratio, λ_{FDR-w} is a non-linear effect of the relative wage gap on departures, and $\lambda_{FDR-LSD}$ is the effect of the relative labour demand to supply ratio gap on labour departure. In ERRE, both non-linear relationships have a neutral effect on departure (multiplier as 1) when they match initial conditions, and are assumed to decrease non-linearly while the inputs increase. This shows that the more a sector demands labour, and pay higher wages than average, the less workers would be willing to leave those sectors. In addition, the effect of demand for workers is assumed to be stronger than that of relative wages.

Thus, each sector would have its own attractiveness for arrivals $LAttr_i$, such that:

Eq. 1.220 – Attractiveness of labour

$$LAttr_i = FL_i(t) \times NAR_i \times \lambda_{FAR-w}\left(\frac{\frac{w_i(t)}{Avew}}{\frac{w_{0_i}(t)}{Avew_0}}\right) \times \lambda_{FAR-LSD}\left(\frac{\frac{LD_i}{LS_i}}{\frac{LD_{0_i}}{LS_{0_i}}}\right)$$

Where $FL_i(t)$ is the fractional arrival rate, NAR_i is the normal fractional arrival rate parameter, λ_{FAR-w} the non-linearity describing the impact of wage gap $\frac{w_i(t)}{Avew}$ on attractiveness, and $\lambda_{FAR-LSD}$ the non-linear effect of the labour demand to supply ratio $\frac{LD_i}{LS_i}$ on attractiveness. For simplicity, these two latter non-linear effects are assumed to be the inverse of those capturing the effects of wage gaps and labour demand to supply ratio on departures, and the normal fraction of arrivals NAR_i is chosen to be the same as the normal fractional departure rate NDR_i. This corresponds to the assumption of pretending that most people leaving a business would move in the same sector, with little interest from workers to move between sectors.

This allows the calculation of the fractional arrival rate FAR_i such that:

Eq. 1.221 – Fractional arrival rate

$$FAR_i = \sum_{\Pi+H} FDR_i \times \frac{LAttr_i}{\sum_{\Pi+H} LAttr_i}$$

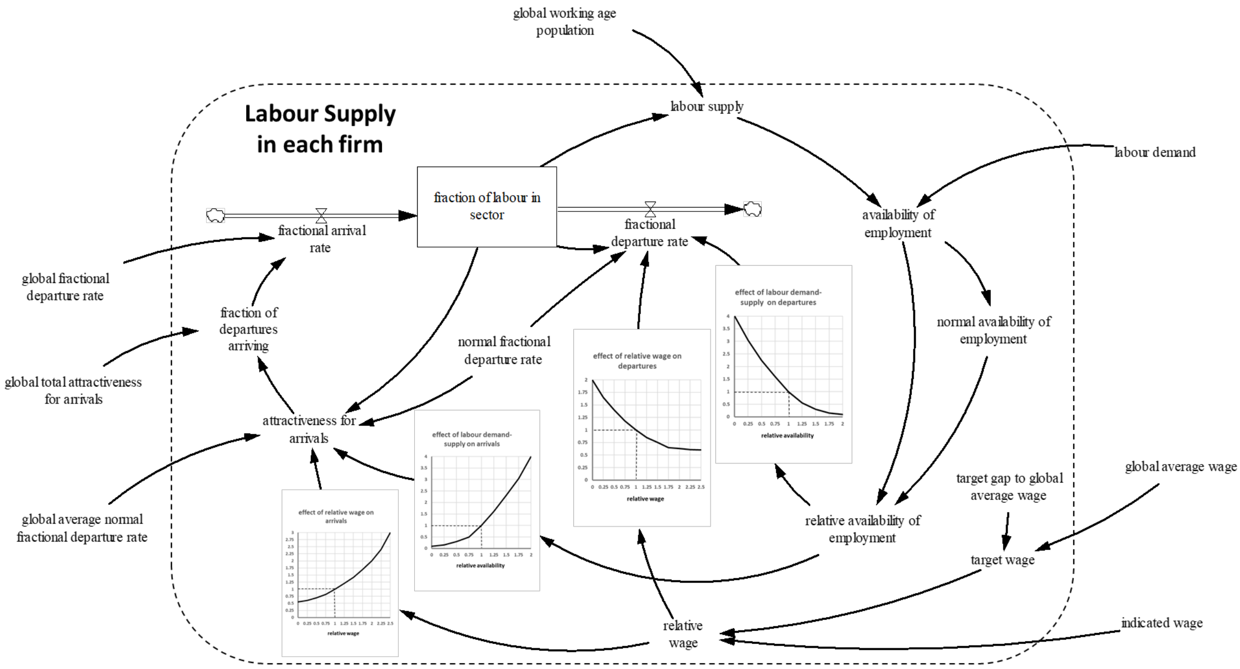
Where $\sum_{\Pi+H} FDR_i$ is the total departure rate, and $\frac{LAttr_i}{\sum_{\Pi+H} LAttr_i}$ the fraction of the total departure arriving in each sector. It is worth noting the equality between the total of all those departing in every sector $TFDR$ and the total of all those arriving in those sectors $TFAR$, such that:

Eq. 1.222 – Total departure and total arrivals for labour

$$TFDR = \sum_{\Pi+H} FDR_i = TFAR = \sum_{\Pi+H} FAR_i$$

Where FDR_i is the fractional departure rate and FAR_i is the fractional arrival rate per sector.

Figure 1.40 – Labour supply per each sector



VOLUNTARY UNEMPLOYED

Figure 1.41 shows the voluntary unemployed workers who provide utility to the household sector. Despite not being paid, their structure is similar to the one of the labour supply of every other sector. In fact, voluntary unemployed labour $VUL_H(t)$ is determined as follows:

Eq. 1.223 – Labour Supply

$$VUL_H(t) = FL_H(t) \times WAP$$

Where $FL_H(t)$ is fraction of workers in a household, and WAP the working age population. Similarly to the other sectors, workers in households are assumed to constantly depart FDR_H and arrive FAR_H from the job market.

Despite the similarities, workers present important idiosyncrasies that are fundamental for the structure of the entire labour market. First of all, they take their decisions based on reference labour market values, both for wages and labour demand to supply ratio. Secondly, and differently to all other sectors, households' debt is assumed to be a third determinant to join the labour market. Third, the non-linear influence of such values to departure and arrival rate is assumed to mirror the ones seen in the firm sector. This is important to assure that, during market upturn, more people would be willing to move into active labour force, and return as unpaid workforce in the opposite case.

The reference wage (marginal opportunity cost of labour mcL_H) considered as a metric of comparison to the value obtained while not working is the global average $Avew$ corrected by the income tax rate τ_H as:

Eq. 1.224 – Marginal opportunity cost of labour

$$mcL_H = Avew \times (1 - \tau_H)$$

This allows the determination of the exponent for labour on utility ε_{H_L}, as previously seen with the other utility factors UF, as:

Eq. 1.225 – Exponent for labour effect on utility

$$\varepsilon_{H_L} = \frac{mcL_H \times L_H}{\sum_{UF} AnExp_{UFj}}$$

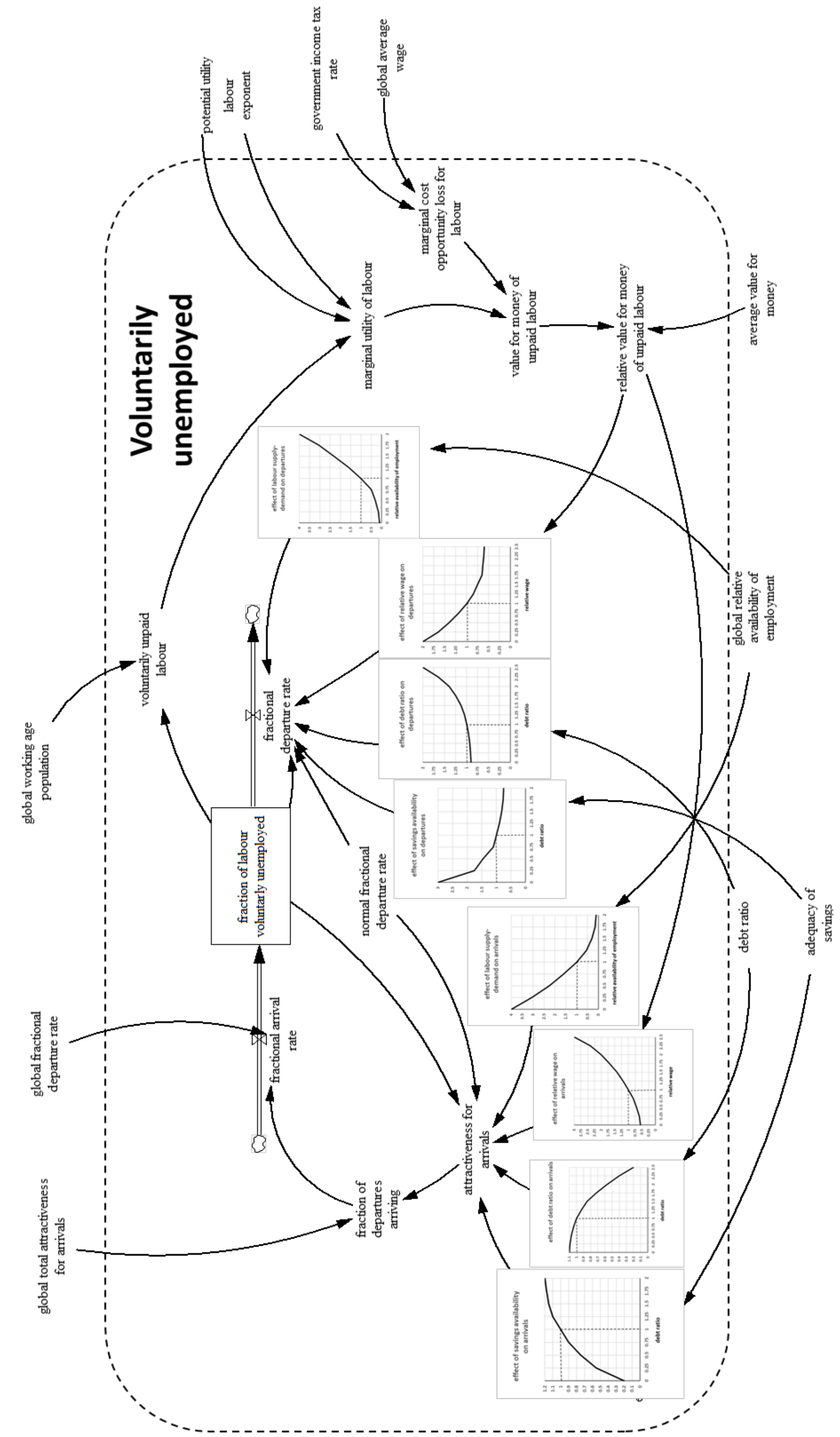
Where L_H is the total voluntary unemployed, $mcL_H \times L_H$ represents the annual cost of people not earning any income, and $\sum_{UF} AnExp_{UFj}$ the total annual expenditure for utility.

In order to determine their effect on both departures and arrivals, a relative value for money of labour $rvfML_H$ is determined as the ratio between the marginal value of labour $\frac{\partial PU}{\partial L_H}$ per unit of cost mcL_H, and the average value of utility $AVfM_H$ as follows:

Eq. 1.226 – Value for money of unpaid labour

$$rvfML_H = \frac{\frac{\frac{\partial PU}{\partial L_H}}{mcL_H}}{AVfM_H}$$

Figure 1.41 – Unpaid labour in the Household sector



The reference value for labour demand to supply ratio $LDSr_H$ is used as the aggregate of all producers, such that:

Eq. 1.227 – Reference labour demand to supply ratio

$$LDSr_H = \frac{\sum_{\Pi} LD_i}{\sum_{\Pi} LS_i}$$

Where $\sum_{\Pi} LD_i$ is the total labour demand, and $\sum_{\Pi} LS_i$ the total labour supply. This allows the determination of the fractional departure rate FDR_H as follows:

Eq. 1.228 – Fractional departure rate in households

$$FDR_H = FL_H(t) \times NDR_H \times \lambda_{FDR-w_H}(rvfML_H) \times \lambda_{FDR-LDS_H}\left(\frac{LDSr_H}{LDSr_{H_0}}\right) \times \lambda_{FDR-D_H}\left(\frac{D_H}{PD_H}\right)$$

Where $FL_H(t)$ is the fraction of workers in households, NDR_H is the normal departure rate from households to the job market, λ_{FDR-w_H} is the non-linear effect of the relative value for money of labour $rvfML_H$ on departures, $\lambda_{FDR-LDS_H}$ is the non-linear effect of the relative labour to supply demand ratio $\frac{LDSr_H}{LDSr_{H_0}}$ on departures, and λ_{FDR-D_H} the non-linear effect of the debt ratio on departures. The non-linearity λ_{FDR-w_H} assumes that the more they perceive unpaid workers as giving positive value to them (i.e. global average wage decreases), the lower the departure rate would be, thereby keeping more workers in households. On the other hand, $\lambda_{FDR-LDS_H}$ indicates that the more the labour demand increases, the more workers would be willing to leave the household sector to support a growing economy. Finally, λ_{FDR-D_H} indicates that the larger the debt D_H in comparison to permissible value PD_H, the more workers would need to go seeking a job, to earn money and pay for that debt.

Similarly to the producers, an attractiveness for household labour $LAttr_H$ is determined as:

Eq. 1.229 – Attractiveness for arrivals in households

$$LAttr_H = FL_H(t) \times NAR_H \times \lambda_{FAR-w_H}(rvfML_H) \times \lambda_{FAR-LSD_H}\left(\frac{LDSr_H}{LDSr_{H_0}}\right) \times \lambda_{FAR-D_H}\left(\frac{D_H}{PD_H}\right)$$

Where $FL_H(t)$ is the fraction of labour in households, NAR_H is the normal fractional arrival rate, and the three non-linear relationships λ_{FAR-w_H}, $\lambda_{FAR-LSD_H}$, and λ_{FAR-D_H} have an effect on arrivals that is the inverse as that for departures. As a result, every increase in perceived value for money of household workers would retain more people in the sector, and every increase in debt above permissible level, and in demand for labour in the economy, would motivate people to leave the household sector in search of jobs.

The fractional arrival rate FAR_H is finally determined as follows:

Eq. 1.230 – Fractional arrival rate in households

$$FAR_H = \sum_{\Pi+H} FDR_i \times \frac{LAttr_H}{\sum_{\Pi+H} LAttr_i}$$

Where $\sum_{\Pi+H} FDR_i$ is the total departure rate, and $\frac{LAttr_H}{\sum_{\Pi+H} LAttr_i}$ the relative attractiveness of households.

DIVIDENDS

The final decision that every firm can take, after having paid all energy and capital suppliers, service their debt to banks, paid taxes to government, and wages to their workers, is on how much of the net profit should be left as liquidity for reducing their risk of default, and how much it should be distributed as dividends among shareholders.

Figure 1.42 shows the dividend payment Φ_i as composed of two elements:

1. Normal dividends $N\Phi_i$ – the normal payments based on profit gains, and propensity of firms to distributed or retain their earnings
2. Bonus dividend $B\Phi_i$ – the additional payments that are provided in times of liquidity abundancy in relation to their goals for return on investments.

The income measured to distribute dividends $NI\Phi_i$ is determined based on expected performance as follows:

Eq. 1.231 – Net Income for dividends

$$NI\Phi_i = smooth(AdjNI_i, AdjTNI_i) \times (1 + TRNDJ(AdjNI_i) \times AdjTNI_i))$$

Where $AdjNI_i$ is the inflation adjusted net income, $AdjTNI_i$ is the time used to perceive change in net income, and $(1 + TRNDJ(AdjNI_i) \times AdjTNI_i)$ represents the trend adjustment for dividends distribution.

The indicated dividend pay-out ratio $\varphi_i(t)$ is a ratio between 0 and 1 that determines the desired fraction of income normally distributed to shareholders. In ERRE, it is assumed that liquidity availability is the force of disequilibrium for dividend payment expectations, whereas the gap in return on investment, and a normal expected value for dividend payments generates convergence and balance. Thus the desired dividend pay-out ratio ${\varphi^*}_i$ is determined as

Eq. 1.232 – Desired dividend pay-out ratio

$${\varphi^*}_i = \varphi_i(t) \times \left(1 + \lambda_{roi}(GapROI_i) + \lambda_{\Lambda}\left(\frac{\Lambda_i}{{\Lambda^*}_i}\right) + \lambda_{N\varphi}\left(\frac{\varphi_i(t)}{N\varphi_i}\right)\right)$$

Where $\varphi_i(t)$ is the current dividend pay-out ratio, λ_{roi} represents the fractional change due to the return on investment performance, λ_{Λ} represents the fractional change due to liquidity performance, and $\lambda_{N\varphi}$ the fractional change due to normal desires for dividend payments. Whereas λ_{Λ} assumes that the larger the liquidity, then firms would tend to pay their dividends, λ_{roi} assumes if the performance of a firm goes beyond their goal for returns, they would generally reduce payments to balance towards their goal. Finally, it is assumed that the ratio between dividend pay-out and normal dividend pay-out $\frac{\varphi_i(t)}{N\varphi_i}$ would generate balance to the equation. In fact, based on the non-linearity $\lambda_{N\varphi}$, every time dividends would register a discrepancy in comparison to normal values, there would be a tendency to move payments back to normal.

The indicated dividend pay-out ratio $\varphi_i(t)$ is the smooth on desired, calculated as:

Eq. 1.233 – Dividend pay-out ratio

$$\varphi_i(t) = smooth(\varphi^*{}_i, AdjT\varphi_i)$$

Where $\varphi^*{}_i$ is the desired dividend pay-out ratio, $AdjT\varphi_i$ the time required to adjust dividends.

The normal dividend payments $N\Phi_i$ are determined as follows:

Eq. 1.234 – Normal Dividends

$$N\Phi_i = NI\Phi_i \times \varphi_i(t) \times \mathfrak{f}_{N\Phi}\left(\frac{\Lambda}{\Lambda^*}\right)$$

Where $NI\Phi_i$ is the net income for dividends, $\varphi_i(t)$ the indicated dividend pay-out ratio, and $\mathfrak{f}_{N\Phi}\left(\frac{\Lambda}{\Lambda^*}\right)$ the constraining decision to payments when liquidity is below desired values.

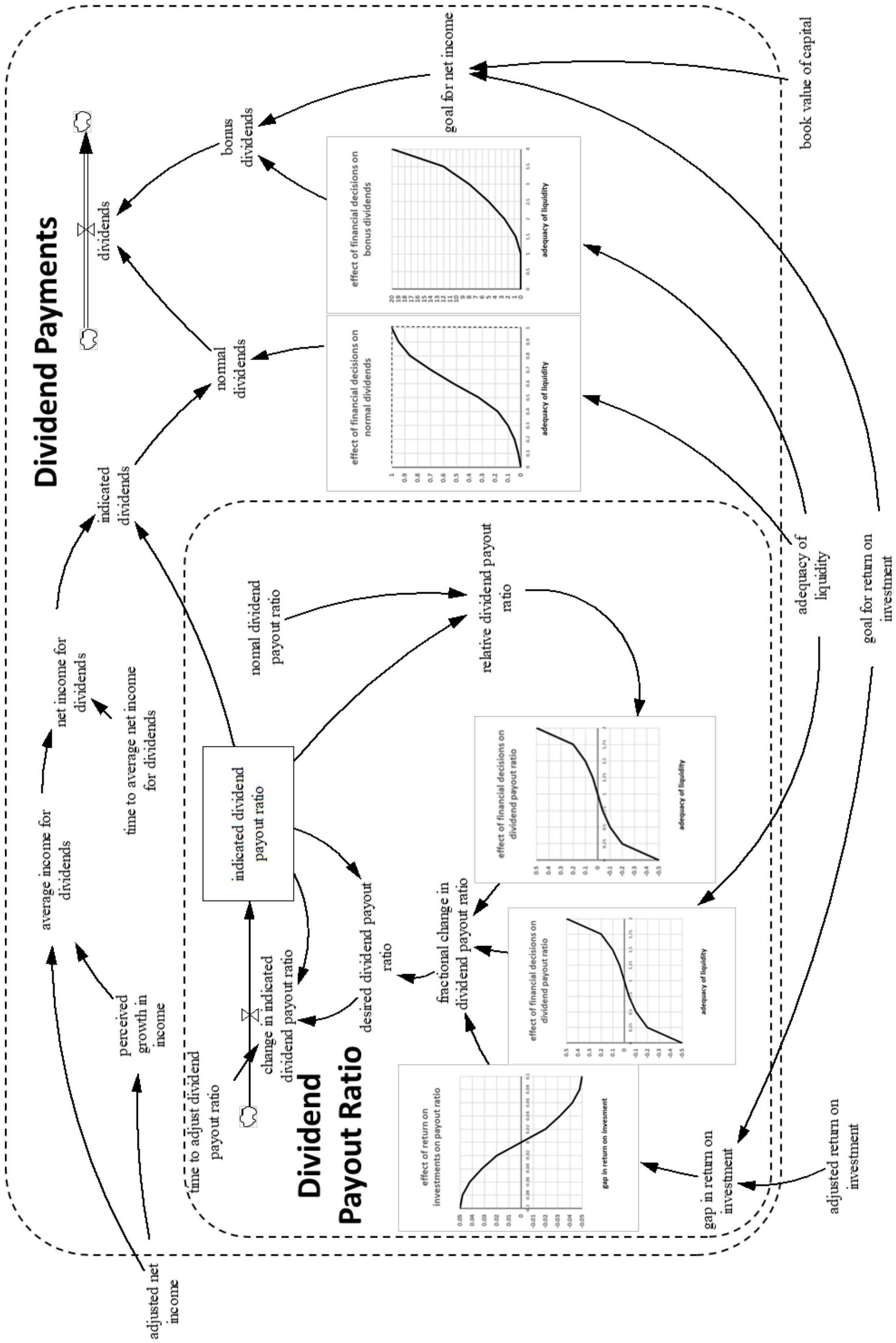
On the other hand, bonus dividends $B\Phi_i$ is determined as follows:

Eq. 1.235 – Bonus Dividends

$$B\Phi_i = gROI_i \times AdjA_i(t) \times \mathfrak{f}_{B\Phi}\left(\frac{\Lambda}{\Lambda^*}\right)$$

Where $gROI_i \times AdjA_i(t)$ represents the desired bonus dividends from a shareholders' perspective, and $\mathfrak{f}_{B\Phi}$ determines whether or not these desires can be fulfilled based on liquidity availability. This latter non-linear relationship based on liquidity adequacy, has a very different meaning in comparison to all others used to control cash flows. In fact, $\mathfrak{f}_{B\Phi}$ is assumed to stop any bonus payments when liquidity is below desired levels, but exponentially support bonuses until liquidity increases to four times above desired levels, generating a $20\times$ multiplier on bonus dividends.

Figure 1.42 – Dividends



The dividend payment to households Φ_i, as registered to the balance sheet of a firm, is calculated as the sum between normal $N\Phi_i$ and bonus dividends $B\Phi_i$ as follows:

Eq. 1.236 - Divided payments

$$\Phi_i = N\Phi_i + B\Phi_i$$

This allows the calculation of the retained earnings/losses REL_i from firms as the difference between all inflows $\sum_{IN-CF} \mathrm{cf}_i$ and outflows $\sum_{OUT-CF} \mathrm{cf}_i$ as:

Eq. 1.237 – Retained earnings and losses

$$REL_i = \sum\nolimits_{IN-CF} \mathrm{cf}_i - \sum\nolimits_{OUT-CF} \mathrm{cf}_i$$

As in Eq. 1.47, the sum of dividends distributed from all firms, closes the loop to determine the income before tax to households.

Ecological limits in ERRE

The previous sections have outlined how the Government, Banking, Firms and Households are modelled. Whereas both government and banks are represented as a global aggregated accounting system of all public and banking systems, Firms and Households have been represented in much finer detail. These are separated among six sub-sectors (fossil fuels, renewables, agriculture, capital, goods and services, households), and represented from the top down perspective of their balance sheets and financial obligations, to the detailed description of their decisions and human biases based on their perceived performance. As part of the fossil fuel and agriculture firm sectors, elements of their ecosystems (i.e. fossil fuel reserves, and agricultural land limits) are also taken into account.

These two latter sub-systems are represented in order to link the effect of economic performance to the evolution of the ecosystems in relation to physical limits in the ERRE model, and should be seen from the global aggregated system perspective, accounting for important imminent feedback loops such as depletion and cost increases under business as usual conditions.

In the chapters 1, 2 and 3 of Pasqualino and Jones (2020), the Limits to Growth model has been explored and compared, in terms of its simulations, to the evolution of real systems up until the present time. In particular, it has been shown how the limits of persistent pollution, mineral resources and agricultural land evolved from 1972 until today such that:

1. Mineral resources have been proven to be available in abundance, pushing the pressure on limits on fossil fuel energy resources constraints
2. Agricultural land erosion and urban land development persisted over the years, but land fertility is in better shape than forecasted in the standard run of the Limits to Growth. This includes the decrease in the effect of Non-Persistent organic pollutants on food production at the global scale.
3. Persistent pollution had a lower impact than forecast in the Limits to Growth, mostly due to changes in industrial practice. These include the banning of chemicals (such as DDT) in most countries of the world in the 1970s, and the plateau in radiation emitted from nuclear energy waste after the Chernobyl tragedy of 1986.

It is worth noting that the effect on the social awareness around global limits that Limits to Growth galvanized has contributed to the reduction of impact of those limits, potentially allowing today's world to avoid, or at least delay, some of the scenarios provided from the World3 model (Meadows et al. 1972). Unfortunately, because growth has not stopped, and population has kept growing, new limits have emerged, alongside the continued physical limits of fossil fuels and land erosion, such as the problem of global warming.

Thus, in the ERRE model, three ecological limits to growth are modelled:

1. Fossil fuel reserves limitation and their impact on the global economy
2. Agricultural land limits, including forest land and land erosion
3. Climate change and the possible impact of a temperature anomaly to food production.

In the previous section, the economic modelling of ERRE has been described, presenting the structure of fossil fuel depletion (see Figure 1.23) and agricultural land limits (see Figure 1.28 and Figure 1.29). In this section, we shall step back to the global perspective on systems, describing the potential implication of the negative feedback loop generated from economic growth, with implications back to the economy due to climate change.

CLIMATE CHANGE AND TEMPERATURE ANOMALY

In the ERRE model, greenhouse gases are assumed to be emitted to the atmosphere through four channels:

1. Production from burning fossil fuels
2. Direct emissions from agriculture due to production
3. Indirect emissions from agriculture due to net deforestation
4. Emissions from the ocean as a result of the carbon cycle.

The first three can be considered to sum up as the anthropogenic greenhouse emissions, whereas the latter is the result of natural processes involving carbon. In so doing, important feedback loops are neglected such as the carbon cycle with the soil. However, for the simplicity, such an effect can be considered embedded among agricultural capital production, deforestation and ocean carbon cycle. The ultimate purpose is to test possible consequences of the hot house effect (Steffen et a;. 2018) and provide a preliminary assessment of climate change consequences on financial risk and the real economy.

TEMPERATURE ANOMALY AND IMPACT ON FOOD

Greenhouse emissions $ghgP$ from production of agriculture and fossil fuels is calculated as follows:

Eq. 1.238 – Greenhouse gases emissions from production

$$ghgP = gP_N \times \frac{ghg_{N_0}}{gP_{N_0}} + K_{F+F_e} \times \frac{ghg_{F+F_{e_0}}}{K_{F+F_{e_0}}}$$

where gP_N is the gross production of fossil fuels, $\frac{ghg_{N_0}}{gP_{N_0}}$ is the conversion factor from fossil fuel units to greenhouse equivalent units initialized to historical data, K_{F+F_e} is the amount of capital in use in the agriculture sector (inclusive of chemicals and equipment), and $\frac{ghg_{F+F_{e_0}}}{K_{F+F_{e_0}}}$ the conversion factor from agricultural production to emissions, initialized to historical data.

The greenhouse gas emissions from net deforestation $ghgNDef$ is calculated as follows:

Eq. 1.239 – Greenhouse gases emissions from net deforestation

$$ghgNDef = (ForLDef_{F+F_e} - ForLAff_{F+F_e}) \times \frac{ghgNDef_0}{(ForLDef_{F+F_e} - ForLAff_{F+F_e})_0}$$

Where $ForLDef_{F+F_e}$ represents the real deforestation (or agricultural land development) in the agricultural sector, $ForLAff_{F+F_e}$ is the forest land afforestation, consisting of regeneration from fallow land to forest land, and $\frac{ghgNDef_0}{(ForLDef_{F+F_e} - ForLAff_{F+F_e})_0}$ the conversion factor of every hectare of net forest land lost into greenhouse gas equivalent.

These greenhouse gases are assumed to accumulate in the atmosphere as $ghgAtm(t)$ and directly impact the temperature anomaly $TA(t)$ as follows:

Eq. 1.240 – Temperature anomaly to preindustrial levels

$$TA(t) = ghgAtm(t) \times \frac{ppmCO2Eq_0}{ghgAtm_0} \times \frac{TA_0}{ppmCO2Eq_0}$$

Where $ghgAtm(t)$ represents the greenhouse gases in the atmosphere, $\frac{ppmCO2Eq_0}{ghgAtm_0}$ is a conversion factor from of the carbon in the atmosphere in parts per million of CO2 equivalent, and $\frac{TA_0}{ppmCO2Eq_0}$ the conversion factor to generate the temperature anomaly. This type of conversion was necessary given the difficulty in estimating an initial value of total greenhouse gases in the atmosphere $ghgAtm_0$, and allowing the calibration to historical data to generate a meaningful value for such a parameter.

As shown in Figure 1.43 the temperature anomaly has a non-linear negative impact on food production in progress $\chi PiP_{F+F_e}(t)$ with larger consequences the bigger the temperature increase, and can be calculated as:

Eq. 1.241 – Effect of Temperature anomaly on Food production Loss

$$\chi PiP_{F+F_e}(t) = PiP_{F+F_e} \times \lambda_{GHG}(TA(t))$$

Where PiP_{F+F_e} is the production in progress of food, and λ_{GHG} the non-linear effect of climate change on food production. This effect is assumed to have no impact until the temperature rise is above +1 degree Celsius on preindustrial levels (multiplier as 1), and slowly increasing the higher the temperature. Two scenarios are considered in which λ_{GHG} decreases until reaching 80% of current production at +4 degrees Celsius and remaining constant for higher temperatures, and a scenario in which production in progress would reach the 70% of total at +4 degrees and keep decreasing until reaching 60% of production at temperature increases of +6 degrees. All these assumptions have been extrapolated from IPCC (2014).

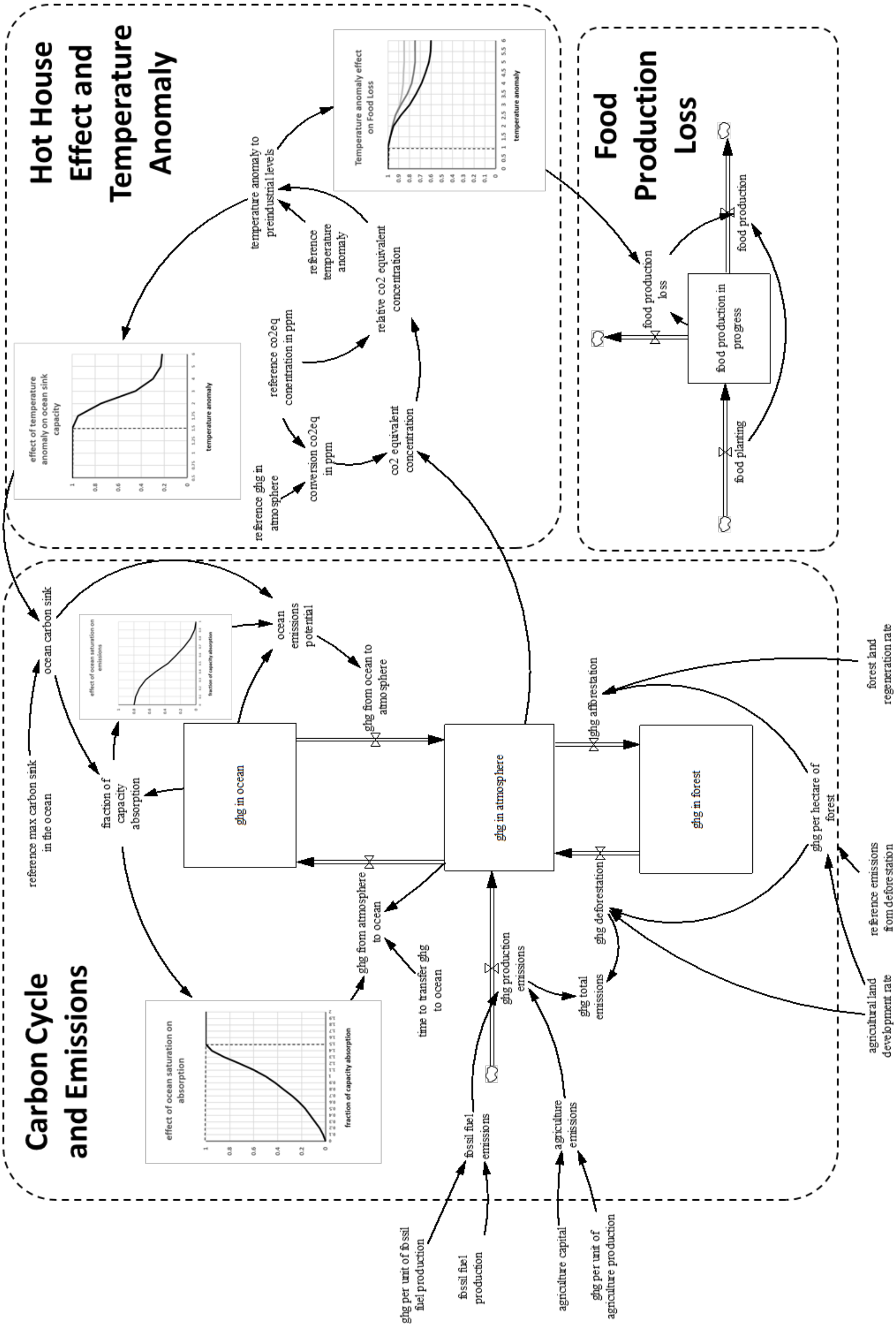
Such lost in production would result in lower production P_{F+F_e} reaching the inventory and the market as described in the following equation:

Eq. 1.242 – Food production from climate change

$$P_{F+F_e} = Delay3(Pl_{F+F_e}, FPT^*) - Delay1(\chi PiP_{F+F_e}(t), FPT^*)$$

Where Pl_{F+F_e} is the total food and biofuel planting, FPT^* is the food production time, and $\chi PiP_{F+F_e}(t)$ the production loss due to temperature rise.

Figure 1.43 – Climate change and temperature anomaly



HOT HOUSE EFFECT FEEDBACK LOOP AND UNCERTAINTY FOR FOOD SYSTEM

In the business as usual scenario, the maximum carbon sink from the ocean $MaxghgOce$ is assumed to be very large in comparison to carbon in the atmosphere $ghgAtm(t)$ and thus does not alter its absorption capacity of CO2 equivalent from the atmosphere. However, two scenarios are tested to address the hot house effect hypothesis (Steffen et al. 2019):

1. The maximum carbon sink from the ocean is a variable that decreases non-linearly with temperature rise, and after a certain threshold stops absorbing greenhouse gases from the atmosphere.
2. In addition to the previous scenario, an additional effect is introduced, in which the tipping point on carbon emissions is reached, and the ocean becomes a net emitter of greenhouse gases.

Due to the large uncertainty in estimating these parameters and non-linear effects, scenarios are run as a sensitivity analysis to show the potential of the ERRE model. Thus the maximum carbon sink from ocean $MaxghgOce(TA)$ is calculated as follows:

Eq. 1.243 – Maximum carbon sink from ocean

$$MaxghgOce(TA) = MaxghgOce \times \lambda_{TA}(TA(t))$$

Where $MaxghgOce$ is a constant value representing the normal maximum carbon sink, and the λ_{TA} is a non-linear relationship similar to the one used in the Limits to Growth, showing that below a +1.5 degrees rise in temperature, the effect would be neutral, and decreasing non-linearly reaching 25% with a temperature rise of +4 degrees.

A second relationship describing the absorption capacity of ocean from atmosphere $ghgAtmToOce$ is modelled as follows:

Eq. 1.244 – GHG from atmosphere to ocean

$$ghgAtmToOce = \frac{ghgAtm(t)}{OceAbsT} \times \lambda_{Oce-Abs}\left(\frac{MaxghgOce(TA)}{ghgOce(t)}\right)$$

Where $\frac{ghgAtm(t)}{OceAbsT}$ represents the normal absorption rate from atmosphere $ghgAtm(t)$ at a average constant time $OceAbsT$, and $\lambda_{Oce-Abs}$ is a non-linear relationship that assumes that, when the maximum capacity of absorption $MaxghgOce(TA)$ would be greater than 1.5 times the actual carbon in the ocean $ghgOce(t)$, the effect would be neutral, and decreases non-linearly reaching 50% of absorption capacity when the two elements equals each other, and decreases to zero in the case when maximum absorption is reduced to zero.

In a similar way the ocean emissions to the atmosphere $ghgOceToAtm$ are modelled as follows:

Eq. 1.245 – GHG Ocean to ATM

$$ghgOceToAtm = \frac{ghgOce(t)}{OceEmiT} \times \lambda_{Oce-Emi}\left(\frac{MaxghgOce(TA)}{ghgOce(t)}\right)$$

Where $\frac{ghgOce(t)}{OceEmiT}$ is the normal emission capacity from ocean and $\lambda_{Oce-Emi}$ a non-linear relationships that reduces to zero till the carbon sink $MaxghgOce(TA)$ is equal or greater than the actual carbon in the ocean $ghgOce(t)$, and starts increasing non-linearly until reaching 80% of normal absorption when the maximum absorption is zero $MaxghgOce(TA)$.

The combinations of these effects generate four areas of interest for the simulation:

1. $MaxghgOce(TA) > 1.5 \times ghgOce(t)$ – where ocean behave as a constant carbon sink
2. $ghgOce(t) < MaxghgOce(TA) < 1.5 \times ghgOce(t)$ – where the absorption capacity of the ocean slows down
3. $0.5 \times ghgOce(t) < MaxghgOce(TA) < ghgOce(t)$ – where absorption capacity decreases, and then oceans start emitting carbon to the atmosphere
4. $0 < MaxghgOce(TA) < 0.5 \times ghgOce(t)$ – where the ocean emission is very high with very limited absorption capacity

All these effects are worsened by a temperature rise, as this decreases the maximum carbon sink $MaxghgOce(TA)$.

2. Tests towards validation

Appendix summary

This appendix is supplementary material to Chapter 5 and 6 of Pasqualino and Jones (2020). In particular, it provides a series of behavioural tests to demonstrate the behaviour of the model emerging from the structure explained in Chapter 5 and supports its validity to the analysis performed in Chapter 6. Most important it allows to evaluate the behaviour of the model in terms of the economic theory, thus supporting the conclusion provided in Chapter 7.

Tests

Forrester and Senge (1980), provides a list of twenty-one tests to support building confidence in dynamic computer models. Sterman (2000) summarises the same tests as twelve, despite giving more emphasis to statistical analysis of the differences between model behaviour and historical data, and showing concern for numerical integration techniques in system models. As described in Barlas (1989), Barlas and Carpenter (1990), Barlas (1996), validation in a system dynamics model is a process that starts when the model development begins, encompassing the merely technical aspect of testing. In particular, models should not be assessed for their validity, but for their ability to fulfil a specific purpose. Sterman (2000) argues that because all models are wrong, there is no meaning in the arguing if a model can be considered valid. Validity is synonym of truth, and no model can never be considered true, due to its infinite gap to the reality it tries to capture. Models should be assessed for their degree of usefulness, relatively to the purpose of the model, ultimately being capable of influencing decisions towards better functioning of real world systems.

The number of tests that should be applied to a dynamic model multiplies with the size of the model. In the case of ERRE, many of those tests have been constantly performed as part of model development. In fact, the modelling process should be considered an iterative cycle involving (i) formulation of a dynamic hypothesis, (ii) definition of the structures that underpin that hypothesis, (iii) the simulation test of the model, and (iv) the comparison between the mental expectation of the model builder (or the client) and the actual model performance. This would generate correction in the dynamic hypothesis or in the model itself, initiating a cycle that ends when all desired tests are considered to be passed in relation to the purpose the model was created for.

The purpose of the ERRE model is to address the difficulties of the financial sector in dealing with a finite planet in the long term, providing structures that could capture the effects of short term shocks cascading through the dynamics of the system, and fill the gap between this model and the economic theory.

In this section, not all tests as proposed by Forrester and Senge (1980), Barlas (1996) and Sterman (2000) can be presented due to the size of the model and its scope. However they have all been discussed and addressed both qualitatively and quantitatively whenever possible. Additional tests to the one present in the literature have been considered in order to compare the model characteristics to economic theory and strengthen the gap existing between system dynamics models and influence to decision making.

In so doing, five classes of tests are defined, some requiring qualitative assessment of the model, other requiring both qualitative and quantitative (e.g. simulation) assessment, and some merely quantitative assessment as follows:

1. Structural validity (qualitative)
2. Dynamic disequilibrium behaviour (both qualitative and quantitative)
3. Stock and flow and technical consistency (quantitative)
4. Data definition and base run formulation (both qualitative and quantitative)
5. Policy scenarios, extreme tests and shocks (both qualitative and quantitative)

These classes should not be seen as sequential one to another, but iterative, each one dependent on the others. In particular, if a model is considered as not being structurally consistent for its purpose, edits in the structure have to be performed. This would lead to change in the dynamic behaviour of the system, which, if not convincing, would be demanding additional changes in the structures until the model can generate the required dynamics convincingly. Because the model is a computer model, numerical calculation and economic consistency in the structure have all to be considered in each simulation. These tests, are fundamental to assess if the model is technically correct, and if flaws in the system are found these have to be corrected, leading back to structural validity and relative dynamic behaviour. Calibration and historical behaviour reproduction represent additional elements, that when not passed would need to recall structural changes. Finally, if the model does not behave consistently with reality once policies are implemented, changes have to be adopted, leading back to the entire chain of tests.

In the attempt of reusing existing structures from the Energy Transition and the World3-03 models, all these tests have been performed cyclically, until the model was able to pass all those tests simultaneously. In this section, all these tests and resulting scenarios are described.

Structural validity

This first set of tests take the same meaning of those proposed as 'Tests of model structure' in Forrester and Senge (1980). These tests require human judgement and comparison between the mental model of the potential users of the model, and the model structure itself. As a result, while targeting the economic and policy community as final beneficiaries of this model, the model is tested against economic theory, and made consistent to physical reality in the most rigorous way. Despite these steps in the validity requiring a lower set of technical skills, it requires the engagement of the client in the modelling process, and often generates difficulty in the agreement between certain communities.

For the scope of ERRE the structural validity is defined in five steps:

1. Knowledge base validity
2. System boundaries validity
3. Parameterization validity
4. Extreme conditions and non-linear effects validity
5. Dimensionality check

KNOWLEDGE BASE VALIDITY

With the term knowledge base, it is meant the set of beliefs on how the world actually works, based on the understanding of such systems, and represented via numerical equations in the computer model. As a result, both the relationships among variables, their mutual effects and linkages, parameters and non-linearities represent all elements to be used as a metric for the knowledge base test of the model structure.

The knowledge base of the ERRE model is fully formalized in Chapter 5 of Pasqualino and Jones (2020), and represents the foundation for addressing system policies and other tests that follows in this appendix. It is the result of the iteration of model testing and structure adjustment towards a complete theory of the world system.

SYSTEM BOUNDARIES VALIDITY

The system boundaries test consists in evaluating if the ERRE model contains the correct set of elements, as well as capturing their granularity such that it can fulfil its purpose. Similarly to the Knowledge base structural test, the structure proposed in Chapter 5 of Pasqualino and Jones (2020) is assumed to be sufficiently extended to fulfil such a purpose.

However, more specific research questions might involve restructuring the model to be considered valid in other contexts. For example, the model does not disaggregate countries as separate entities, therefore it presents constraints in the study of the dynamics across countries. Because households are not distinguished across groups, the ERRE model is not currently able to study inequality between people and between countries in the world. All these questions would require structural changes in the current model. However, it is considered that the ERRE provides a family structure such that its application to diverse questions would require lower effort than the one used to develop the ERRE itself.

PARAMETERIZATION VALIDITY

In the ERRE model, all the parameters have been chosen and set in realistic ranges based on the modeller’s knowledge of real world systems. This is a big difference from regression type models, in which the value of structural parameters is given by the analytical solution of an equation while fitting the model to historical data. In ERRE, the approach has been the opposite. The model has been constructed based on the observation of the structure of the real world systems and decision making, and the parameters have been placed in ranges based on such an understanding. While keeping parameters within those ranges, the model is assumed to portray some world dynamics correctly within a certain degree of confidence.

As a result, all quantitative tests in the model are constrained by the realistic value of those parameters, determining all possible results both in policy recommendation and calibration to historical data.

EXTREME CONDITIONS AND NON-LINEAR EFFECTS VALIDITY

As the parameters need to be placed within plausible ranges, all non-linear relationships in the model, resulting in the non-linear feedback among system elements, must be plausible in terms of their extreme ranges and feedback forces. Such a test can be applied to every non-linear decision variable, including all financial decisions, borrowing decisions, and production orders decisions among others. Despite the requirements to perform some sensitivity analysis, it is felt that all non-linear relationships defined within ERRE pass this test.

A particular case of extreme condition test at the structural level involves the inclusion of the climate change and resource constraints modules with the relative feedback structures to the economy. Despite data about the precise extreme impacts of these constraints not being well studied (and not having been historically observed), their structure has been provided and tested during sensitivity and extreme scenario analysis.

DIMENSIONALITY CHECK

The dimensionality check involves the assurance that the unit of measurement applied to every single element and variable in the model, is consistent among each other and with the real world. Despite being often neglected in modelling work, this test is helpful to reveal inconsistencies in model formulation, and results in being particularly useful in the context of large models such as the ERRE. Thus the model has been tested in terms of each unit of measurement successfully.

Dynamic disequilibrium behaviour

The second group of tests involves the quantitative assessment of model behavioural consistency, and requires skills that go deeper in the understanding of systems. These involve simulation, behavioural tests, and demonstration of disequilibrium. The tests performed in this section represent the minority of those captured under the category of 'Tests of Model behaviour' in Forrester and Senge (1980). The focus is on those fundamental leverage points that generate behaviour in the context of the ERRE calibration, and show the dynamic behaviour to address the fit of the ERRE in economic literature. In this section these effects are captured via sensitivity analysis on key disequilibrium factors in the model, and briefly discuss the additional tests proposed in Forrester and Senge (1980) for system dynamics models. This section is divided in the categories of:

1. Exogenous disequilibrium
2. Endogenous disequilibrium
3. Exogenous short term shock
4. Other behavioural tests

It is worth noting that the demonstration of disequilibrium of the ERRE model, requires starting the analysis from the equilibrium condition in order to show the differences lying in comparison to the general equilibrium theory. However, the analysis and policy assessment remain based in disequilibrium dynamics as follows.

EQUILIBRIUM AS BASE FOR TESTING

The equilibrium (or balance) condition as base for testing the ERRE model was adopted to demonstrate with the simplest degree of clarity the sources of disequilibrium, both endogenous and exogenous, in the model. Such a condition is often used in System Dynamics models to demonstrate behaviours when perturbing the system with exogenous shocks. This condition is different from the equilibrium philosophy generally adopted by the neo-classical school in defining general equilibrium as foundation for their models.

In ERRE, equilibrium is a condition of perfect dynamic balance between every in- and out-flow for each stock in the model. Because sectors behave deterministically, it is possible to set the model such that every agent spends as much as they receive as income, and purchases assets as much as those assets discard. Such a test allows to show the mathematical consistency of the system architecture that can be reached with a deterministic model such as the ERRE. Failing to pass this test would reveal structural inconsistencies in its formulation.

Such an equilibrium can be broken with exogenous elements such as population growth or exogenous government debt creation to stimulate consumption and growth, technology improvement or energy efficiency, and others. It is worth noting that the application of these exogenous elements is not sufficient condition to classify the ERRE model as a disequilibrium model, as every general equilibrium model can be exogenously unbalanced towards growth in the same way.

The reason why the ERRE model is a disequilibrium model can be established via endogenous structural elements, that, when placed in an out-of-balance condition, would generate dynamics of disequilibrium without any other perturbance in the system. These are demonstrated by (i) varying the behaviour of firms in distributing dividends against investments, (ii) the sensitivity analysis of the behaviour of households in accumulating savings against consumption, and (iii) applying ecological constraints in the areas of resources, land and climate in the model. Other structural disequilibrium elements could be applied by varying the balance point in every non-linear relationship in the model, including both financial decisions and investment decisions of each agent. In addition to this, the model can be tested against noise and stochastic components. Despite not being considered here for the purpose of the analysis, such a test would reveal that, despite the attempt to place the model in balance position, noises could generate disequilibrium, with possibilities to determine multiple long term equilibria due to the time lags and path dependencies in the system.

Table 2.1 – Sources of disequilibrium in the ERRE model

Exogenous	**Endogenous**
Population	Propensity for savings
Technology – Labour Productivity Growth	Dividend pay-out ratio
Energy efficiency	Fossil Fuel Depletion
Exogenous money creation	Climate change impact on food
Government debt money creation	Agricultural land erosion and cost

The next section compares the equilibrium condition with every exogenous disequilibrium and endogenous disequilibrium.

EXOGENOUS DISEQUILIBRIUM

In this section the effects of the exogenous disequilibrium elements in the ERRE model are assessed. Each disequilibrium is applied in isolation in comparison to equilibrium condition. The four elements are:

1. Population growth
2. Money creation (both government debt and exogenous money creation)
3. Technology change
4. Energy efficiency

POPULATION CHANGE

Figure 2.1 and Table 2.2 show the population change input in comparison to equilibrium case. In particular four scenarios where population is assumed to change at an exponential growth rate from 2010 are tested. The sensitivity involves four levels of growth rate from -2% to +2% with 1% step increase between scenarios.

Figure 2.1 – Test 1 - Population input for testing

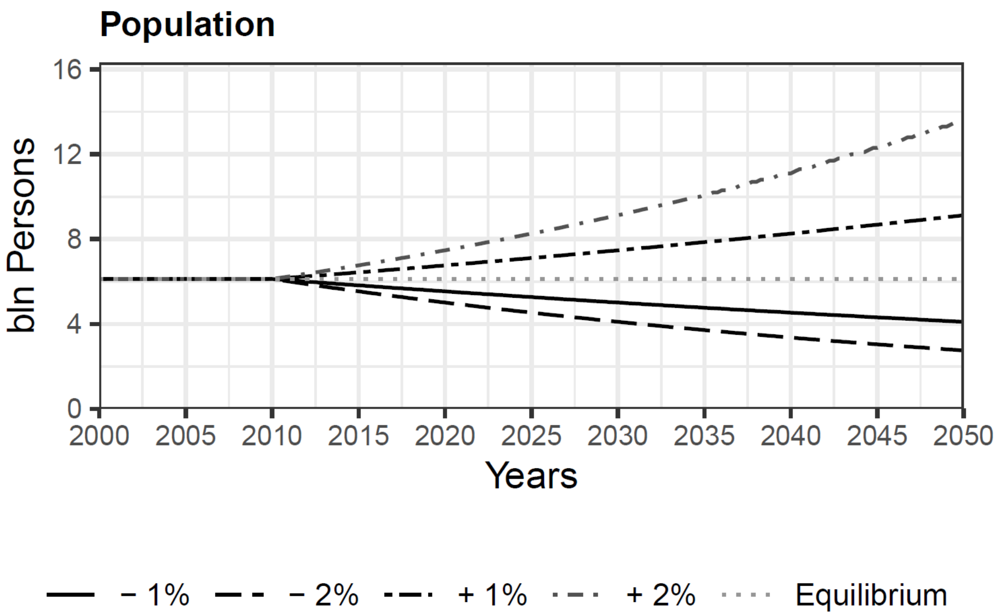
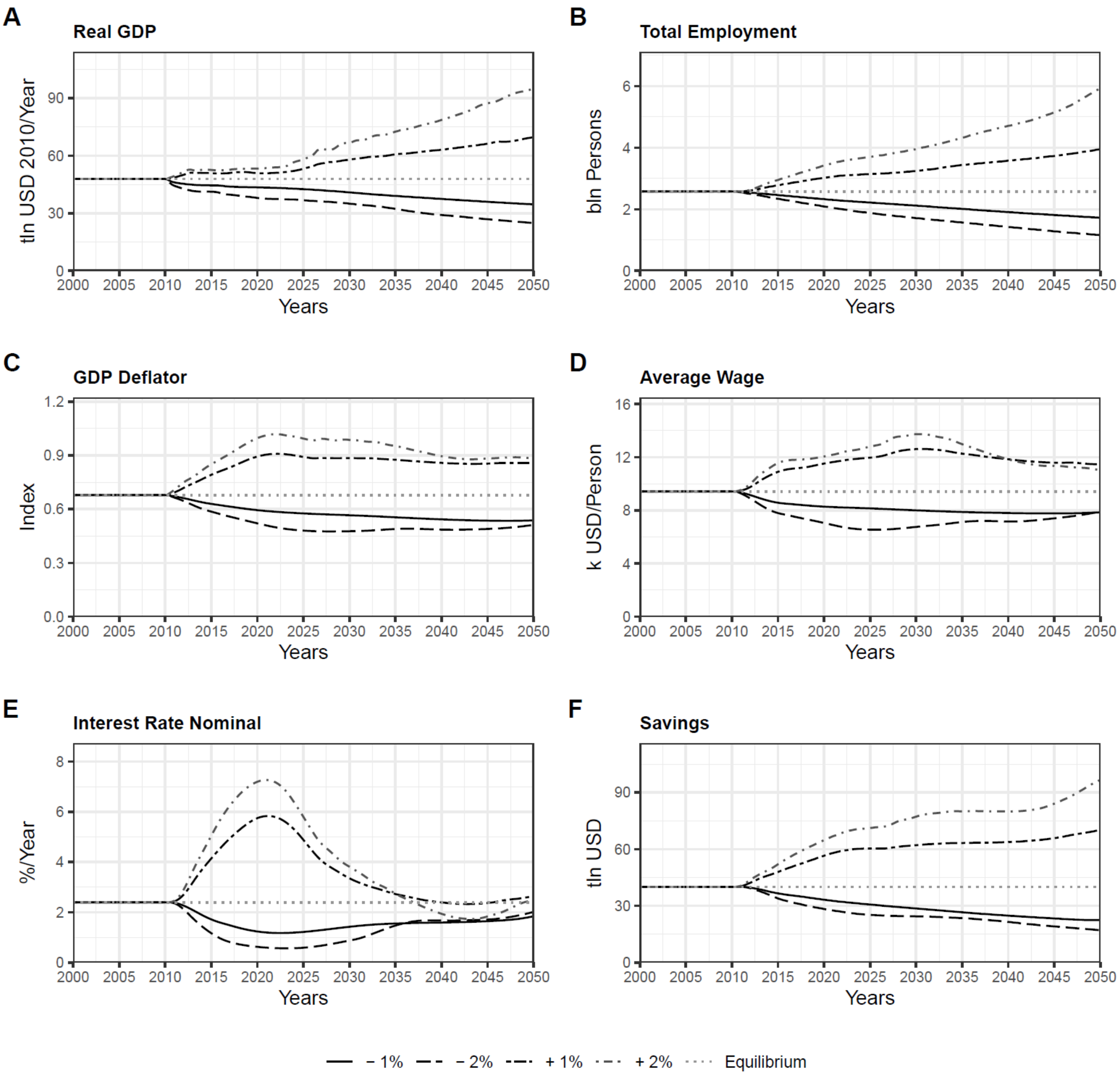


Table 2.2 – Test 1 - Population input for testing

Test 1	Parameter	Value
+2% Exponentially increasing population from 2010	Population exponential growth rate	+2%
+1% Exponentially increasing population from 2010	Population exponential growth rate	+1%
Equilibrium	Population exponential growth rate	0%
-1% Exponentially decreasing population from 2010	Population exponential growth rate	-1%
-2% Exponentially decreasing population from 2010	Population exponential growth rate	-2%

Figure 2.2 shows the dynamic impact of population change on six selected variables.

Figure 2.2 – Test 1 - Impact of population on key variables



As it is possible to see, population change is a clear and direct determinant for economic activity in the model. Ceteris paribus, and starting from balance conditions, the larger the population the higher the household demand, and the higher the labour force. This triggers real output growth almost instantaneously. The increased demand generates pressure for labour growth, resulting in increased wage per person and inflation over time. The combined effect of increased employment and wage per person allows for a positive feedback loop which generates more wealth, supporting real demand growth. It is worth noting that the profiles of average wage and GDP deflator curves are similar in shape. This is due to the mutual effect of labour payment as the largest cost component of prices in the ERRE model (approximately 70% for each sector), as well as the assumed positive impact of inflation wages. While savings increase, the banking system pushes nominal interest rate up until the economic growth catches the trend in demand generated by population increase. Such an effect reduces the speed of growth by constraining investments. The peak of interest is reached approximately ten years after the population increase has started, after which it can return toward initial values. This allows growth to speed up even further, recording an acceleration in real output growth supported by lower interest rates.

All these dynamics are inverted in the case in which population decreases. This indicates that the overall assumption behind the ERRE model for population change is that the economy would work as usual supplying output for those people, providing no grounds for testing hypothesis of population degrowth driven by any underlying cause. Additional features would be necessary to study population change dynamics.

TECHNOLOGY GROWTH AND PRODUCTIVITY INDEX

In the ERRE model, technology growth is represented via the variable labour productivity, indicating the ability of the same amount of labour to produce more output. Labour productivity is introduced as an exogenous element in each sector of the economy, with strong implications for the functioning of the model. In the calibration of the model, each sector is given a specific labour productivity curve which differentiates their behaviour. In this section it is shown the sensitivity of two cases in which:

1. All sector have the same labour productivity curve
2. The capital sector can increase labour productivity while all other sectors do not alter it

General technology and growth in the economy

Figure 2.3 and Table 2.3 show the inputs for the sensitivity analysis of global labour productivity. This indicates that all sectors in the economy (energy, food, capital and goods and services) are subject to the same output productivity increase per worker over time. This simulation runs from the year 2000 to the year 2100. The three scenarios compare the differences between exponential growth rate (multiplicative), ramp growth rate (linear) and growth with plateau (less than linear) at +3% growth rate.

Figure 2.3 - Test 2 – Global technology input for testing

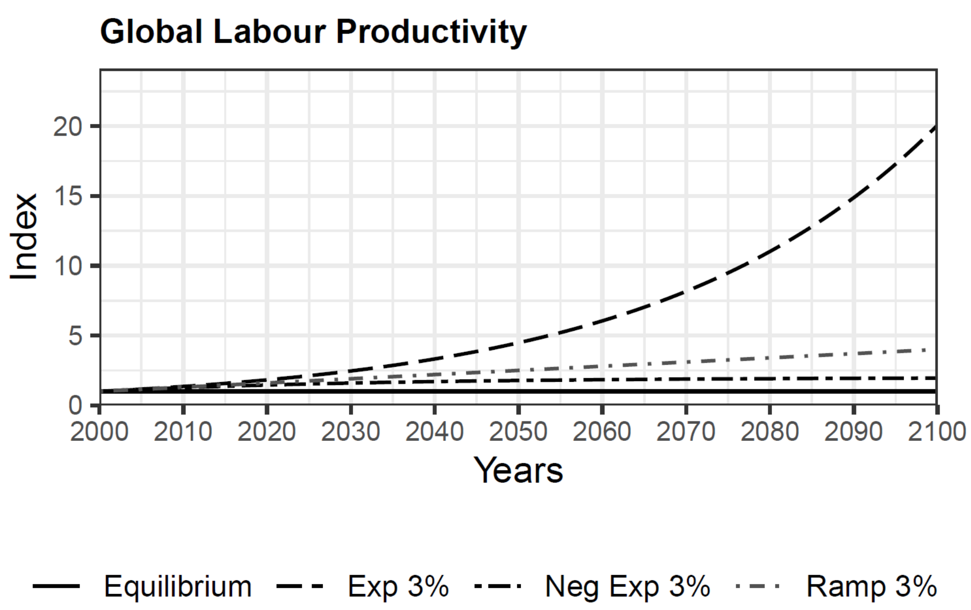


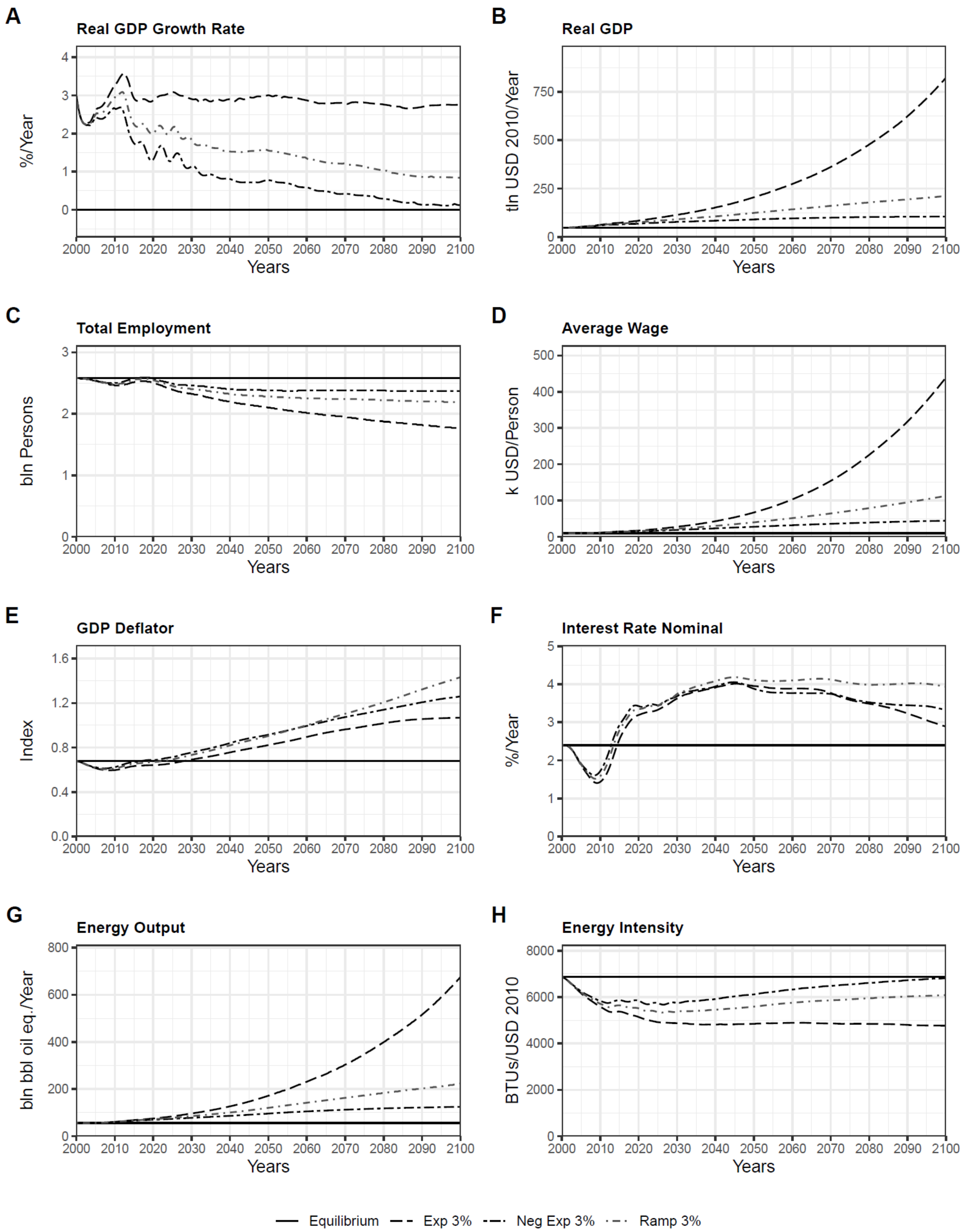
Table 2.3 - Test 2 – Global technology input for testing

Test 2 – Global technology	**Parameter**	**Value**
+3% Exponentially increasing labour productivity starting 2000	Global labour productivity exponential growth rate	+3%
+3% Linear increase in labour productivity starting 2000	Global labour productivity ramp growth rate	+3%
+3% Decreasing growth in labour productivity starting the year 2000	Global labour productivity degrowing growth rate	+3%

Figure 2.4 shows the sensitivity of selected variables for the global labour productivity test.

An interesting dynamic emerges in the area of energy consumption and energy intensity due to labour productivity growth. While energy output grows, the ratio between energy consumption and real GDP (i.e. energy intensity) decreases sharply for approximately thirty years in the simulation, and then this dynamic changes, ranging from stability in the case of exponentially growing labour productivity to increasing energy intensity. This dynamics can be explained by understanding the application of labour productivity growth in the firm sector. These assume implicitly that more output will be generated with the same amount of capital and energy resources. This explains the steady decline for the first part of energy intensity at the beginning of the simulation. However, because the household sector is not assumed to generate any change in their utility, their energy intensity per capital and goods remains the same over the time. In this test, it appears that households express preferences over commodities which consume energy, thus increasing their energy intensity over the longer term. This pushes the energy intensity of the total economy up after the year 2030.

Figure 2.4 – Global technology sensitivity on key variables



Relative technology in the capital sector

In this test, the application of three types of labour productivity growth (exponential, linear and degrowth) relative to the capital sector, while keeping the labour productivity growth of the other sectors flat, are shown. Other tests on labour productivity applied to one single sector only are presented both for resources and agriculture, to address the impact of technology growth on depletion. Figure 2.5 and Table 2.4 show the input in capital productivity at +2% all starting at the year 2010.

Figure 2.5 – Capital Technology growth for testing

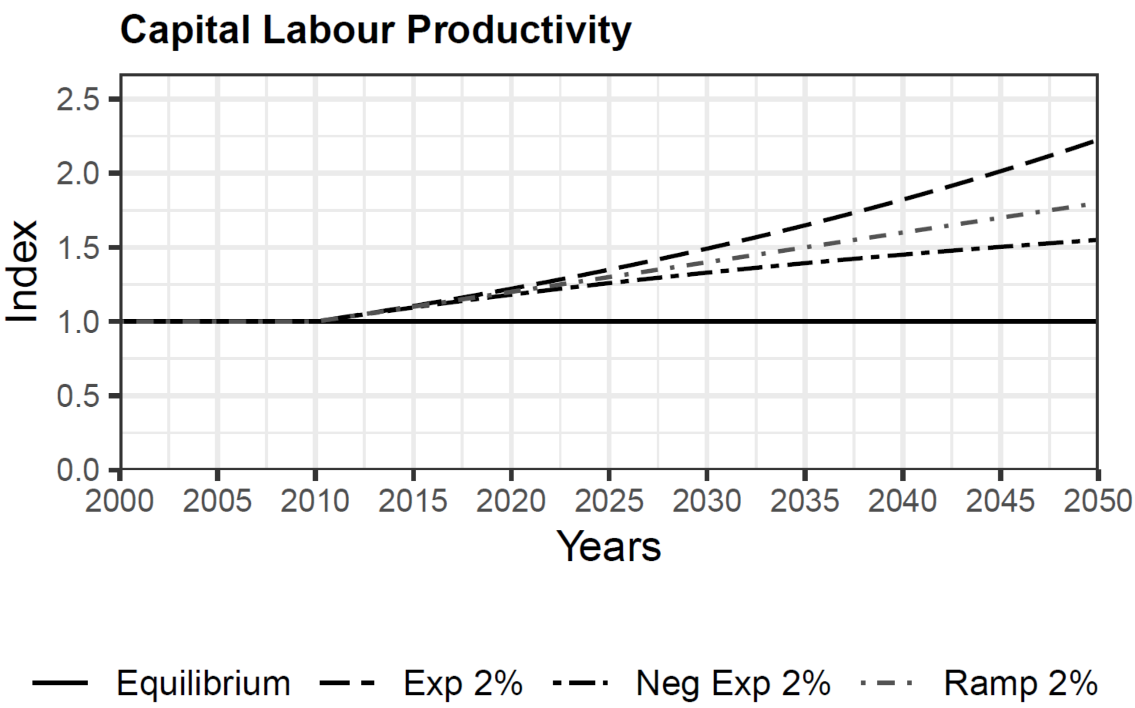
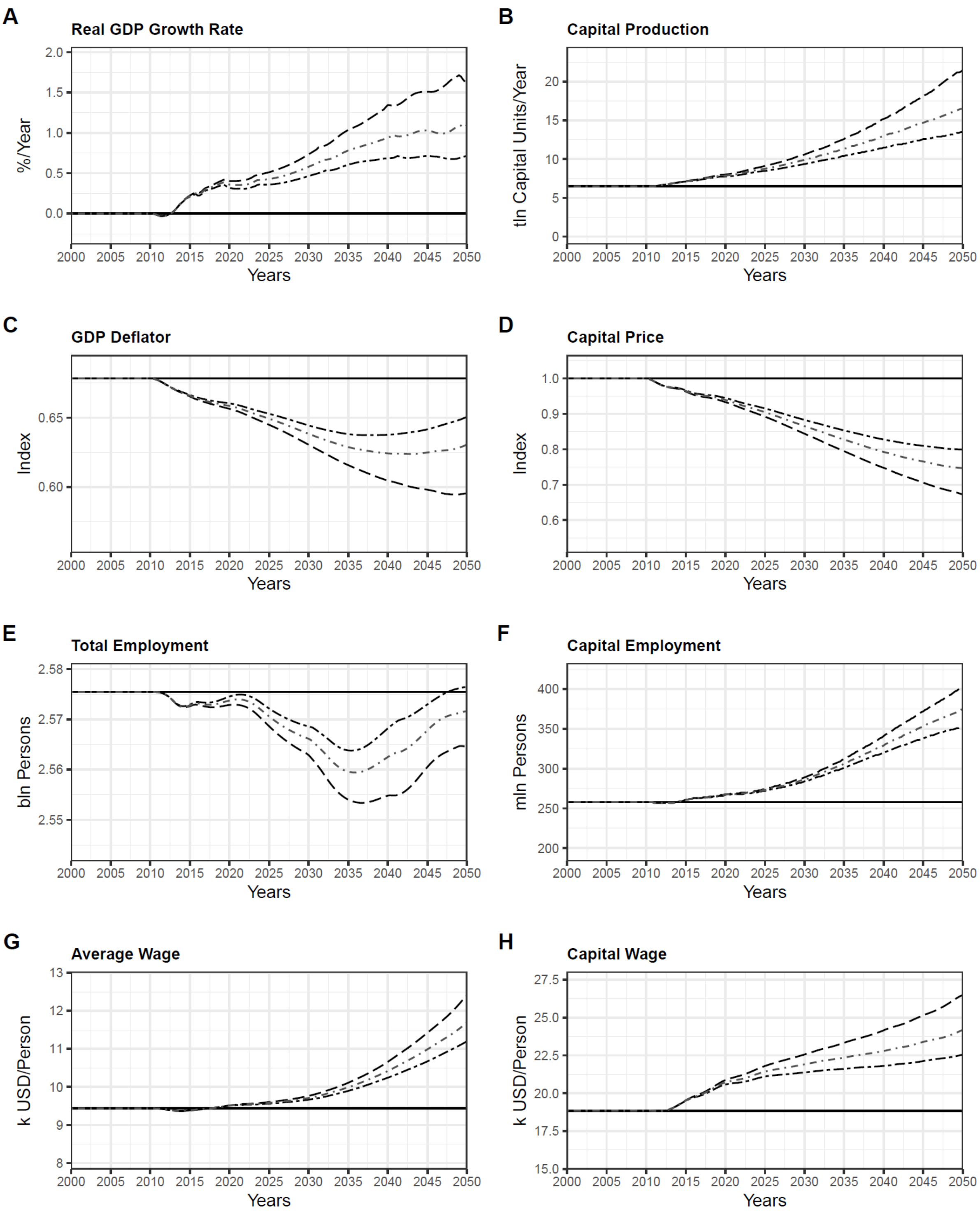


Table 2.4 - Test 2 – Global technology input for testing

Test 2 – Global technology	Parameter	Value
+2% Exponentially increasing labour productivity starting 2000	Capital labour productivity exponential growth rate	+2%
+2% Linear increase in labour productivity starting 2000	Capital labour productivity ramp growth rate	+2%
+2% Decreasing growth in labour productivity starting the year 2000	Capital labour productivity degrowing growth rate	+2%

Figure 2.6 shows the sensitivities of capital technology growth on selected variables.

Figure 2.6 – Impact of capital technology change on key variables



As Figure 2.6 shows, the result is different than the previous case. The major implications of the growth in capital productivity (i.e. generation of more output with the same amount of people) are (i) economic growth, (ii) increased production of capital output, (iii) increased energy production driven by capital growth, (iv) decreased inflation and capital price, (v) increased energy price.

In fact, as the capital sector is the base for the entire economy, the price of capital decreases together with technology growth, dragging down inflation for the entire economy. Counterintuitively, labour productivity growth does not reduce labour force in the capital sector, but it rather increases indefinitely. In the meantime the employment for the total economy lowers in a similar way to the previous test. Despite inflation decreases, wages do rise up both for the capital and rest of the economy. In addition, the real price of fossil fuels receives pressures generating increases in price with an important oscillation due to the time delays of the fossil fuel sector in adapting to rising demand.

The apparently counterintuitive behaviour can be explained by simple business principles captured in the ERRE that are driven by choice of agents in ordering of productive factors. In the ERRE model, increases in labour productivity increases wages for the capital sector, while at the same time increasing output per worker. Labour force and capital require time to adjust, while supply gradually increases at the exogenously defined rate. The result is that, over the short term, supply tends to be above demand for the time labour productivity rises. The relative effect on prices is to decrease from one time step to the next. Because every sector of the economy (including capital) generates preferences for purchase of each productive factors based on the marginal value for money in comparison to the others, the lower capital price pushes towards preferences for more capital to generate output. Such a behaviour generates a growth in capital production for all sectors of the economy. Due to the continuous increase in productivity, the capital price keeps decreasing. Because capital represents the basic components for the output of every other sector, the price of all commodities tends to decrease over time (i.e. decreasing GDP deflator).

The resulting rise in demand for the capital sector is sufficient to raise employment for that sector despite population being kept flat, and thus increasing wages in the capital sector. In the ERRE model, every sector adjusts (with a delay) on the average global wage to determine the payments to their labour force. As a result, the increasing wage for the capital sector triggers a rise in the wage of all other sectors. However, because these lag behind and no technology is assumed to rise their wages, there is always a wage gap which pushes more people to be attracted to work in the capital sector rather in the others. The pressures to the capital sector cascade as pressures to the energy sector, which, subject to no productivity growth, can answer the demand rise by increasing their capital and labour force levels. This rises costs and pushes real energy prices up.

ENERGY EFFICIENCY

In the ERRE model, energy efficiency is the reduction of energy required to operate a certain amount of capital. As the structure of the ERRE model implies, this test assesses the option of a reduction in energy intensity of capital on three levels for the goods and services sector as shown in Figure 2.7 and Table 2.5. The goods and services sector has been chosen since it involves a lower amount of feedback loops in comparison to the capital sector. In fact, the goods and services sector is characterized as the only sector with one customer only (Households) while ordering output from all other productive sectors. In addition, it is the largest sector of the economy in real terms.

Figure 2.7 – Test 3 - Sensitivity on energy efficiency scenario

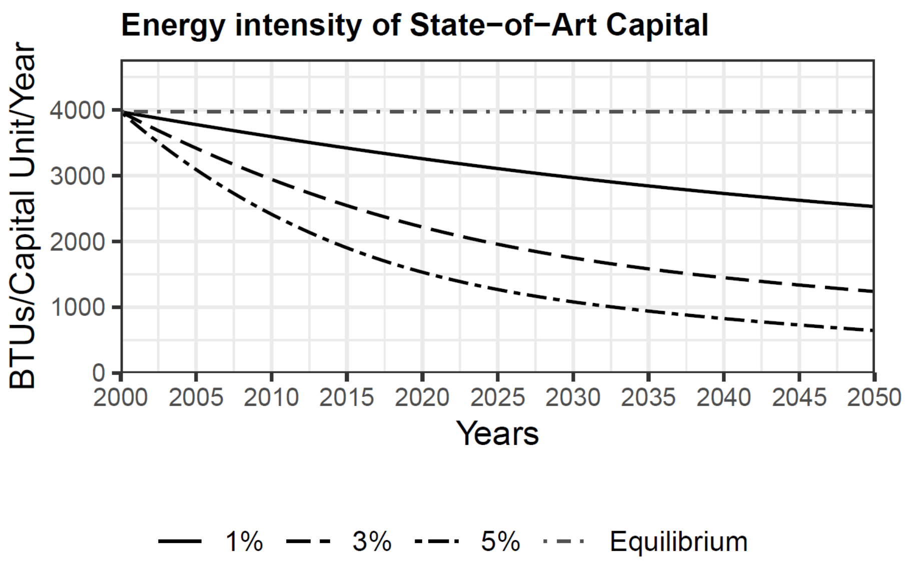
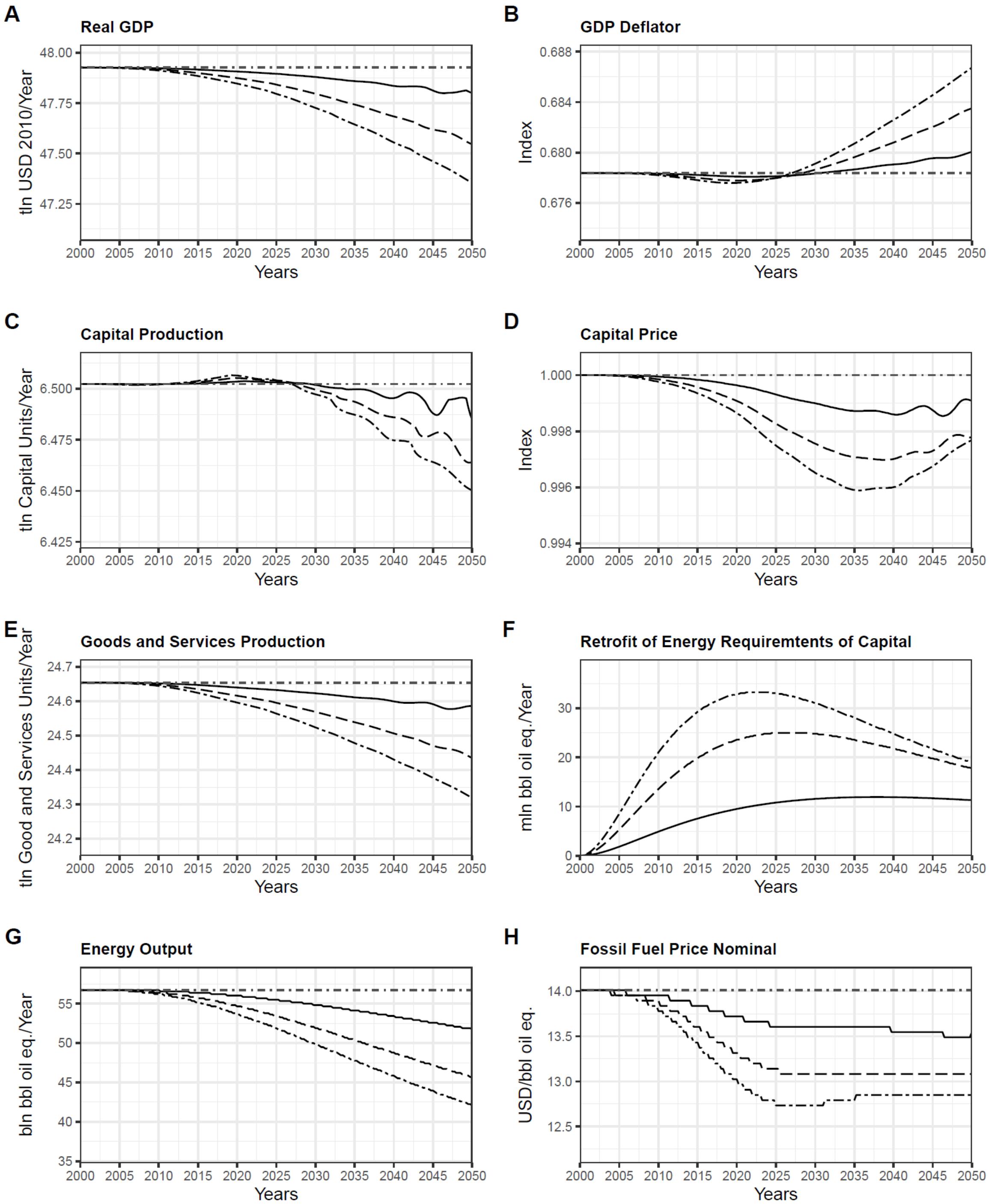


Table 2.5 - Test 3 – Sensitivity on energy efficiency scenario

Test 2 – Global technology	Parameter	Value
-1% Exponentially decreasing energy intensity of capital starting 2000	Reference reduction rate in energy intensity	+1%
-3% Exponentially decreasing energy intensity of capital starting 2000	Reference reduction rate in energy intensity	+3%
-5% Exponentially decreasing energy intensity of capital starting 2000	Reference reduction rate in energy intensity	+5%

Figure 2.8 shows the impact of change in energy efficiency improvement on eight selected variables in ERRE.

Figure 2.8 – Test 3 - Impact of energy efficiency on selected variables



As Figure 2.8 shows the general dynamic of the system is towards global recession. Without considering direct investments linked to an energy efficiency improvement, a better energy efficiency would require less energy to be produced, cascading in less capital demand, lower income to households and a long term decrease in demand, generating a recession.

In addition, due to the characteristics of the CES production function employed in the model, a decrease in energy intensity in one particular sector, reduces their ability to generate output with the same amount of inputs, thus triggering a constant deficiency in the good and services sector to supply household demand. This explains the high overall inflation rate in the economy, while lowering energy demand leads to lower energy prices.

Such a behaviour demonstrates a weakness in the neo-classical theory of CES production function, and implies serious considerations for using energy efficiency scenarios in the ERRE. In particular, all productive sectors and the household sector, employ a Constant Elasticity of Substitution (CES) production function. The assumption of constant elasticities, and the assurance of their sum to be equal to unity, would imply that every exogenous reduction in the capital energy intensity would correspond to a relative reduction in productivity of that capital by construction. The dynamic implication for this phenomena in the model would be to aim at achieving desired production of the sector while expanding the size of the other productive factors such as labour and capital. Such an effect would have opposite implications in the case of an exogenous increase in energy intensity. This leads to the partially unrealistic behaviours, requiring further investigation with alternative production functions, or potentially relaxing the hypothesis of constant marginal productivity of each factor.

It is worth noting that despite the overall dynamic of the system leading to the reduction of real output (GDP real), both food production and capital sectors show some elements of production increase. This is due to the shift in demand in the household sector, faced by increases in goods prices and difficulty of production. In fact, their behaviour is to shift their preference towards the other sectors, despite this being insufficient to avoid the decline of the general economy.

MONEY CREATION AND GOVERNMENT DEBT

In the ERRE model, money can be generated via central bank monetary policy as an exogenous increase in the money supply. Additionally government can create debt. The two cases present important differences in terms of their impact on the economy.

When a bank prints money out of nothing, these are instantly injected in their balance sheet, allowing for greater availability of cash and redistribution of this cash to borrowers via lending. While pushing the private sector debt above permissible levels, it would support growth via increased expenditure and interest payments.

On the other hand, when government generates debt, it also requires an acquirer of that debt among the financial and household sectors. In ERRE, it is assumed that debt creation would not result in instant injection of those money in the economy, but rather require a time delay due to the bureaucracy interlinking the two institutions. The resulting money issuance would be distributed in the economy via lending to the private sector as in the previous case. This would push households and banking cash availability down by the amount required to purchase that debt in the short term, gradually generating growth via interest payments and increases in expenditure. In the ERRE model, all government expenditure are provided as Government Transfer to the Household sector itself, which is then responsible to increase consumption due to the higher availability of cash. In the real world, it would be the government itself allocating those budgets in the areas that are most interest to them, including education, defence or infrastructure.

This section provides tests for the impact of changes in money supply from monetary policy, and a comparison of one of those scenarios with the ability of government to generate the same amount of cash via deficit creation. In this second case the differences in behaviour in the cases in which the government is also supported via money creation from the central bank or not are shown.

Exogenous money creation via central bank policy

Figure 2.9 and Table 2.6 show the four scenarios where the central bank would impose a shock increase or reduction to the money supply of the economy starting from equilibrium condition at the year 2010.

Figure 2.9 – Test 4A - Sensitivity on shock increase in money supply via money creation from central banks

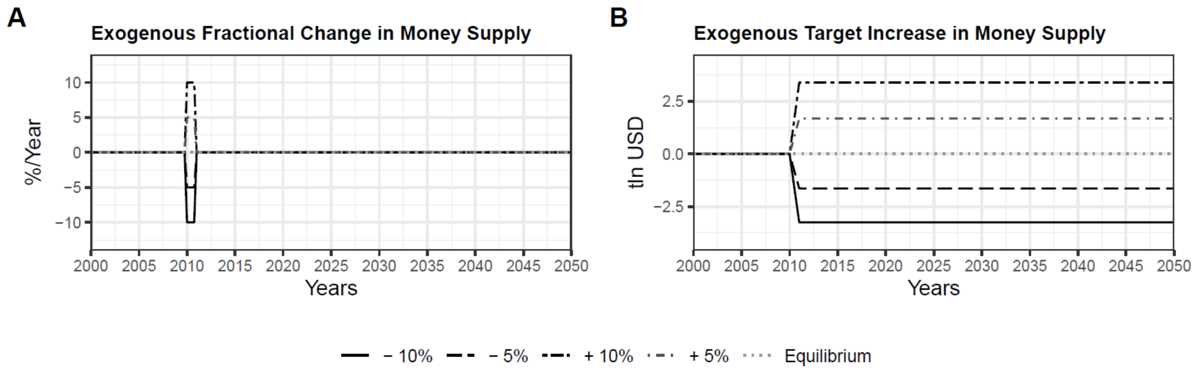
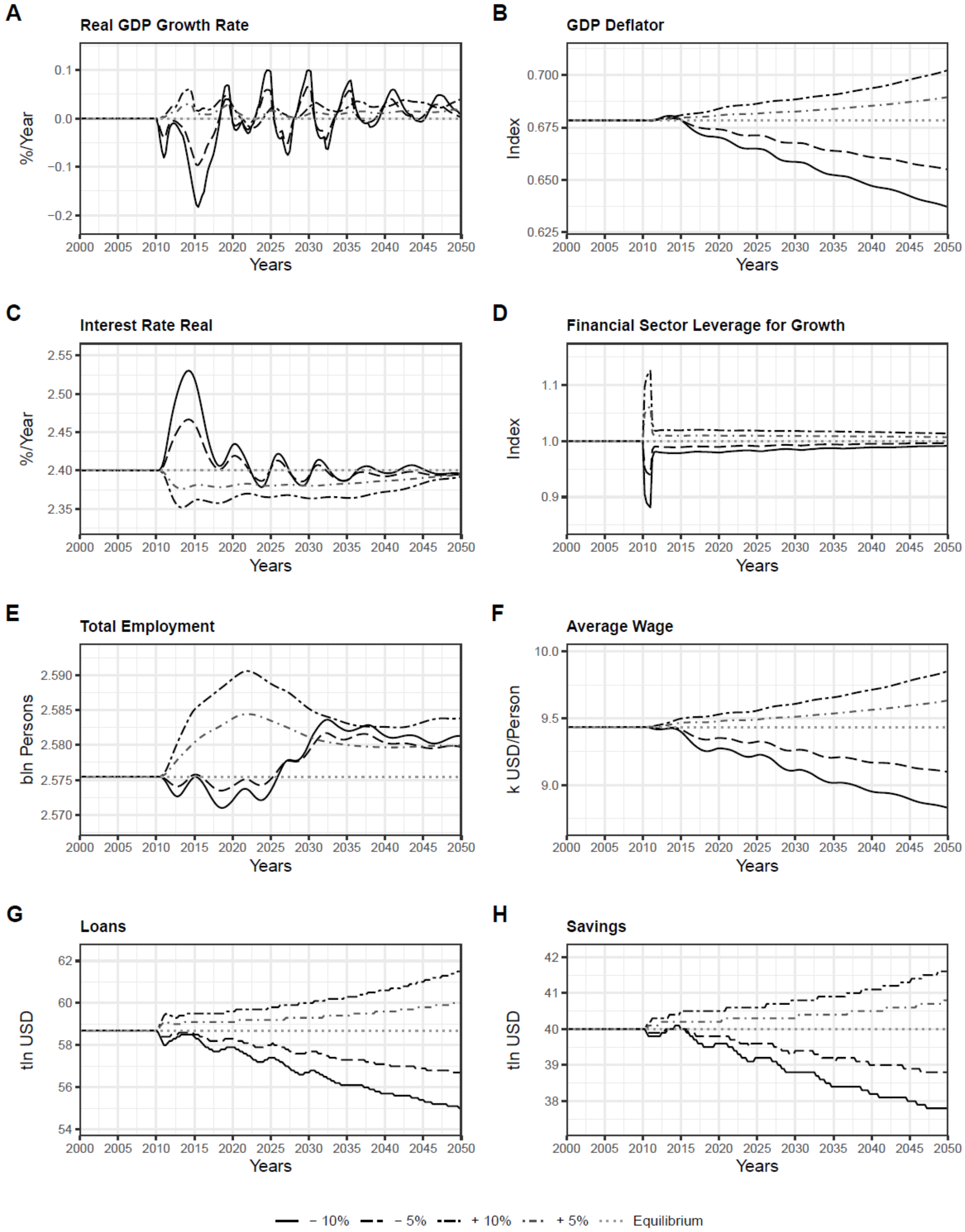


Table 2.6 – Test 4A - Sensitivity on shock increase in money supply via money creation from central banks

Test 4A – Money creation from Central Bank	Parameter	Value
+10% Shock Increase in Money supply in 2010	Pulse increase in money supply	+10%
+5% Shock Increase in Money supply in 2010	Pulse increase in money supply	+5%
-5% Shock decrease in Money supply in 2010	Pulse increase in money supply	-5%
-10% Shock decrease in Money supply in 2010	Pulse increase in money supply	-10%

Figure 2.10 shows the sensitivity of printing money out of money to selected variable in the ERRE model.

Figure 2.10 – Test 4B – Impact of money creation from central banks on selected variables



As Figure 2.10 shows, a single shock in money creation is sufficient to unbalance the ERRE economy towards growth (or degrowth) over the longer time period. This is demonstrated by the changes in inflation, loans, savings and average wage, that are clearly correlated with a positive or negative shock in money creation of the ERRE model. This is mostly due to the endogenous dynamic behaviour of the financial sector in printing further money after a shock occurs. The Financial Leverage for growth in the financial sector, is dependent on the availability of cash in the bank, and impacts the lending rate to the private sector as a multiplier effect. In other words, as far as the Financial Leverage equals 1, then the model would be seeking a dynamic balance over the long term of the simulation.

A positive shock in generating money out of nothing is directly reflected in such a financial leverage. Due to the increase of cash via lending, the private sector would find themselves in a cash surplus position such that they can increase consumption and investments, and support growth. While the financial sector applies endogenous money creation based on the real growth rate of the economy, the initial shock is sufficient to support further money creation over the longer time period, thus pushing the economy towards higher growth. This is also reflected in the higher level of employment. It is worth noting, that the interest rate is dependent on the demand for lending from the private sector. As a result the possibility to lend additional money via exogenous money creation policy is also reflected in higher real interest rates, which stabilizes over the longer time period.

In a similar way, a negative shock in money creation (i.e. money withdrawal from the economy) implies a sudden shock in the reduction of debt of the private that cannot achieve desired borrowing. Such a shock reduces availability of cash in the economy, which is reflected in the sudden reduction in growth rate of the real economy and relative instability. Thus inflation, loans, savings, and average wage start decreasing. The financial sector adds to the difficulty of the real economy by printing less money, while decreasing interest rates to support their growth rate over time. Interestingly, employment rises before the crisis, mostly due to the inability of household to pay their debt, forcing them into the job market, increasing labour supply, and reducing wages even more.

Such a behaviour demonstrates the large differences of the ERRE model from the neo-classical theory and the standard use of general equilibrium models that assume perfect availability of information, and a money generation policy would have little to no impact on the real economy, mostly increasing inflation. The ERRE behaviour is thus in line with the Post-Keynesian school of thought, that accepts the effects of money creation policies on the real economy. Such a behaviour is also enriched by the set of non-linear relationships that are most characteristics of the Behavioural Economic School, and the path dependent disequilibrium dynamic of the Evolutionary Economics School of thought.

Exogenous money creation via government debt creation

Figure 2.11 and Table 2.7 show the sensitivity parameters used to test government debt creation as an equivalent of a +10% increase in money supply (previous scenario at maximum level). The tests are performed under the three hypothesis of 0%, 50% and 100% of government debt created as new money within banks. In all cases the Government creates the same amount of debt as a shock in the year 2010 and keeps it stable from then on. The attempt of the bank to print that debt on these three levels correspond to the attempt of creating 0%, +5% and +10% of money directly out of nothing.

Figure 2.11 – Test 4B - Sensitivity on shock increase in money supply via money creation from government debt

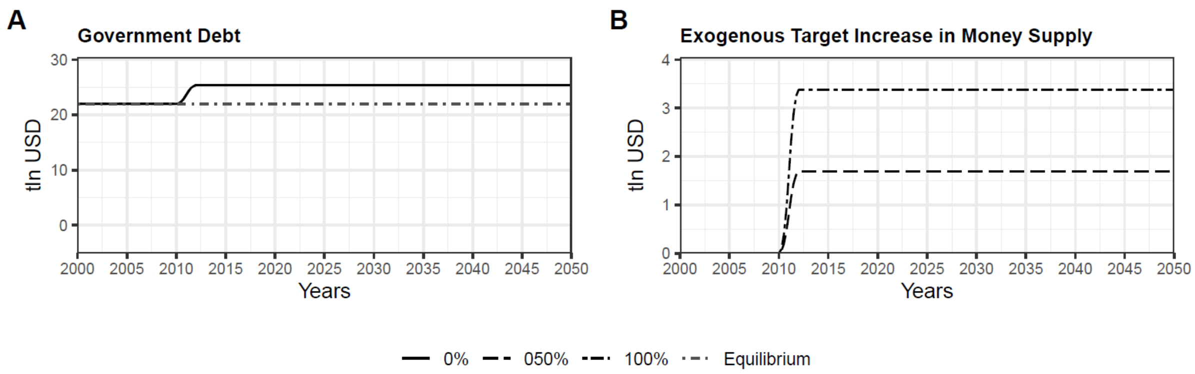


Table 2.7 - Test 4B - Sensitivity on shock increase in money supply via money creation from government debt

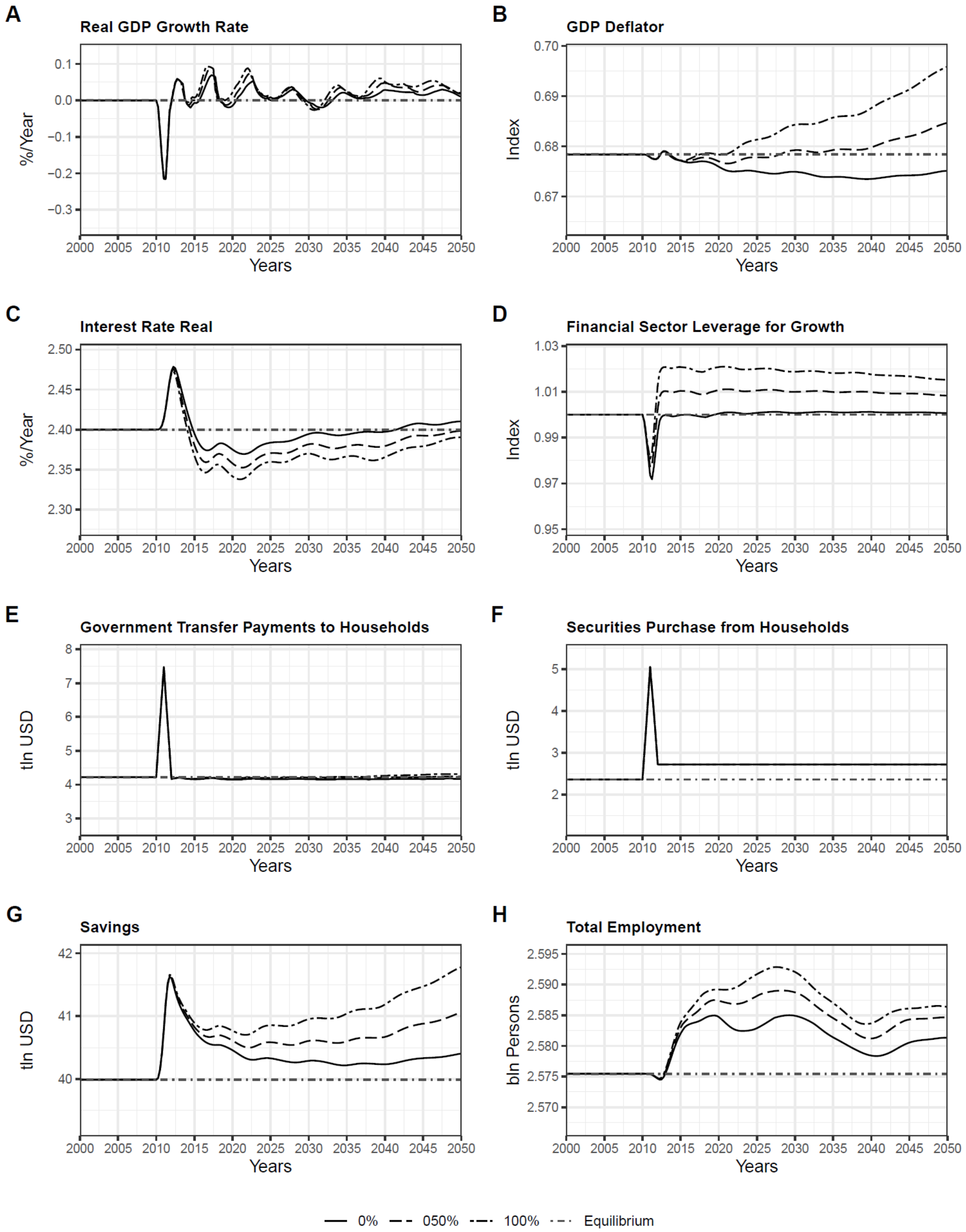
Test 4B – Government debt	Parameter	Value
+10% Shock Increase in Money supply in 2010	Pulse increase in money supply	+10%
+10% Shock Increase in Money supply via government debt in 2010	Pulse increase in Government deficit	+76.4% (+10% money creation)
	Money creation fraction via government deficit	100%
+5% Shock Increase in Money supply via government debt in 2010	Pulse increase in Government deficit	+76.4% (+10% money creation)
	Money creation fraction via government deficit	50%
+0% Shock Increase in Money supply via government debt in 2010	Pulse increase in Government debt	+76.4% (+10% money creation)
	Money creation fraction via government deficit	0%

Figure 2.12 shows the sensitivity of those three cases on selected variables in the model, and compares their results to the +10% money supply increase scenario as proposed in Test 4A.

As Figure 2.12 shows, the effect is very different than in the previous case. In particular, real GDP growth is sacrificed in the short term to raise funds from the government both when new money are created and when they are not. This is mostly due to the sudden decrease in financial resources for the banks, who are forced to reduce their financial leverage in the short term, decreasing the ability of the private sector to borrow and consume. All the debt taken from banks and households is returned to the Household sector itself as government transfer. Due to the decreased ability of the private sector to borrow, the household is forced to keep most of the money as savings, and slowly increase expenditure. The long term result is similar to the previous case in terms of economic growth and employment, while sacrificing the short term gains.

Interestingly, an application of both policies (Test 4A and 4B) in conjunction can improve the economic performance as a whole, as it is described in the scenarios section of this text.

Figure 2.12 – Test 4B - Impact of government debt money creation on selected variables



ENDOGENOUS DISEQUILIBRIUM

In this section the sensitivity of endogenous disequilibrium structures in the ERRE is tested model. These can be divided among disequilibrium due to human choice and ecological constraints.

<u>*HUMAN BEHAVIOUR DISEQUILIBRIUM*</u>

In ERRE both the propensity for savings and the dividend pay-out ratio structures are modelled based on similar principles. In particular, they both receive unbalancing pressures due liquidity adequacy, and income generation, while at the same time present balancing forces towards their desired (or normal) value. In the equilibrium scenario, those balancing values are fixed to 1 in both cases. This means that all net income generated in firms is distributed as dividends, and all income for households is spent as consumption. Such an equilibrium condition is actually not possible, since the average firm would be inclined to keep retained earnings while distributing a small fraction of income to their shareholders. On the other hand, many households can be differentiated among those who consume all their income (thus savings propensity towards zero), and those who save the majority of their income, thus providing investments in assets or companies. These disequilibrium tests show how the model reacts when households and firms take more realistic decisions than equilibrium.

Dividend pay-out ratio

If dividend pay-out ratio is greater than 1, it means that investors would receive larger dividends than a firm can generate as income, pushing the firm to generate liquidity via borrowing from the financial sector. In the case in which the dividend pay-out ratio is lower than 1, firms behaviour would be to distribute less dividends to households, holding more cash in the firm, reducing risks of failure and generating increased expenditure via investments and payments for labour. Figure 2.13 and Table 2.8 shows the different dynamic of the dividend pay-out ratio in the capital sector, when pressured to keep their normal values at 0.7,0.9, 1.1 or 1.3, and compared to the equilibrium case.

Figure 2.13 – Test 5 - Sensitivity on initial normal dividend pay-out ratio for the capital sector

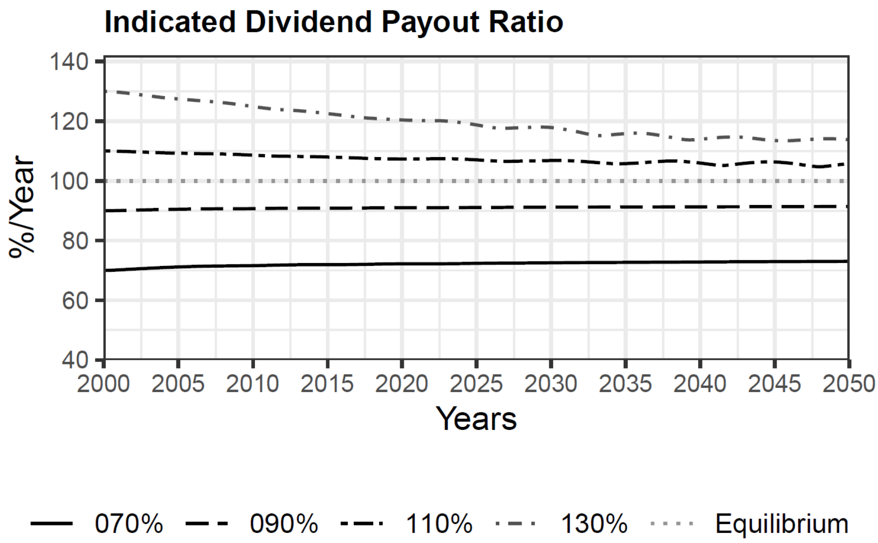


Table 2.8 – Test 5 - Sensitivity on initial normal dividend pay-out ratio for the capital sector

Test 5 – Preference for Dividends distribution	Parameter	Value
+30% Increase from the year 2000	Normal Dividend Pay-out Ratio in Capital	1.3%
+10% Increase from the year 2000	Normal Dividend Pay-out Ratio in Capital	1.1%
-10% decrease from the year 2000	Normal Dividend Pay-out Ratio in Capital	0.9%
-30% decrease from the year 2000	Normal Dividend Pay-out Ratio in Capital	0.7%

Figure 2.14 shows the sensitivity of dividend pay-out change on the dynamics of the system.

Figure 2.14 – Sensitivity of change in dividend pay-out ratio on selected variables

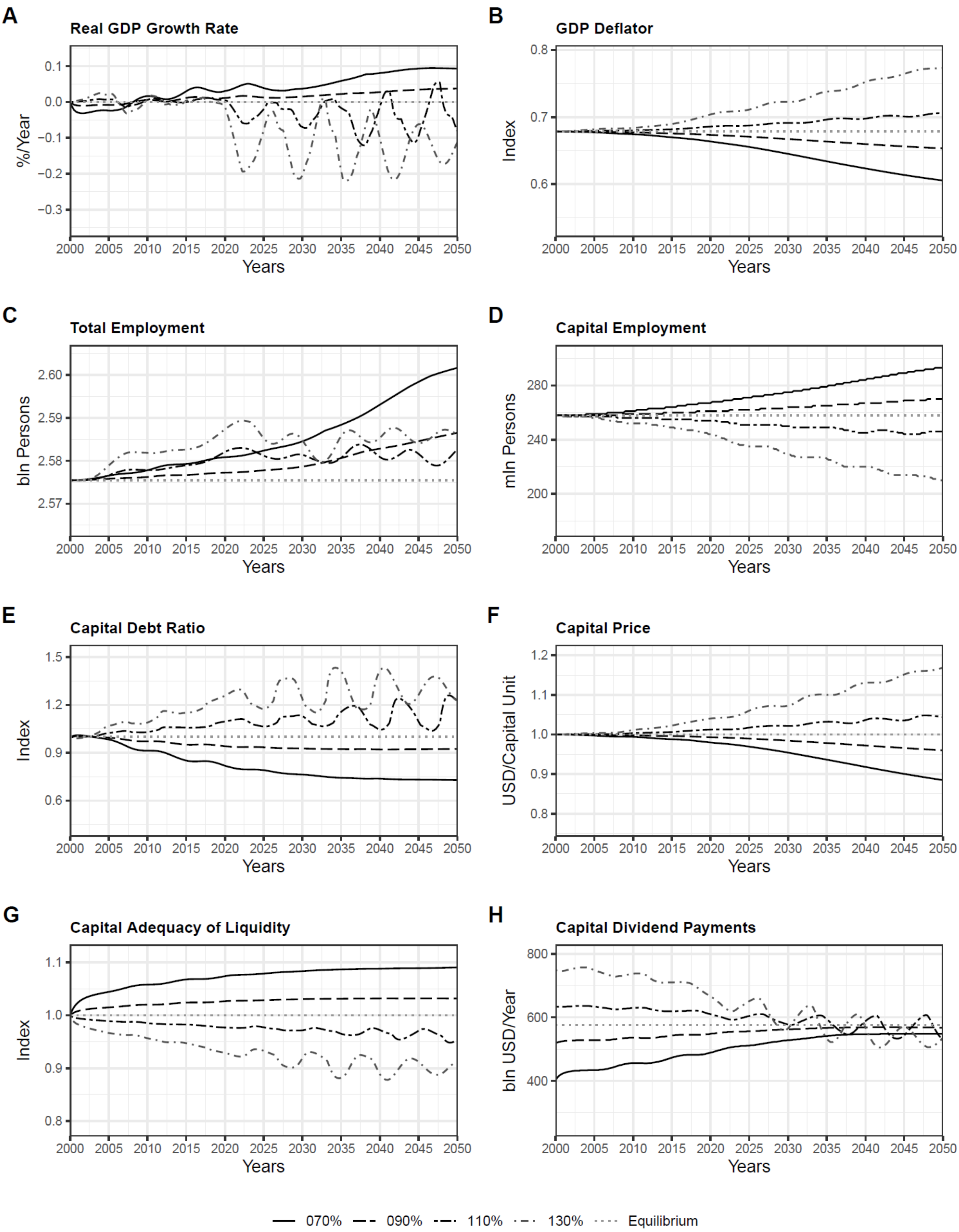


Figure 2.14 shows an important change in behaviour in the two cases, from increased stability and real output growth in the case the firms hold more cash and distribute less dividends, to instability, decline and high inflation when investors demonstrate greed at the expenses of the firm. Despite the change being done on one single sector of the economy (capital), its effects could both support and benefit the dynamics of the entire economy. This shows that two types of companies, one of which supports the better risk management of the firm, and the second prioritizing the investors income at the expense of firm performance could have percussions on the overall economic activity in both positive and negative ways.

The first element of correlation is the dynamic of capital price and its linkage with the dynamics of inflation in the entire economy. It appears that the capital sector, while providing the fundamental input to every other sector of the economy acts as control for the inflation of the entire economy. For example, if price of capital rises, all the costs of production will increase, thus increasing prices in every part of the economy. Also the opposite case holds true. The other figures help to understand how the change in dividend pay-out pushes the prices of capital up or down, or generate instability in the economy.

If firms distribute less dividends than their total income, the firms hold additional cash that can be spent in productive activities. This generates a surplus of cash in the vault of firms (adequacy of liquidity increases). The resulting effect is to decrease debt to reach the same level of output (debt ratio decreases thus decreasing costs), and a general decrease in prices. The price reduction increases demand in relative terms to other productive factors, thus pushing the economy to employ more people, and the general dynamics of real output growth. It is worth noting that, at the beginning of the simulation, the real output slightly decreases, to start increasing over the long term in the simulation. This is due to initial reduction in disposable income to households, who reduce their expenditure. However, because of the dynamic increase in output growth, the firm creates more wealth such that it increases income in the future. It is worth noting that dividend payments, that are initially reduced by firm policy, have the tendency to increase supported by large availability of cash in the firm, as well as the economic growth.

When the investor collects more dividends than that which is generated as income from the firm, the general dynamic leads to difficulty for the entire economy, including instability generated from the interaction with the banking system and borrowing. In fact, their adequacy of liquidity is pushed towards deficiency over time, pushing the firm to generate more debt than that which is normally permissible for their assets. This increases costs, as well as reducing production since capital remains a requirement for firms. This pushes capital price to rise together with the overall inflation in the economy. The real output grows in the short term (households have more liquidity to increase consumption) but the continuous push of investors, eventually pushes the economy towards decline due the inability of firms to perform their activities in low risk conditions. Despite the employment in the capital sector decreasing (as a result of higher prices and lower production), the employment of the entire economy rises, despite the instability. This is mostly due to the change in preferences of households towards goods and food sector, as well as pushing households to engage in economic activity to pay for their debt on assets. Because of the dynamics of decline, the initially high dividends eventually reduce until becoming lower than in the case in which more cash was left in the vaults of firms. In relative terms to inflation, it would be possible to observe that dividends would be greater in real terms when firms distribute less of it.

This is an important result of the ERRE model, demonstrating how the basic behaviour of firms of distributing less income as dividends can be sufficient to generate growth in the global economy.

Propensity for savings

In a similar way to the previous case, when a propensity for savings is higher than 1, it would indicate the high propensity of households to save money in the bank (thus receiving income via interest payments) rather than spending in consumption. In the case in which propensity for savings is lower than 1 it would push households to save less, benefitting their consumption rate. Figure 2.15 and Table 2.9 show the different sensitivity input of propensity for savings and compares those to equilibrium case.

Figure 2.15 – Test 5 - Sensitivity on initial normal propensity for savings ratio

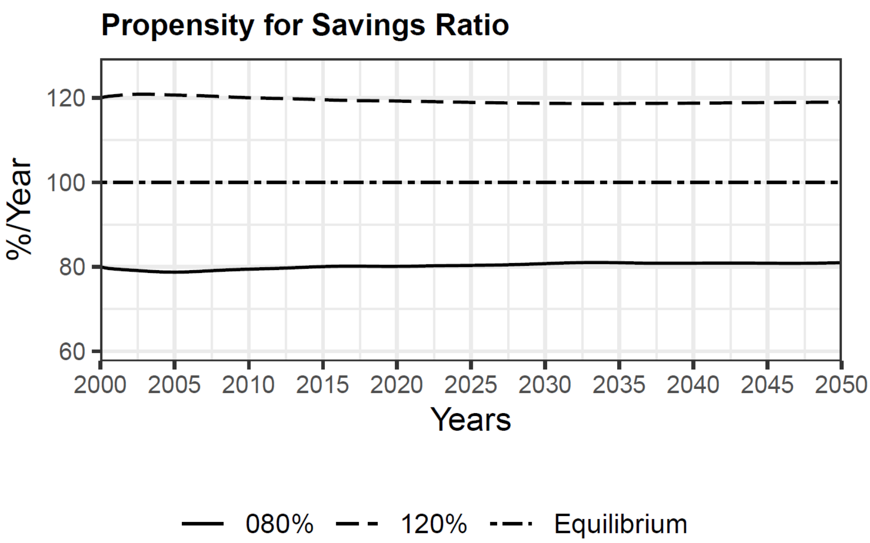
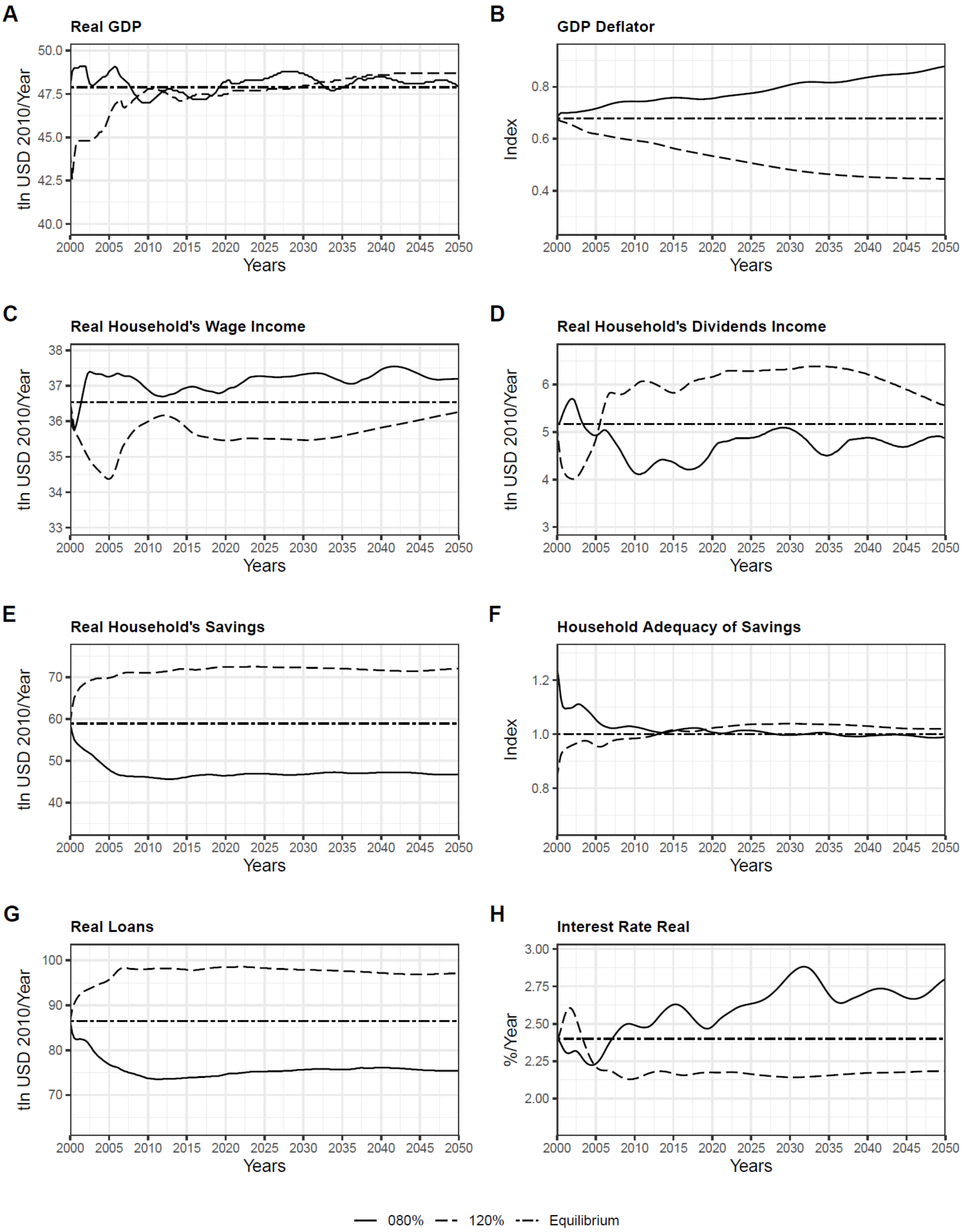


Table 2.9 – Test 5 - Sensitivity on initial normal propensity for savings ratio

Test 6 – Preference for Propensity for Savings	Parameter	Value
+20% increase in propensity for savings ratio from year 2000	Normal propensity for savings ratio	1.2
+10% increase in propensity for savings ratio from year 2000	Normal propensity for savings ratio	1.1
-10% decrease in propensity for savings ratio from year 2000	Normal propensity for savings ratio	0.9
-20% decrease in propensity for savings ratio from year 2000	Normal propensity for savings ratio	0.8

Figure 2.16 shows the sensitivity of change in propensity behaviour on selected variables in the model.

Figure 2.16 – Test 5 - Sensitivity of change in propensity for savings ratio on selected variables



This test demonstrates interesting and apparently counterintuitive insights in the economic system. The first element to consider is that household propensity to save (or consume) would have little impact on economic growth, but would be an important driver for inflation throughout the economy in ERRE. In particular, if households would be willing to spend more than their income, they would be pushing up demand, initiate production activity and increase inflation. In the case in which all households would rather be risk adverse and constantly save a fraction of their income, the result would be lowering demand and decreasing inflation over time. However, despite this dynamic impacting production in the short term, their long term effect would result in relative stability in the real economy, perturbed by oscillations and small business cycles. In fact, when households are willing to spend more than their income, the equilibrium value of savings deposit rate would be felt as too high for them, thus resulting in using all the surplus savings as consumption, while decreasing their debt. Due to the non-linearity determining the behaviour of payments in case of surplus in adequacy of liquidity, this effect would have no initial impact on Real GDP, pushing it up for some time, and oscillating around the equilibrium value when debt would be reduced to the desired level. In the case, in which households would feel like savings more than their income, the non-linear effect on payments would push them to spend less starting at the initial time in the simulation, resulting in Real GDP being lower than equilibrium value. Perceiving their savings adequacy as too low, they would raise debt to maintain their desired deposits within banks.

Despite the change in nominal values, both cases would stabilize the economy at two different levels of real savings and loans in the economy. In particular, when households prefer to save more, their real value for savings would increase (low inflation), while in the opposite case the real value of their savings and debt would decrease (high inflation). Both cases determine differences in the financial sector in terms of monetary policy around real interest rate. In particular, low interest rates would be applied in the case of higher propensity for savings to stimulate their consumption level. On the other hand, increases in real interest rates would be applied when households tend to consume more than their income, thus rising inflation rates.

An interesting result is shown between the dynamics of income generation for households. In particular, it appears that high consumption rates would support higher real wages and lower real dividends, whereas higher propensity for savings would result in reducing wage income and rising dividend rates.

This test, applied in isolation to other disequilibrium factors, appears to support the idea that an economy characterized with low saving deposit rate, would support inflation increase and real wage income rise, whereas an economy characterized with high propensity savings rate would push wages down potentially increasing inequality between the wealthy and the poor. Interesting analysis could be applied while splitting the household sector between those who receive wages and those who receive dividends, and test the hypothesis of different propensity for savings/consumption between the two.

It is worth noting that the application of those endogenous behaviours linked with the option of firms to distribute lower dividends, and potential for increases in labour productivity could reach a different system dynamics as we shall see in the scenario analysis of chapter 6 of Pasqualino and Jones (2020). The next section shows the endogenous ecological non-linear elements applied in the ERRE model.

ECOLOGICAL DISEQUILIBRIUM

In this section the sensitivity of the three major ecological constraints, as in common use under the Limits to Growth approach to define the economy within planetary boundaries, are provided. These include resource depletion and energy transition, agricultural land erosion and forest land limits, climate change and feedback impact on the economy. All tests are performed under the condition of population disequilibrium growth (using UN Population division historical data and forecast), to support the meaningfulness of this analysis. In fact, the higher the growth the greater the effect of those limits.

Resource depletion and transition sensitivity

The energy transition was one of the major concerns in the Sterman (1981), which reports various sensitivity tests on both the energy system and policy analysis. In ERRE, many edits were performed in every area of the model that intersect with the dynamics of the energy transition. In addition to removing the OPEC sector for the scope of global aggregated analysis, (i) the fossil fuel depletion curve and fossil fuel production structure has been changed, (ii) technology growth curves were differentiated among all sectors of the economy, and (iii) the structure of the energy market and energy shift was modified to reflect possible scenarios and constraints in the ability of renewables to substitute fossil fuel beyond a mere price gap between the two. In addition, new indicators such as the EROEI, have been considered and included as decision variables in the feedback structure of energy production. This section provides sensitivity tests in relation to these edits.

Figure 2.17 – Inputs to the sensitivity test on the energy transition scenario

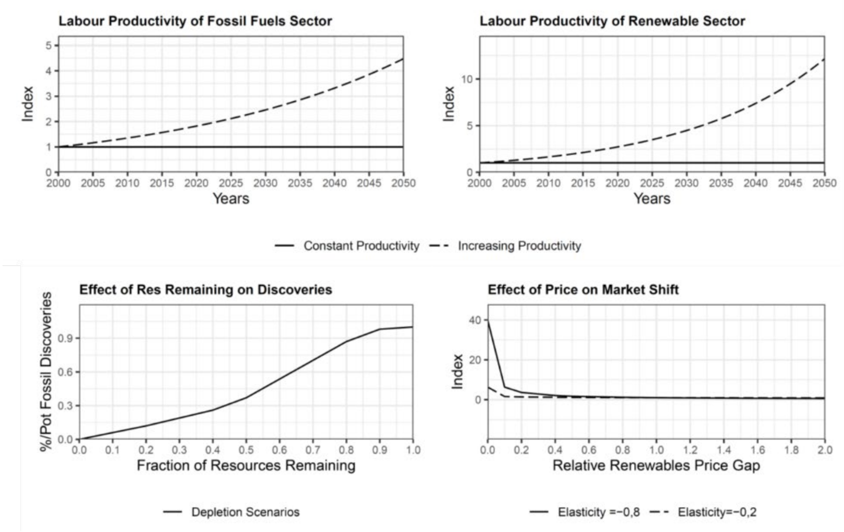


Table 2.10 – Test 7 – Scenarios for the fossil fuel depletion sensitivity analysis

Test 7 – Fossil fuel depletion	**Parameter**	**Value**
Case 0 – Population Growth	Population growth case	**UN Population division global projection**
	Fossil fuel depletion curve	OFF
	Fossil Fuel Exponential Labour Productivity	0%
	Renewables Exponential Labour Productivity	0%
	Elasticity of Renewables Energy shift	-0.2 (slow adaptation of Renewables in comparison to Fossil fuels)
Case 1 – Fossil fuel depletion	Population growth case	**UN Population division global projection**
	Fossil fuel depletion curve	**ON**
	Fossil Fuel Exponential Labour Productivity	+0%
	Renewables Exponential Labour Productivity	+0%
	Elasticity of Renewables Energy shift	-0.2 (slow adaptation of Renewables in comparison to Fossil fuels)
Case 2 – +3% Exponential growth in Fossil Fuel Labour Productivity	Population growth case	**UN Population division global projection**
	Fossil fuel depletion curve	**ON**
	Fossil Fuel Exponential Labour Productivity	**+3%**
	Renewables Exponential Labour Productivity	+0%
	Elasticity of Renewables Energy shift	-0.2 (slow adaptation of Renewables in comparison to Fossil fuels)
Case 3 – +5% Exponential growth in Renewables labour productivity	Population growth case	**UN Population division global projection**
	Fossil fuel depletion curve	**ON**
	Fossil Fuel Exponential Labour Productivity	**+3%**
	Renewables Exponential Labour Productivity	**+5%**
	Elasticity of Renewables Energy shift	-0.2 (slow adaptation of Renewables in comparison to Fossil fuels)
Case 4 – More elastic energy shift towards Renewables	Population growth case	**UN Population division global projection**
	Fossil fuel depletion curve	**Off**
	Fossil Fuel Exponential Labour Productivity	**+3%**
	Renewables Exponential Labour Productivity	**+5%**
	Elasticity of Renewables Energy shift	**-0.8 (as elastic as the fossil fuel sector)**

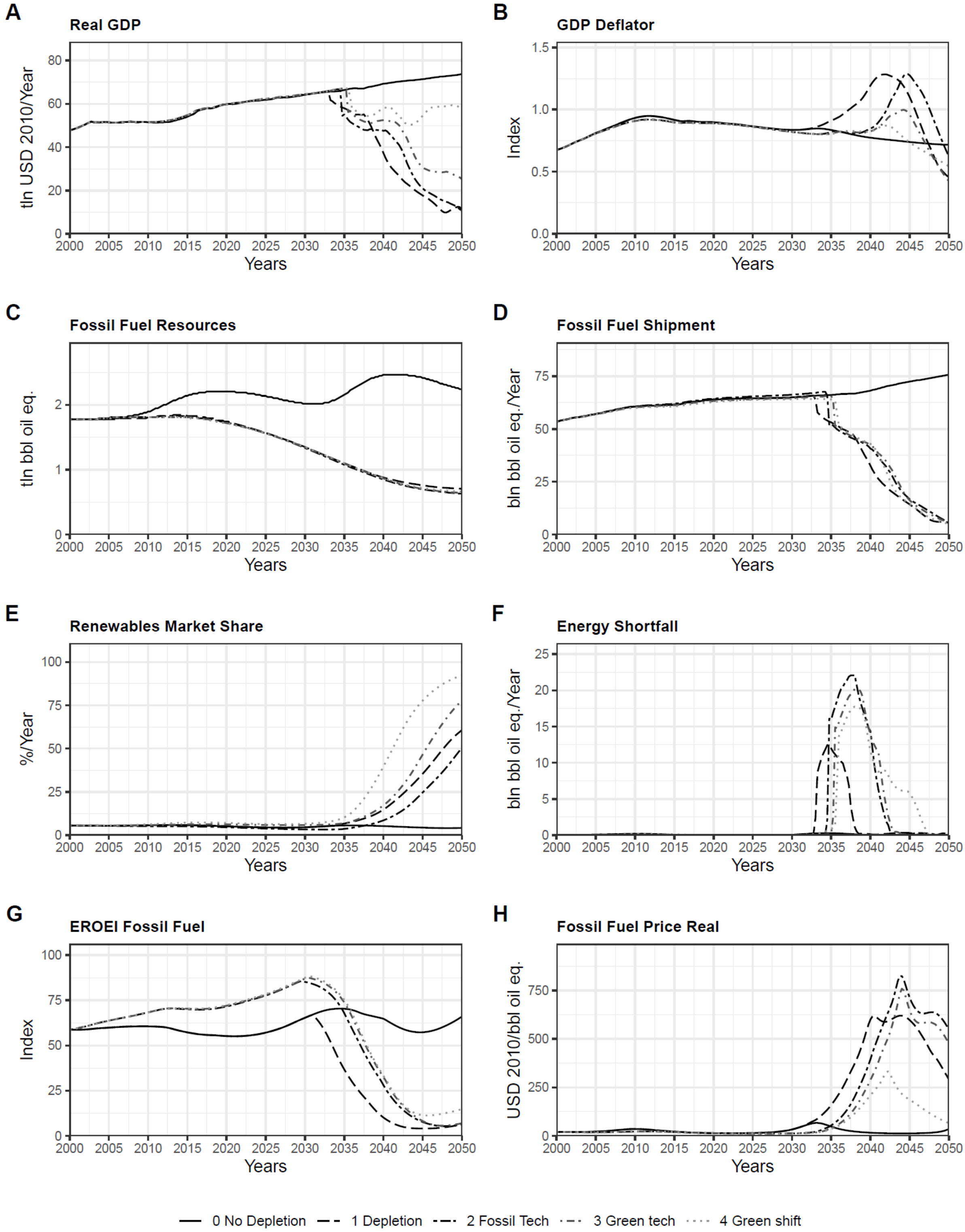
Figure 2.17 and Table 2.10 show the four scenarios considered each one building on top of the previous one. In particular, Case 0 assumes no resource limits, and Case 1 accounts for resource depletion with such a low amount of resources that it drives economic collapse. Case 2 shows the possibility of increasing productivity growth in the fossil fuel sector, and Case 3 provides a sensitivity on the possibility of increasing renewables productivity growth. Last, Case 4 applies on top of all the other possibilities the assumption in which the renewable sector can find a low barrier to the energy transition as the fossil fuels do, thus assuming perfect substitutability between the two resources. In fact, the base run scenario of ERRE assumes that not only productivity and price can drive the change towards green growth, but rather technological characteristics and issues with applicability of green energy to today's economy would slow down green growth by a factor of four times in comparison to fossil fuels (see bottom right graph in Figure 2.17). Thus the final scenario places the elasticity of substitution between the two at the same level. All scenarios run under the hypothesis of population growth.

Case 0 shows the behaviour of the model in the absence of depletion while affected by population growth. As the Figure 2.18 shows, the economy grows supporting an increasing population, inflation is relatively stable, proven reserves and fossil fuel shipments keep rising while affected by inherent business cycles, no energy shortfall is present, the market share between renewables and fossil fuels remain constant at approximately 95% to 5% over the time of the simulation, EROEI is relatively stable, and fuel prices remain low despite small rises every 25 years due to business cycles.

As soon as the depletion scenario is applied (Case 1), we see important dynamics emerging form the ERRE model. Due to the low amount of initial resources, proven reserves start decreasing below the level necessary to supply energy to the economy earlier than when green energy would be mature enough to support a smooth energy transition. This determines a sudden stop in fossil fuel shipments around the year 2035 with drastic reduction in Real GDP. Fossil Fuel EROEI drops quickly to very low levels such that no more proven reserves are viable for extraction. Energy price skyrockets generating hyperinflation. Energy orders move towards green energy that, without technology improvement and low flexibility in becoming a viable solution for the energy system, leads the entire economy to collapse

Figure 2.18 shows the impact of these sensitivities on selected variables in the model.

Figure 2.18 – Impact of resource depletion and technology on key variables



Case 2 (Fossil Fuel productivity growth at +3%) does not show particular improvements apart from seeing the EROEI increasing until the peak of production is reached. Higher productivity

growth supports lower prices and slightly delays the economic collapse. However, without discovering additional resources, improvement in productivity in energy resources generate an even sharper collapse of the economy. Case 3 (Renewable productivity growth at +5%) provides greater benefits to the economy, slowing down collapse, keeping energy prices lower for longer time, and supporting a faster shift towards green energy. Unfortunately, this results in being not sufficient due to the too low level of resources in the ground, which leave not enough time for the green sector to develop sufficient capacity to manage the transition smoothly.

In Case 4, the sensitivity on the Renewable shift elasticity parameter are shown. In ERRE, fossil fuels and renewables are not perfectly substitutable. In the base run, instability in the fossil fuel market can support the expansion of the green sector, but this is not true the other way around. In other words, decreasing the cost of renewables to become lower than fossil fuels is not sufficient condition to support their application to the economy at the same level of productivity. Many sectors would find it difficult to switch to green energy (e.g. aviation). Thus this scenario assumes the friction of the renewable sector is as low as the one of fossil fuels, such that the economy can choose between the two interchangeably. The result is a much better scenario, where the economy suffers but does not collapse, inflation and energy prices do not rise to unsustainable levels, and energy orders supports the transition towards renewable much faster than in the previous case.

These tests show the viability for the energy sector to adapt to stresses due to energy depletion and technology growth. Despite these parameters being hard to measure, sensitivities can be applied based on user requirements to test every scenario within these ranges of possibilities.

Agricultural land depletion

In the ERRE model, the agricultural land structure has been adapted starting from the World3-03 model, thus accounting for (i) impact of land yield on land erosion rate, (ii) impact of available land to convert to agricultural purposes on cost of land development. Whereas the first curve was based on the World3-03 calibration as in Pasqualino et al. (2015), the application of the cost curve based on depletion required particular attention for the sensitivity analysis.

In the World3-03, the cost curve indicates the increased marginal cost of agricultural land development, as measured in industrial output units, relatively to a simulation starting in the year 1900 and ending in the year 2100. This resulted in a very low level cost for agricultural land due to the large availability of land all over the globe at the beginning of the twentieth century, with a steep rise in cost when land availability was below 10% of total (see Figure 2.19 low curve). In the ERRE model, the simulation starts in the year 2000, indicating a different position of the steepness of that cost curve (see high cost in the Figure 2.19). This requires understanding the behaviour of the model while applying the two curves with possible implication for the calibration of the ERRE model.

In this section five scenarios are tested, where in Case 1 it is assumed the cost curve as in the World3-03, and in Case 2 the cost curve as applied for the scenario analysis of ERRE. Case 3 and 4 demonstrate the impact of exogenous increases in agricultural land and agricultural workers productivity on land erosion. In the Case 5, it is shown how the combined effects of both productivity curves while testing the hypothesis of no recovery of agricultural land lost due to erosion processes. It is worth noting that the technology curves used in the case of the agricultural sector follow a decreasing marginal productivity shape (rather than exponential growth) over time. This implies that the increased productivity would be reached with the application of more fertilizer rather than new methods of expanding production. Figure 2.19, Figure 2.20 and Table 2.11 show the sensitivity inputs to generate those scenarios.

Figure 2.19 – Test 8 – Sensitivity of agricultural land cost curve

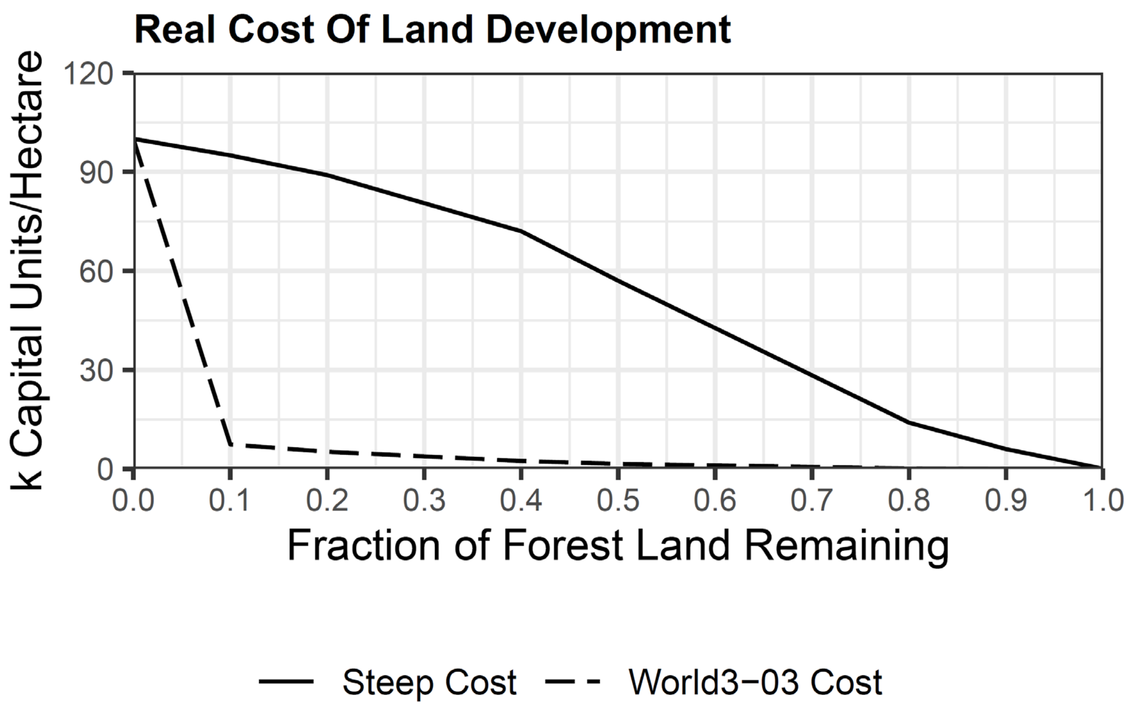


Figure 2.20 – Test 8 - Sensitivity inputs on technology growth, land erosion and forest land depletion

Figure 2.21 shows the sensitivity of agricultural depletion tests on selected variables. The six variables presented above show important impacts of the cost curves and technology

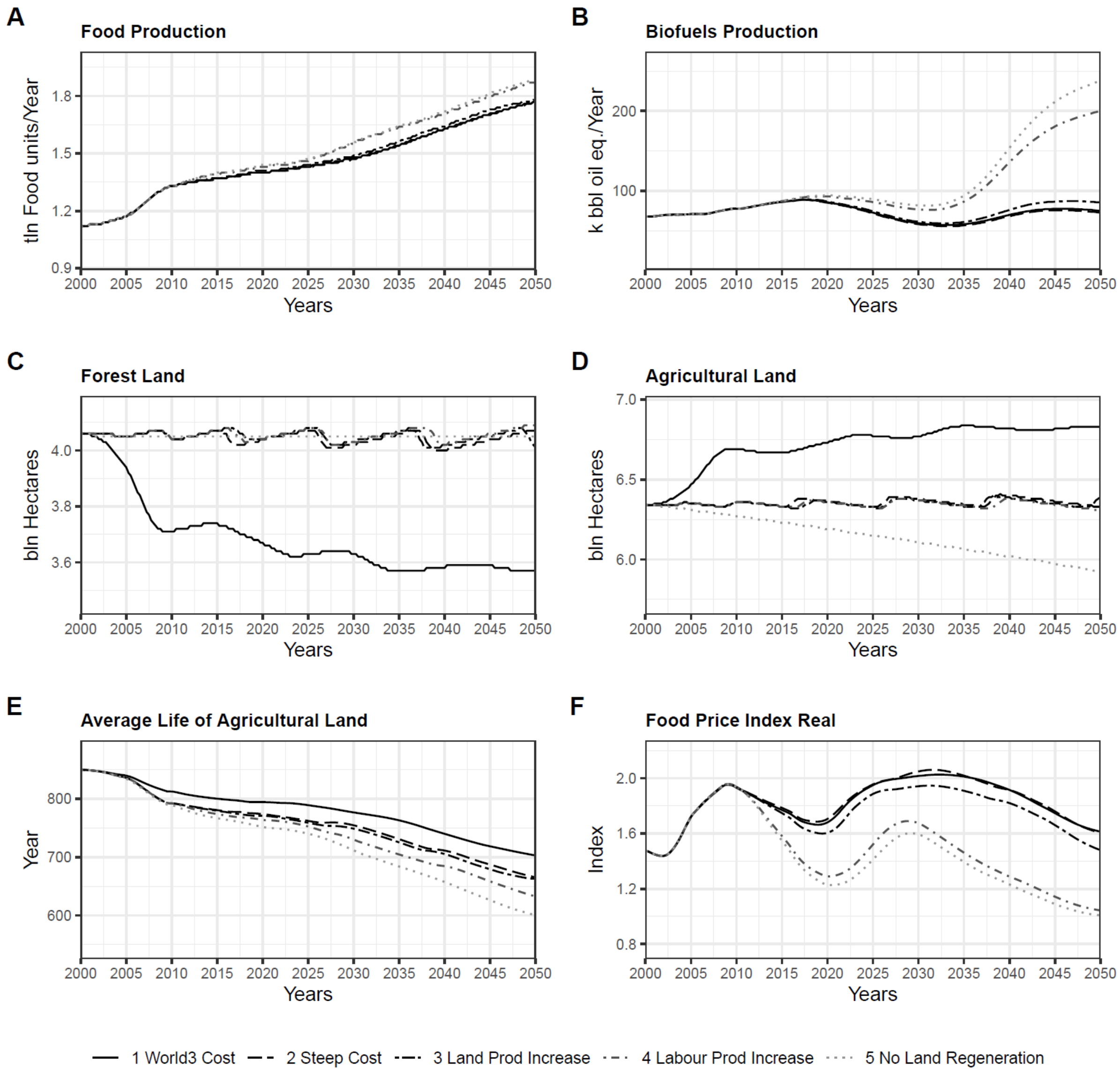
scenarios in the behaviour of the agricultural sector, with important implications for the model calibration and base run formulation. As the Figure 2.21 shows technology growth has direct consequences for all variables. The productivity increase, both for land and labour, supports lower prices and greater production both for food and biofuel output. The sensitivity of the biofuel sector is much higher than for the food sector due to absolute size differences between the two, and the different markets in which they operate. In fact, while food output is directed to the households only, biofuels competes as a substitute for both renewables and fossil fuels, despite being the smaller among the three.

Table 2.11 – Test 8 - Sensitivity inputs on technology growth, land erosion and forest land depletion

Test 8 – Agricultural land depletion	**Parameter**	**Value**
Case 1 – World3-03 cost depletion curve	Population growth case	UN Population division global projection
	Cost curve	World3-03
	Agricultural land marginally decreasing productivity index	+0%
	Labour Linear Productivity Index	+0%
	Regeneration of Eroded land	ON
Case 2 – Steeper cost depletion curve	Population growth case	UN Population division global projection
	Cost curve	Steeper increase curve
	Agricultural land marginally decreasing productivity index	+0%
	Labour Linear Productivity Index	+0%
	Regeneration of Eroded land	ON
Case 3 – +3% Marginally decreasing Agricultural land productivity index	Population growth case	UN Population division global projection
	Cost curve	Steeper increase curve
	Agricultural land marginally decreasing productivity index	+3%
	Labour Linear Productivity Index	+0%
	Regeneration of Eroded land	ON
Case 4 – +3% linearly increasing Agricultural labour productivity index	Population growth case	UN Population division global projection
	Cost curve	Steeper increase curve
	Agricultural land marginally decreasing productivity index	+0%
	Labour Linear Productivity Index	+3%
	Regeneration of Eroded land	ON
Case 5 – Assume no regeneration of Fallow land	Population growth case	UN Population division global projection
	Cost curve	Steeper increase curve
	Agricultural land marginally decreasing productivity index	+3%
	Labour Linear Productivity Index	+3%
	Regeneration of Eroded land	OFF

Significant effects are registered in relation to (i) the cost curve and (ii) neglecting the hypothesis of recovery of fallow land after erosion occurs. In the first case, we see that agricultural land rises quickly at the beginning of the simulation to decrease its growth rate around the year 2010. Such land is taken from forest land. This behaviour is due to the initial low cost of agricultural land based on the World3-03 curve which supports farmers to expand agricultural land to achieve production among the other production factors. When looking at the FAO data (2018), both agricultural and forest land show relative stability or slight decrease in their areas, thus demonstrating how this first scenario cannot be considered realistic based on today's state of the world systems.

Figure 2.21 – Impact of land erosion and technology growth on selected variables



The application of the steeper cost curve (Case 2) improves the behaviour of agricultural land abruptly. Given the high marginal cost increase, farmers will find it inconvenient to expand on land, aiming at keeping it constant while expanding the other productive factors (fertilizers, equipment, labour). The cases 2, 3, and 4 present important oscillations due to business cycles for both forest and agricultural land. As far as regeneration is assumed, bringing eroded land back to forest land with a certain time delay, there is a cyclical return of forest land which decreases the cost of developing new land. As a result of the regeneration process, farmers would cyclically find it more convenient to expand newly regenerated land out of the other productive factors, thus generating those oscillations.

When regeneration of land is neglected, agricultural land and forest land stop oscillating. Agricultural land slowly decreases while forest land remains flat and stable. Farmers employ better technology to produce the required amount of food, and stop expanding on their land due to the high cost of developing new land. This scenario supports the view of the agricultural sector of the World3-03, where no regeneration was considered and accounting for land erosion as one of the limits to growth in the model. While data seems to confirm the behaviour of reduction in agricultural land due to erosion, they show a fallacy in this structure given that forest land remains constant rather than decreasing (FAO 2018). Given the purpose of the ERRE model, such a structure is sufficiently detailed. However, further work can be applied to address new elements of the agricultural sector, such as the hypothesis of forest land being lost without any use in agriculture, or the hypothesis of agricultural land being voluntarily left as fallow land by farmers due to other reasons rather than erosion rate and urban land development.

Climate Change

In this section five sensitivities of the climate effect in the ERRE are compared. The first scenario involves no climate feedback to the economy thus ignoring such a structure for the general dynamics of the system. The second scenario assumes a negative impact of temperature rise on food production, the more the temperature increases. Such a scenario is tested on three additional levels of hot house effect hypothesis:

1. Ocean carbon sink reaches maximum capacity and stops absorbing carbon from the atmosphere, thus supporting higher concentration of carbon, temperature anomaly, and impact
2. Ocean carbon sink stops absorbing and becomes an emitter of carbon to the atmosphere
3. Both previous scenarios, and assuming that overall carbon sink capacity in the ocean reduces non-linearly due to temperature rise while melting of glaciers.

Figure 2.22 and Table 2.12 show the application of impact curves as sensitivity inputs to the model.

Figure 2.22 – Test 9 – Sensitivity of impact curves from climate to food sector

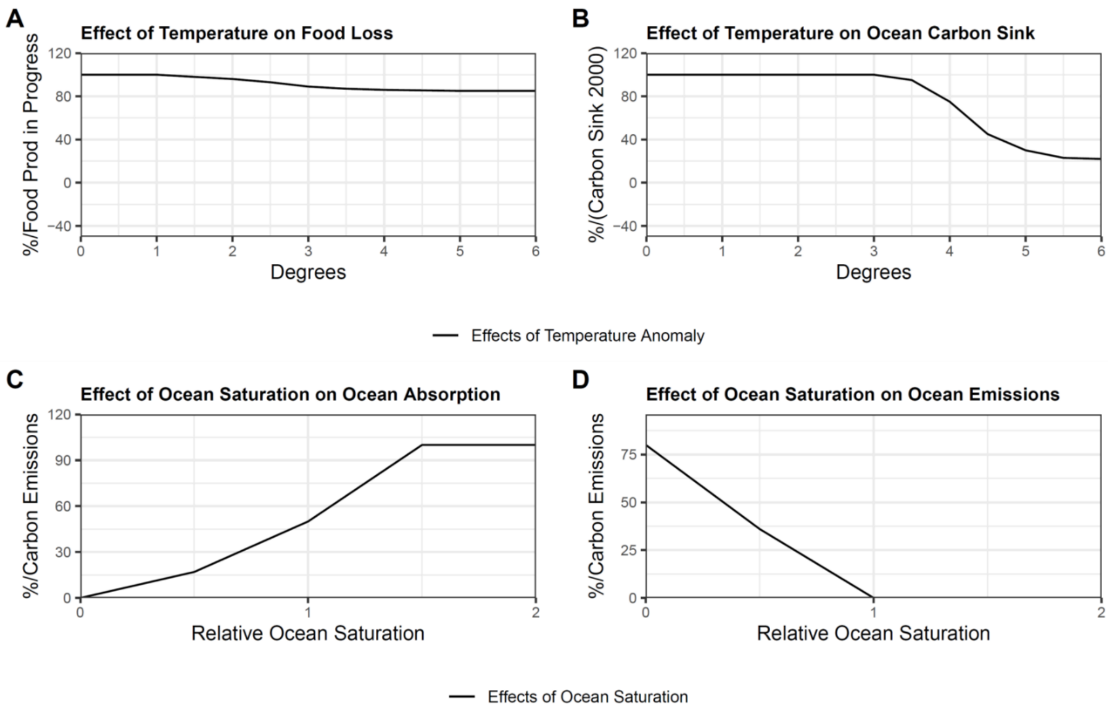
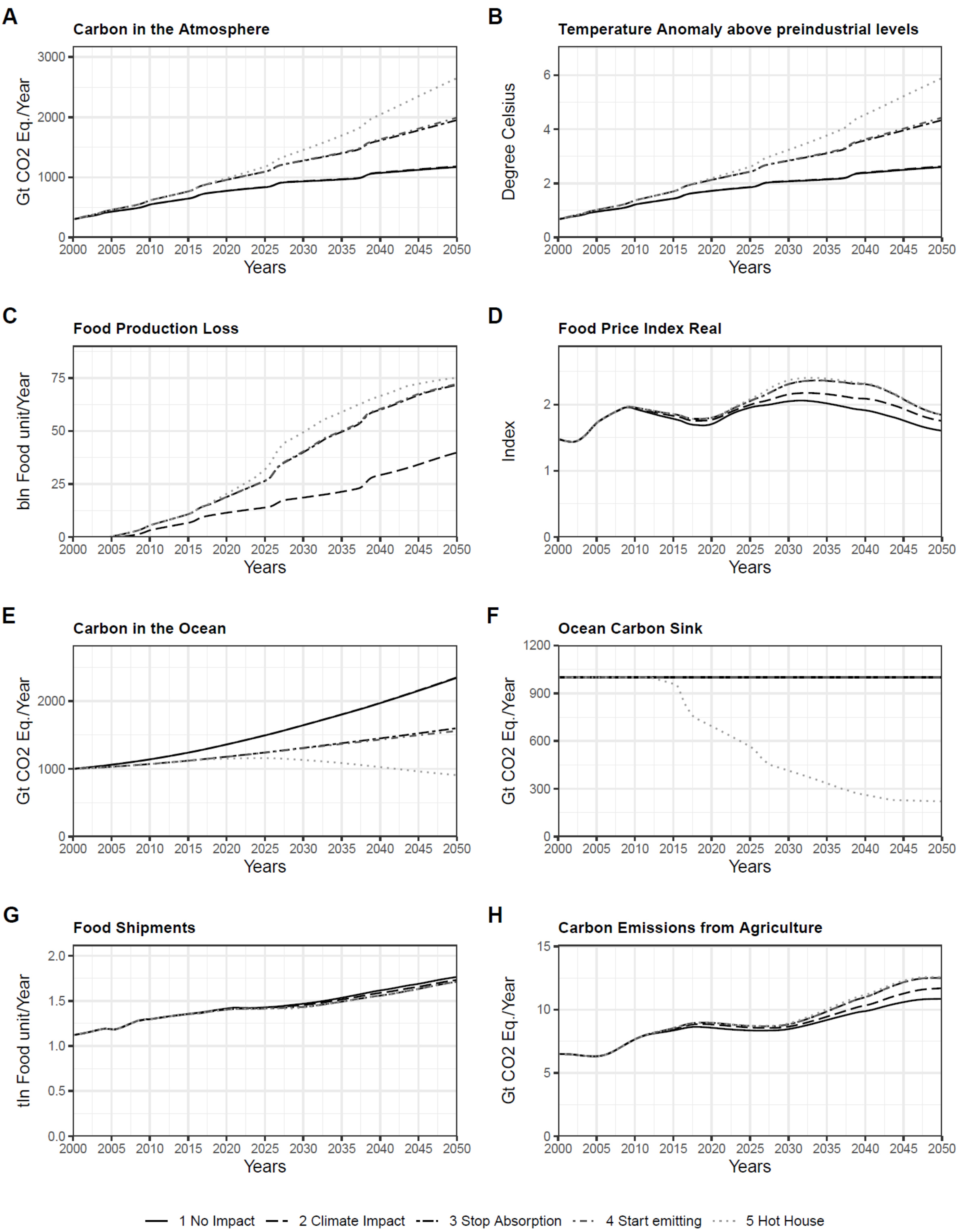


Table 2.12 – Test 9 – Sensitivity of impact curves from climate to food sector

Test 9 – Climate impact on Food loss	**Parameter**	**Value**
Case 0 – Climate impact	Impact of climate on food curve	OFF
	Impact of ocean saturation on absorption curve	OFF
	Impact of ocean saturation on ocean emissions curve	OFF
	Impact of temperature anomaly of reduction in ocean carbon sink	OFF
Case 1 – Climate Impact	Impact of climate on food curve	ON
	Impact of ocean saturation on absorption curve	OFF
	Impact of ocean saturation on ocean emissions curve	OFF
	Impact of temperature anomaly of reduction in ocean carbon sink	OFF
Case 2 – Stop Absorption	Impact of climate on food curve	ON
	Impact of ocean saturation on absorption curve	ON
	Impact of ocean saturation on ocean emissions curve	OFF
	Impact of temperature anomaly of reduction in ocean carbon sink	OFF
Case 4 – Start Emitting	Impact of climate on food curve	ON
	Impact of ocean saturation on absorption curve	ON
	Impact of ocean saturation on ocean emissions curve	ON
	Impact of temperature anomaly of reduction in ocean carbon sink	OFF
Case 5 – Hot House	Impact of climate on food curve	ON
	Impact of ocean saturation on absorption curve	ON
	Impact of ocean saturation on ocean emissions curve	ON
	Impact of temperature anomaly of reduction in ocean carbon sink	ON

Figure 2.23 shows the sensitivity of selected variable in the model to temperature anomaly effect.

Figure 2.23 – Test 9 - Sensitivity of climate effect on selected variables



As Figure 2.23 shows, the five scenarios considered gradually increase the impact of climate change on food production, with important amplification due to feedback processes in the food system. The first case assumes no impact of climate on agriculture, and that ocean behaves as an infinite carbon sink (maximum increase in carbon in the ocean scenario). Given that temperature anomaly and carbon in the atmosphere are modelled as interrelated, their profile is the same. Food price is the lowest, thus supporting greater shipments. This is achieved with lower agricultural inputs thus maintaining the lowest carbon emissions from agriculture among the five scenarios.

In the second case, not much difference is seen in carbon in the atmosphere and temperature anomaly. However, the food loss generates higher pressure on agriculture, raising prices, lowering shipments, and increasing carbon emission from agriculture due to the higher capital intensity. Case 3 and 4 (ocean stopping absorption and starting emitting) have a similar profile. In these scenarios, ocean carbon remains lower than in the previous cases pushing carbon in the atmosphere to higher levels. The impact of food is even higher, pushing prices up and increasing carbon from the agriculture itself. The case 5 (Hot house) assumes that the temperature rise can generate a drastic reduction in the ocean carbon sink, thus pushing the carbon accumulated over millennia back to the atmosphere. This last scenario provides the highest level of carbon in the atmosphere, the temperature anomaly reaching +6 degrees Celsius on preindustrial levels, highest impact on the food system, which, while trying to balance the inefficiency due to food loss, will increase capital thus generating even more carbon.

Despite the high uncertainty in modelling these impact curves in the ERRE, this test demonstrates the validity of the behaviour of the ERRE model in this domain. In particular, the shape of each curve should be assessed with care. It is worth noting that in the ERRE model, climate impact is registered for the agricultural sector only, while it will impact other sectors of the economy as well. More sophisticated results are presented in the scenario analysis following in this section.

OTHER ENDOGENOUS DISEQUILIBRIUM (FINANCIAL DECISIONS)

The elements described above represent the key endogenous disequilibrium used in the ERRE model calibration (as follows). In addition to these, there are many more options for generating disequilibrium in the system. In particular, every non-linear relationship indicating a decision point which describes the impact of fluctuations in the system has the tendency to balance out those elements towards the equilibrium point (1,1). This indicates that the multiplier effect on all financial decisions would be equal to 1 (no effect) when cash available would be exactly as much as desired for making a payment (see adequacy of liquidity, or adequacy of savings as an example). It would be sufficient to slightly change that multiplier away from the (1,1) equilibrium point to generate dynamic disequilibrium in the system (for example to (1,0.9) to assume conservative behaviour decision makers, or (1, 1.1) to assume their tendency towards disequilibrium and growth). Due to the scope of this work, such assumptions have not been considered leaving space for future work in this direction.

OTHER BEHAVIOURAL TESTS

In Forrester and Senge (1980) there are more tests necessary to address the behavioural validity of a system dynamics model. In the section presented above some of those have been covered, and others have been neglected. In particular, the tests shown in the previous section demonstrate problematic symptoms can be generated (interaction between growth and limits), but considerations on periodicity of cycles and multiple mode tests have been adopted only partially. The ERRE model can be used to generate cycles, but such behavioural tests remain of higher interest in lower scale systems such as a firm sector or the interaction between supply and demand within a nation. As the scale of the ERRE model is global, these tests are not useful for addressing the validity of the ERRE model. For example, and as seen in the calibration section, the model is capable of generating the characteristic behaviours of cycles within the food and energy system, but such behaviours have been tested only partially for the scope of ERRE.

In the model, family behaviour tests have been largely adopted, due to the application of similar structures of different firms based on a similar generic structure underpinning them all. This shows how the same firm structure is capable of addressing different dynamic behaviours thus capturing the specific requirements of capital or energy producers among others, by varying their parameters.

Other tests such anomaly tests and surprise behaviour tests have been performed many times during the model development and are shown during sensitivity of effects such as resource depletion or climate impact on agriculture in the following sections. Additional surprise behaviours tests could be implemented when looking at lower scale systems, thus allowing a direct comparison between model output and management experience within organizations.

In addition, many modellers (Sterman 2000, Radzicki 2011, Morecroft 1988) have opted for partial model testing when models become large, in order to isolate behaviours of interest and address model capabilities within the wider framework. For the scope of ERRE the model was tested as a fully aggregated system model, due to the size of the model and the scope of the analysis. However, such a test would surely provide value for future work emerging out of the ERRE structure.

Stock and Flow and Technical consistency

The third group of tests listed in this section, can be seen as a mix between what Forrester and Senge (1980) referred to as behavioural, extreme scenarios, and structural tests, most useful to address the technical and mathematical consistency of the model. Among those we find what Sterman (2000) considers the Integration (or dt) test. In addition two ad-hoc tests aimed at demonstrating the dynamic stock and flow consistency of the model and the architectural precision of the model, while dealing with the scalability of the input variables are provided.

These are listed as:

1. Physical Stock and Flow consistency
2. Economic stock and flow consistency
3. Integration test
4. Scalability test

PHYSICAL STOCK AND FLOW CONSISTENCY

The physical stock and flow consistency test requires that under whatever conditions, the model results are robust and assures that every stock and flow variable with a real world counterpart do not reach negative values, and meaningful behaviour depending on how unrealistic the model inputs can be. In particular, all the stocks accounted in the ERRE model represent quantities and accumulations that can be only positive in the real world. The model has been tested for robustness and is considered to be consistent with reality for what concerns physical stock and flow consistency of its elements.

ECONOMIC STOCK AND FLOW CONSISTENCY

The economic stock and flow consistency of the model is a condition that is dependent on the architecture and structure of all the cash flows and balance sheets in every sector of the model. The check of stock and flow consistency is an important test that needs to be done in the ERRE model to reveal inconsistencies, and, if any is to be found, to remove those.

Thus, additional variables to compare all assets and liabilities of each single sector, to demonstrate they are always equal one to another, have been added. In addition, it is expected that the cash in every sector must flow while accumulating as Liquidity or physical assets, or rather spent. As a result additional variables have been used to check that all Cash stocks remain at zero for the entire duration of the simulation, thus resulting in cash in and cash out being equal one to another at all time. The only single sector that does not reproduce this latter condition is the banking sector, simply because the decision rule determining their accumulation of cash in their vaults must accumulate proportionally to Loans and Securities as consistent with the Basel III regulatory approach. While the model grows, loans grow as well, requiring a certain gap between the amount of liquidity received and the amount of liquidity distributed out of banks, resulting in liquidity accumulation over time for banks.

The conclusive demonstration of the stock and flow consistency must be done at the level of the entire economy. Because banks are not assumed to own any assets (e.g. capital or land), their liabilities must equal to the sum of Cash available in the entire economy. While Savings deposit within banks are matched by the savings from the household sector, the stock of Debt Money has to match the total Money Supply in the model, that corresponds to the sum of liquidity from every other sector in the economy.

In form of equation:

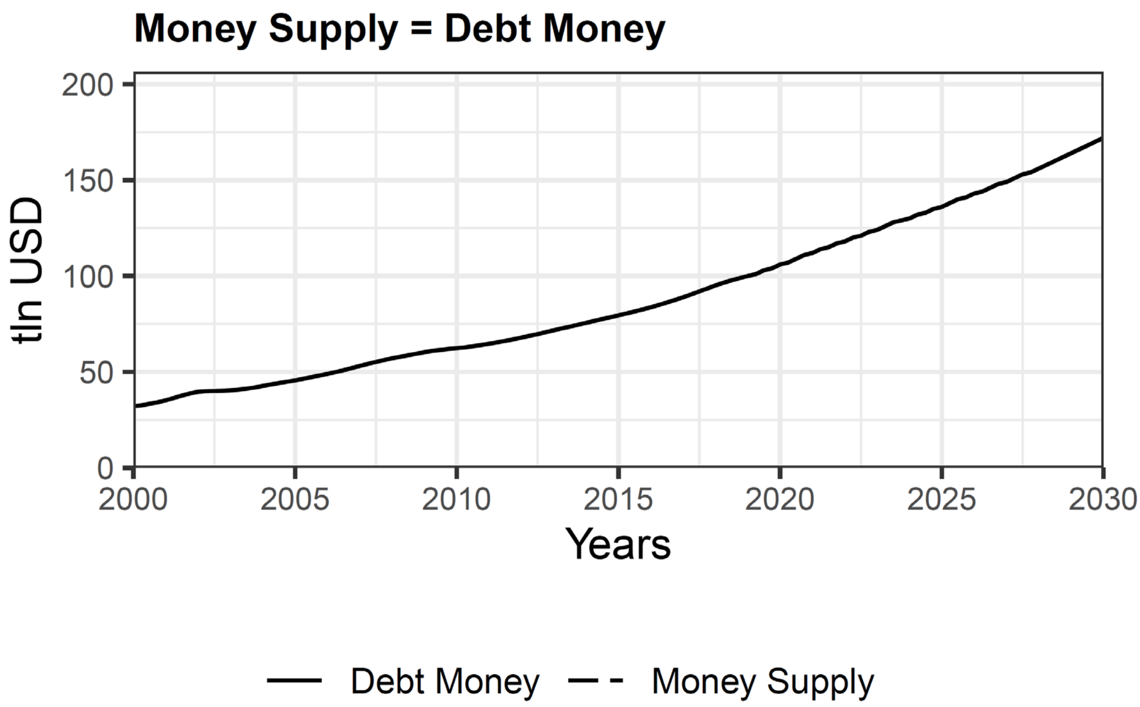
Eq. 2.1 – Stock and Flow consistency equivalence in ERRE

$$MS(t) = \sum_{\Pi+H+B} \Lambda_i + Res(t)_B \equiv DM_B(t)$$

Where $MS(t)$ is the money supply of the entire economy as described in Figure 1.2, $\sum_{\Pi+H+B} \Lambda_i$ is the sum of liquidity stocks in each sector of the economy, $Res(t)_B$ the reserves in banks, and $DM_B(t)$ is the debt money stock as liability of the banking sector, and foundation for the creation and distribution of money in the economy (see the balance sheet of the banking sector in Figure 1.2.

Figure 2.24 demonstrates the stock and flows consistency in the ERRE model, and the dynamic equivalence between debt money $DM_B(t)$ and money supply $MS(t)$ in the base run of the model.

Figure 2.24 – Stock and flow consistency in the simulation



INTEGRATION TEST

The integration (or dt or time step) test consists in the assurance of mathematical consistency in the model in relation to continuous variables and correct accounting of time delays. Forrester (1961) suggested that every simulation should consider the smallest time step possible. Experience has shown that such a solution often increases the computational cost of simulation (slower time to simulate and increased size of files produced) giving no value to the focus of the analysis. In fact the general rule adopted by system dynamicists, is to use a time step in the simulation that is at least three times smaller than every single time delay used in the model. In fact, every smaller time step would result in the same numerical result despite increasing the computational effort.

The time step in continuous time modelling determines the size of the step the integrator will need to make to provide a numerical value consistent with the mathematical formulation of the model. Nowadays, software adopts different and more sophisticated numerical integration methods at the same time step at the cost of computational power. These include the Euler method (lower computation) and different orders of numerical integration based on the Runge-

Kutta method (higher computation the higher the order of integration). This latter is often considered beneficial for the high level precision of modelling machine systems, and generally rejected for system dynamics models that are meant to be focused on the social science and capture of general dynamics within systems.

As a result, the correct choice for the time step and integration method for the ERRE model consisted in the TIME STEP = 0.03125 years (approximately 11 days) and Euler method as integration method. The integration (or dt) test consisted of the comparison between the simulation of the ERRE model using these latter characteristics, and the same simulation adopting smaller TIME STEP and more sophisticated numerical integration method. The result is that both simulations should be numerically equivalent. The test is very important because it highlights the presence of mathematical inconsistencies often generated by the adoption of non-continuous non-linear relationships or exogenous discrete events that can generate problems with the integration calculation. The ERRE model passes this test.

SCALABILITY TEST

The scalability test consists in the attempt to demonstrate the mathematical consistency of the model while creating a new parameter (SCALE) that would allow to scale up, or down, all real data directly input to the simulation in absolute terms, without altering the dynamics of the system. Such a test demonstrates the ERRE model as a starting point for a family of models of different sizes. In fact, the ERRE model, despite being global in scale, was drawn from a national modelling effort. As such, the scalability test demonstrates how the model could be used as a starting point to calibrate on data at the national level supporting the concept of regionalization of the current model structure. The model passes the test, such that when the parameter SCALE is increased or reduced, the size of all absolute variables (e.g. population, GDP, energy production) in the model reduces proportionally without altering the relative variables in the model (such as prices. wages or interest rates).

References

Arrow, K. J., Chenery, H. B., Minhas, B. S., & Solow, R. M. (1961). Capital-labor substitution and economic efficiency. The review of Economics and Statistics, 43(3), 225-250.

Barlas, Y. (1989). Multiple tests for validation of system dynamics type of simulation models. European journal of operational research, 42(1), 59-87.

Barlas, Y. (1996). Formal aspects of model validity and validation in system dynamics. System Dynamics Review: The Journal of the System Dynamics Society, 12(3), 183-210.

Barlas, Y., & Carpenter, S. (1990). Philosophical roots of model validation: two paradigms. System Dynamics Review, 6(2), 148-166.

Cincotti, S., Raberto, M., & Teglio, A. (2012). The Eurace macroeconomic model and simulator. Paper presented at the Agent-based Dynamics, Norms, and Corporate Governance. The proceedings of the 16-th World Congress of the International Economic Association, Palgrave.

Forrester, J. W. (2003). Dynamic models of economic systems and industrial organizations. System Dynamics Review: The Journal of the System Dynamics Society, 19(4), 329-345. Recovered from 1956.

Forrester, J. W. (1961). Industrial dynamics. 1961. Pegasus Communications, Waltham, MA.

Forrester, J. W., & Senge, P. M. (1980). Tests for building confidence in system dynamics models. System dynamics, TIMS studies in management sciences, 14, 209-228.

IPCC (2014). IPCC fifth assessment report: climate change 2014: synthesis report. Contribution of Working Groups I. II and III to the Fifth Assessment Report of the intergovernmental panel on Climate Change. IPCC, Geneva, Switzerland, 151.

Meadows, D. H., Meadows, D. L., & Randers, J. (1992). Beyond the limits: global collapse or a sustainable future: Earthscan Publications Ltd.

Meadows, D. H., Meadows, D. L., & Randers, J. (2003). The limits to growth: the 30-year update: Routledge.

Meadows, D. H., Meadows, D. L., Randers, J., & Behrens, W. W. (1972). The limits to growth. New York, 102, 27.

Meadows, D. L., Behrens, W. W., Meadows, D. H., Naill, R. F., Randers, J., & Zahn, E. (1974). Dynamics of growth in a finite world: Wright-Allen Press Cambridge, MA.

Morecroft, J. D. (1988). System dynamics and microworlds for policymakers. European journal of operational research, 35(3), 301-320.

Pasqualino, R., Jones, A., Monasterolo, I., & Phillips, A. (2015). Understanding global systems Today—A calibration of the World3-03 model between 1995 and 2012. Sustainability, 7(8), 9864-9889.

Radzicki, M. J. (2011). System dynamics and its contribution to economics and economic modeling. Complex Systems in Finance and Econometrics, 727-737.

Sato, K. (1967). A two-level constant-elasticity-of-substitution production function. The Review of Economic Studies, 34(2), 201-218.

Shaikh, A. (1974). Laws of production and laws of algebra: the Humbug production function. The review of Economics and Statistics, 115-120.

Steffen, W., Rockström, J., Richardson, K., Lenton, T. M., Folke, C., Liverman, D., . . . Crucifix, M. (2018). Trajectories of the Earth System in the Anthropocene. Proceedings of the National Academy of Sciences, 115(33), 8252-8259.

Sterman, J. (1981). The energy transition and the economy: A system dynamics approach (2 vols.): MIT Alfred P. Sloan School of Management, Cambridge, MA.

Sterman, J. (2000). Business dynamics: systems thinking and modeling for a complex world: Jeffrey J. Shelstad, Indianapolis, IN.

Turner, G. M. (2008). A comparison of The Limits to Growth with 30 years of reality. Global environmental change, 18(3), 397-411.

Turner, G. M. (2012). On the cusp of global collapse? Updated comparison of The Limits to Growth with historical data. GAIA-Ecological Perspectives for Science and Society, 21(2), 116-124.

Turner, G. M. (2013). The limits to growth model is more than a mathematical exercise. Gaia, 22(1), 18.

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